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A Comparison of Problem-Centered Learning Model and Guided-Practice Model on High-School Students' Mathematics Performance and Attitude

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A COMPARISON OF PROBLEM-CENTERED LEARNING MODEL AND GUIDED-PRACTICE MODEL ON HIGH-SCHOOL STUDENTS' MATHEMATICS PERFORMANCE AND ATTITUDE

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Chapter I

Introduction

Statement of the Problem

What mathematics skills should be most central in the high-school curricula? What type of mathematics curricula is needed to ensure successful learning of these most central skills? Reports from the National Council of Teachers of Mathematics (NCTM, 1980, 1989) indicated that problem solving should be the focus of school mathematics and declared the 1980s as the "decade of problem solving" (NCTM, 1980). Lester (1994) wrote that " problem solving has come to be regarded as a fundamentally important aspect of mathematics education" (p. 662). More important, Lester argued that most mathematics educators agree that the development of students' problem-solving abilities is a primary objective of instruction.

Research studies, however, indicate that high-school students have not developed appropriate problem-solving skills. For example, a published report of the National Assessment of Educational Progress (Dossey, Mullis, & Jones, 1993) indicated that on extended constructed-response tasks, which required students to solve problems and then explain their solutions, the average percentage of students producing satisfactory or better responses was 16 % at grades 4 and 8 and 9% at grade 12. Lester wrote, " the situation in the American schools with respect to student performance in mathematical problem solving is desperate. Although the conference reports, curriculum guides, and textbooks insist that problem solving has become central to instruction at every level, the evidence suggests otherwise" (p. 660). Lester argued that no mathematics program has been developed that adequately addresses the issue of making problem solving the central focus of the curriculum. Indeed, there seem to be few examples of effective problem-solving curricula.

Purpose of the Study

The purpose of this study was to compare and correlate the mathematical problem-solving skills and attitudes of high-school students using two different teaching approaches: (a) problem-centered learning approach and (b) teacher- guided approach. This study predicted that using problem-centered learning approach would not only produce better problem-solving students but also students with better attitudes toward mathematics. In this study, problem-solving skills were defined by Polya's four-phase problem-solving model. Polya's model includes four steps: (a) understanding the problem, (b) devising a plan, (c) carrying out the plan, and (d) looking back at the completed solution (Polya, 1957).

Problem-centered learning has three essential components: assigning tasks, working in small groups, and sharing results. In this model, the teacher provides students with a problem. Students, working in small groups, find an answer to this problem. Then the teacher provides some sharing time at which students present their answers to the whole class (Wheatley, 1989).

For the purpose of this study, the teacher-guided approach was defined as lecturing for most of the class time then giving students drill practice exercises to practice what has been lectured. This teacher-guided approach is what is practiced in most classrooms and served as a control in this study. In this study, the instructional strategy will be an independent variable examined at two levels: problem-based learning approach and teacher-guided approach. The two dependent variables that were investigated are as follows: a) mathematics performance as measured by a problem-solving test and b) attitude toward mathematics as measured by a questionnaire.

Definitions of Key Terms

Attitude Toward Mathematics: In this study, the term "Attitude toward mathematics" was defined as a mental position or feeling with regard to a mathematics as a field of study. A Likert scale was used to measure attitude toward mathematics.

Problem Solving: Problem solving refers to the process of moving from a starting point of information toward some goal (Mayer, 1985). In this study, the term problem solving was defined by Polya's four-phase problem solving that includes understanding the problem, devising a plan, carrying out the plan, and looking back (Polya, 1957). Students are expected to communicate their solution strategies to mathematics problems by writing down their explanations on paper using the above four steps.

Mathematics Achievement or Performance: In this study, mathematics achievement was defined as the students' ability to become better problem solvers. Mathematics achievement was measured by a problem-solving test (Van Akkeren, 1995).

Teacher-Guided approach: In this study, teacher-guided approach was defined as lecturing for most of the class then giving students drill and practice exercises to practice what has been learned. The teacher-guided approach was utilized in a first-year Algebra class.

Problem-Learning Centered: In this study, problem-learning centered was defined as a teaching strategy that includes three components: assigning tasks, working in small groups, and sharing results. In this strategy, the teacher provides students with a problem then allows students to work in small groups to find a solution to this problem. Finally, the teacher provides some sharing time at which students present their solution to the whole class (Wheatley, 1989). The problem-centered approach was utilized in an IMP class.

Problem: In this study, the term "problem" was defined as a task that is difficult for the individual who is trying to solve it (Schoenfeld, 1985). According to Schoenfeld (1985), this difficulty should be an intellectual impasse rather than a computational one. Schoenfeld (1985) argued that, if the person has ready access to a solution schema for a mathematical task, the task is an exercise and not a problem.

Heuristics: In this study, heuristics were defined as strategies and techniques for making progress on unfamiliar problems (Scheonfeld, 1985). Examples of how students are expected to use heuristics in this study include drawing figures, making tables, finding patterns, or a combination of these approaches.

Background and Need

Different approaches to teaching mathematical problem solving have been identified in the literature. One purpose of this section is to discuss five of these approaches including (a) real-life situation curricula, (b) process-focused approach, (c) new mathematics approach, (d) Polya and heuristics approaches, and (e) problem-centered learning curricula. Looking at this research literature will provide not only basic understanding of different approaches to teaching mathematical problem solving but also highlight what is lacking in the previous research.

Real life-situations curricula. Research has indicated that mathematical problem-solving curricula has been adapted around life situations since the 1930s (Ornstein & Hunkins,1993). As the name indicates, the emphasis of this teaching approach is on life functions or life situations. In this approach, students learn the mathematics they need around the context of real-world problem solving (Hiebert et al., 1996). Thus, these curricula attempt to make mathematics useful.

Recent reform recommendations also place a heavier emphasis on meaningful applications and connecting mathematics to the real world (NCTM, 1989, 1991). In addition, many real-life problems are proposed as an appropriate context for learning and assessment (Burhardt, 1981; Cognition and Technology Group at Vanderbilt, 1990; Lesh & Lemon, 1992; Romberg, 1992).

This mathematical problem-solving approach was built upon three major assumptions as described by Ornstein and Hunkins (1993). First, persistent life situations such as (a) protecting life and health, (b) getting a living, and (c) improving material conditions are essentials to society successful functioning, thus it is important to organize the curriculum around these situations. Second, the curriculum will be meaningful to students if the content is organized around aspects of community life. Third, the study of social or life situations will not only help students study ways to improve society but also directly will be involved in such improvement. There are three major strengths of this instructional design. First, its focus is on the problem-solving procedures for learning. Both content and process are integrated effectively into the curricular experience. Second, it utilizes the past and the current experiences of learners as a mean of getting them to analyze the basic areas of living. Third, it presents mathematics in an integrated form by cutting across the separate subjects areas and across related categories of social life.

But, like other teaching approaches, this approach has several deficiencies. First, determining the scope and sequence of the essential areas of living is difficult. The scope of the present time is different from the scope of the future. Second, this design does not expose adequately students to their cultural heritage. Third, this design is not appropriate for teachers, because they lack adequate preparation for it. More important, the textbooks and other teaching materials inhibit the teaching of this curriculum (Ornstein & Hunkins, 1993). Therefore, it is difficult to implement this approach unless there is an adequate training for teachers and students and a good mathematical problem-solving curricula that addresses mathematics topics and relates mathematics to real-life situations. An example of the mathematical real-life situations curriculum is the "anchored-instruction" approach implemented by members of the Technology Group at Vanderbilt. The goal of this curriculum is to help students develop the confidence, skills, and knowledge needed to solve problems and become independent thinkers and learners (Cognition and Technology Group at Vanderbilt, 1990).

Members of the Technology Group at Vanderbilt have been influenced by the concept of the *inert knowledge*. This concept is defined as the knowledge that can be recalled when people are asked explicitly to do so but is not used spontaneously in problem solving even though it is relevant (Whitehead, 1929). For instance, a teacher of educational psychology gave her students a long article and told them they had 10 minutes to learn as much as they could about it. Despite of the fact that these students had classes that taught them to skim for main ideas, consult section headings, and so forth, they were not able to recall what they have been learned. Instead, students in this class began with the first sentence of the article and started to read as fast as they could until their time was up. Later, when discussing their strategies, the students acknowledged that they knew better than to simply begin reading. but, they did not use spontaneously this problem-solving strategy they had been taught when it would have been useful (Bereiter, 1984).

To help students overcome the problem of *inert knowledge,* the Cognition and Technology Group at Vanderbilt (1990) has implemented the concept of *anchored instruction*. According to the Cognition and Technology Group at Vanderbilt (1990), the term *anchored instruction* is instructions that are situated in videodisc-based instructions that include problem-solving environments for both teachers and students to explore. Using the concept of anchored instruction, the Vanderbilt Group has created environments that enable both teachers and students to explore and understand the kinds of problems and opportunities that "experts" encounter and the knowledge that experts use as tools.

These adventures were designed for fifth and sixth graders. In these projects, students have to generate the problems to be solved and then have to find relevant mathematical information that was presented throughout the video.

The "Jasper Series" by the Vanderbilt Technology Group (1990) was designed to develop and evaluate a series of videodisc adventures whose primary focus is on mathematical problem formulation and solving. The main character of these adventures is Jasper. In these adventures, Jasper goes to Cedar Creek to look at an old cruiser that he is interested in buying. He sets out for Cedar Creek in his little motorboat. On the video, Jasper is shown consulting a map of the area, listening to his marine radio, and so forth. As the story continues, Jasper stops for gas at Larry's dock. He leaves Larry's after buying gas with his only cash-- a 20-dollar bill-- and sets out up river. He runs into a bit of trouble when he hits something in the water and breaks the shear pin of his propeller. Jasper goes to a repair shop where he pays to have his shear pin replaced. He finally reaches Cedar Creek boat dock where he locates Sal, the cruiser's owner. He and Sal test drive the cruiser and find out the boat's cruising speed. They return to the dock where they fill the cruiser's gas tank. Jasper decides to buy the cruiser, and he and Sal conclude the transaction.

Students are challenged to do three things including (a) identify Jasper's major goal, which is to get home before the sunset without running out of gas, (b) generate subproblems that represent obstacles to this goal such as running out of gas, and (c) devise strategies to deal with and solve these subproblems.

Results of this project indicate that fifth graders can become very good at complex problem formulation on tasks similar to Jasper after working in cooperative learning groups for 4 to 5 class sessions (Cognition and Technology Group at Vanderbilt, 1990). The facilitation of this transfer was enhanced by creating a series of 6 to 10 Jasper discs that provides a foundation for using key mathematics concepts in a variety of realistic settings and opportunities for training for transfer (Cognition and Technology Group at Vanderbilt, 1990). The students were given opportunities to discuss similarities and differences among problem situations that helped in facilitating the degree to which the transfer occurred (Brandsford, Stein, Delcos, & Littlefield, 1986). It also was found that teachers have been enthusiastic about Jasper, because their students seem to be challenged to solve the problem and because even students who normally are not good in mathematics can contribute to problem solving by noticing information in the video that is relevant for solving Jasper's problem.

The situated instruction has three major problems that make it an incomplete approach to teaching mathematical problem solving. First, it is built on the assumption that mathematical procedures and concepts should not be taught as isolated bits of information, and the instruction should be designed so that students build connections with prior knowledge. It is less clear what connections are most important or what kind of instruction is most effective for promoting these connections (Hiebert & Carpenter, 1992). Therefore, it is difficult for mathematics teachers who are implementing the real-life approach to design daily lessons that builds on students' previous knowledge and that is connected to real life.

Second, the research on anchored instruction does not suggest how knowledge is integrated into a fully developed network including concepts, procedures, and symbols of in-school mathematics (Hiebert & Carpenter, 1992).

Third, one of the dangers in attempting to build upon students' knowledge is that students' informal conceptions may be limiting. The problem situations that initially are most meaningful for students may not provide a sufficiently rich context in which to develop a full understanding of a given construct. For example, children as young as first graders can begin to develop an initial understanding of multiplication as a group of sets containing the same number of objects; by the third grade, these students relate multiplication to repeated addition (Kouba, 1989). This narrow conception of multiplication does not extend well to fractions and decimals and results in such misconceptions as "multiplication always makes bigger" (Bell et al., 1989; Fischbein et al., 1985). Thus if you limit all instructions to students' prior knowledge, students might find it difficult to build more complex mathematical concepts on that knowledge. For example, much research on the effect of prior knowledge on learning advanced mathematical topics like proportional reasoning and algebra has analyzed how prior knowledge from arithmetic leads to misconceptions when generalized to more advanced topics (Hart, 1988; Matz, 1980).

Process-focused approach: Expert-novices studies. Expert-novice studies generally provide a model of how knowledge might be connected once it is acquired (Chi, Feltovich, & Glaser, 1981). The goal of the expert-novice studies is to teach students to use the same kinds of strategies that experts use by focusing on the process of expert thinking rather than the products of experts.

Focusing on the process has its roots in the 1950s. The first major attempt to help remedial college students improve their problem-solving performance was carried out by Bloom and Broder at the University of Chicago in 1950. In order to help these students improve their problem-solving performance, Bloom and Broder focused on two major issues: "what to teach" and "how to teach." To determine what to teach, Bloom and Broder made a clear distinction between "Products of Problem Solving" and "Process of

Problem Solving." Products of problem solving were defined as whether the students arrive at the correct final answer or not. Process of problem solving was defined as the strategies people use to get to the answer. Previous research had placed too much emphasis on the product of problem solving, ignoring the processes of problem solving (Bloom & Broder, 1950). So, Bloom and Broder decided that instruction in problem solving should not focus on reinforcing students for getting the correct answer but rather on *problem-solving strategies* that are useful in generating answers. More important, they argued that these strategies could be altered by appropriate training and practice.

To find out what problem-solving strategies were used by successful problem solvers, Bloom and Broder (1950) used think-aloud procedures. These procedures involved asking model solvers to describe what was going on in their heads as they solved given problems. These procedures involved asking model solvers to describe what was going on in their heads as they solved given problems. Broder decided to teach remedial students to imitate and make use of the processes used by model students. The results of the study indicated that students who participated in the training not only scored an average of .49 to .68 points higher than matched groups who did not participate in the training but also expressed high levels of confidence and optimism concerning their newly acquired problem-solving abilities.

Expert-novice research suggests expert problem solvers can be distinguished from worse problem solvers in at least five major respects:

(a) expert problem solvers know more than worse problem solvers, (b) expert problem solvers tend to focus their attention on structural features of problems, while worse problem solvers on surface features, (c) expert problem solvers are more aware than worse problem solvers of their strengths and weaknesses as problem solvers, (d) expert problem solvers are better than worse problem solvers at monitoring and regulating their problem-solving efforts, and (e) expert problem solvers tend to be more concerned than worse problem solvers about obtaining "elegant" solutions to problems (Schoenfeld, 1985, 1987a, 1987b).

A major problem that been identified with the process-focused approach is that because this approach intends to teach students specific symbols and procedures that are not related to real-life situations or with what they already know, students may develop two separate systems of mathematics: (a) an informal system that they use to solve problems that are meaningful to them and (b) a school mathematics system consisting of procedures that they apply to symbols or artificial story problems they are given in school (Carraher, Carraher, & Schliemann, 1987; Cobb, 1988; Ginsburg, 1982; Lave, 1988; Lawler, 1981). These two systems operate independently of one another, thus, studen+ts will not see readily the connections between these two. In addition, because this approach to teaching mathematics does not make contact with what students already know, students may have difficulty relating to the formal mathematical structures (Hiebert & Carpenter, 1992).

The problem of the expert-novice approach is that it attempts to separate abstract mathematical concepts and procedures from the contexts that initially give them meaning. In addition, the emphasis of this approach is on fundamental semantic properties that define similarities and differences among problems rather than on the particular context of the problems (Hiebert & Carpenter, 1992). Thus, learning mathematics using this approach may result in making mathematical problem-solving procedures very difficult to understand.

The new math approach. The new math is a similar approach to discovery math and was implemented in the 1960s as a result of a mathematics reform. Two factors led to implementing the new math curricula including widening discontinuities between the mathematics taught in universities and that taught in the lower schools and growing concern over declining enrollments in university mathematics courses (Cooper, 1985; Howson, Keitel, & Kilpatrick, 1981; Moon, 1986). The major goal of the "new math" approach was to change the focus of mathematics curriculum from rote learning to understanding. Four elements of concern were the major issues in the new math curriculum including learning by discovery, readiness for learning, processes of learning, and aptitude for learning (Shulman, 1970).

The new math was built upon Bruner's principle that you can teach anything to anyone at any age if you do it right (Bruner, 1961). Bruner's theory includes the notion that any complex idea can be reduced into simpler ideas. Bruner's approach makes it possible for a child to learn the foundation of any subject that is presented in a meaningful form at any stage of development.

Bruner argued that regardless of how stimulating and enriching the environment may be, the child must be motivated to do something on his or her own. A child does not necessarily learn because he or she is exposed to information or material. The child, in order to learn, must initiate his or her own action. He or she must generate action within his or her system and operate by his or her own power rather than simply react to what is happening.

In the new math approach, learning is viewed as goal centered. A child is motivated by tension (a special kind of eagerness) to accomplish a certain goal. The better the child perceives his or her goal, the stronger his or her motivation to act toward achieving it. Motivation may be thought of as a way to activate interest that is presented already in a child.

The new math curriculum was enhanced by Bruner's idea that students should be provided with situations that permit them to use their own capacities to pursue self-interests. Students must see the meaning for themselves in order to learn.

The way that the new math implemented this idea at the elementary-school level was to take complex abstractions and make them " grade appropriate" by reducing them to, and teaching them as, simple abstractions (Schoenfeld, 1994). Pupils were not able to cope with the abstractions. In fact, there was a mismatch between what the pupils were capable of grasping psychologically and the mathematical structures they were asked to struggle with. Schoenfeld (1994) argued that in order for students to understand mathematics better, mathematics educators need to look at the world from their point of view as educators design and implement the curriculum. There were two additional reasons that led to the failing of the new math including the fact that elementary-school teachers were uncomfortable with teaching the new mathematics curriculum, and that the new mathematics curriculum appeared alien to parents as well as teachers (Schoenfeld, 1994).

In summary, the New math curricula failed, because it did not take into consideration that cognitive development occurs in stages (Piaget, 1954). In other words, you can not teach a child a concept if he or she is not cognitively ready to learn it.

Polya and heuristics. Polya (1945) introduced his approach to problem solving called *heuristics*. Heuristics is defined as mental operations "typically" useful for the solution of problems (Polya, 1945). Polya's work on problem solving is held in high regard by both mathematicians and mathematics educators (Schoenfeld, 1985). Heuristics have been the focus of much problem-solving research in mathematics education and the foundation for many development efforts in problem-solving instruction (Schoenfeld, 1985).

Polya (1957) argued that planning is the key process in mathematical problem solving. Polya further suggested several strategies for devising a solution plan, such as finding a related problem that one can solve or breaking the problem into parts that one already knows how to solve. Polya's model of problem solving is composed of four steps: understanding the problem, devising a plan, carrying out the plan, and looking back.

Schoenfeld (1985) argued that Polya's heuristics approach has not worked. He also argued that the literature of mathematics is full of heuristics studies. Most of these studies, although encouraging, have little concrete evidence that heuristics have the power that the research hoped they would. For example, studies by Wilson (1967) and Smith (1973) indicate that general heuristics did not transfer, as hypothesized, to new situations. Reports by Kantowski (1977), Kilpatrik (1967), and Lucas (1974), based on examinations of problem-solving protocols, indicate that the use of heuristics is correlated with scores on ability tests and with success on problem-solving tests. Treatment comparison studies have yielded consistently promising but inconsistent results (Goldberg, 1974; Loomer, 1980). Schoenfeld argued that the results are less dramatic than one might expect, because learning to use heuristics strategies is not sufficient to ensure competent problem-solving performance. In addition, inconsistent results have occurred because the complexity of heuristic strategies combined with the amount of knowledge necessary to implement them.

More recent studies, however, indicate the usefulness of Polya's heuristics and provide better results regarding these strategies. For example, two studies at the elementary-school level reported that instruction using the Polya's model enhanced problem-solving performance in mathematics (Charles & Lester, 1984; Lee, 1982). Using first-year algebra students, Conlon (1992) determined that students who received heuristics training did better on a measurement of problem solving than did a control group. Ghunaym (1985) compared two kinds of instructional strategies and found that advanced mathematics students at the secondary-school level who received an explanation for the underlying structure of problem-solving strategies did better on a measure of performance than those who received no explanation but were just told how to solve the problems.

Morgan-Brown (1990) applied modeling of Polya's four-phase approach to problem solving in mathematics with sixth-grade students in an urban middle school. Results indicated that the modeling group (the group that received instructions on Polya's model) outperformed the control group immediately after treatment and again 4 weeks after the conclusion of the treatment. A similar study conducted at the fourth-grade level (Van Akkeren, 1995) provided same results.

The research has provided two contradicting conclusions about Polya's model: some studies showed Polya's model is insufficient to enhance problem-solving performance and other studies showed that Polya's model is indeed sufficient to enhance problem-solving performance.

What the research is lacking is a description of a mathematics-classroom environment that promotes and ensures successful use of Polya's model for enhancing problem-solving.

Problem-centered learning. The problem-centered approach has three components: assign tasks, work in groups, and share results. In preparing for a class, a teacher selects tasks that have a high probability of being problematic to students. The students work on these tasks in small groups. Finally, the class is convened as a whole for a time of sharing (Wheatley, 1991).

An example of problem-centered learning is the Integrative Mathematics Program (IMP). IMP's 4-year program of mathematics problem solving replaces the traditional Algebra I, Geometry, AlgebraII / Trigonometry, and Calculus sequence. IMP integrates algebra, geometry, and trigonometry with probability and statistics and uses technology in order to enhance students' understanding.

The IMP curriculum challenges students to explore open-ended situations. in a way that closely resembles the inquiry method used by mathematicians in their work (Alper, Fendel, Fraser, & Resek, 1995). Unlike the traditional mathematics curricula that have emphasized rote learning of isolated-mathematical skill. IMP calls on students to experiment with examples. think about articulate patterns, and make. test. and prove conjectures (Alper et al., 1995). In IMP classes, students work in small groups trying to solve mathematics problems by communicating mathematics ideas and then present their solutions to these mathematics problems to other students in class.

The IMP curriculum is problem based, consisting of units that require from 5 to 6 weeks of class time and that are organized around a central problem or theme. Motivated by the central problem, students solve a variety of smaller problems, both routine and nonroutine. Solving these problems helps students develop the skills and concepts needed to solve the central problem (Alper et al., 1995). For example, during the second year of IMP, students spend about 5 weeks on a unit called *Cookies*. The goal of this unit is to teach students how to manipulate equations and how to reason using graphs. This unit begins by posing a linear programming problem with two variables. The problem was chosen because it provides a mathematically rich set of circumstances that requires students to solve two equations in two unknowns (Alper et al., 1995). Alper and other IMP developers argued that if the unit had been built around a situation that required solving just one system, students would be able to use guess-and-check or could use the graphing calculator to arrive at a solution. They would not be motivated to find an algebraic procedure that could be used to solve any system of two equations in two unknowns.

The "Cookies" unit encourages students to work in small groups to find solutions to certain mathematics problems, share ideas, and reason about mathematical concepts. Even when students are tested, they are allowed to work together in small groups to answer test questions. Typically, when students are first put in small groups and asked to reason mathematically, they will resist. This resistance is due usually to their being unsure of themselves. It may due also to their thinking that mathematics classes are not supposed to function in this matter (Alper et al., 1995). Alper and other IMP developers argued that in a 4-year program almost all students overcome these feelings.

Presenting the small groups findings to the whole class is an essential part of IMP, because presenting solutions to mathematics problems improves students' understanding and may be used as an assessment tool by the teacher. After finding solutions to a challenging mathematics problem, students present this solution to the whole class.

Research studies have shown that IMP students are staying with mathematics longer than students in traditional mathematics programs. For instance, only 60% of U.S. high-school students graduating in 1993 completed 3 years of college-preparatory mathematics. In contrast, 90% of 1993 IMP graduates completed 3 years of IMP college-preparatory sequence (Webb, Schoen, & Whitehurst, 1993).

Another study has shown that IMP students taking the SAT test had a higher mean mathematics score (545 compared with 531) than Algebra students, even though a higher portion of IMP students took the test (83% compared with 72%), and both IMP and Algebra students had comparable pretest results. Furthermore, of students taking the test, IMP students also had a higher percentage doing "very well" (600 or higher) than the Algebra students (Webb et al., 1993).

More research is needed to determine the impact of problem-centered curricula such as IMP on mathematical problem-solving performance and attitude toward mathematics. Research needs to be conducted to find out if IMP is an effective environment for teaching problem solving. Currently there is not enough studies of a successful high-school mathematical curricula for teaching problem-solving skills. Although it seems reasonable that IMP may accomplish this goal, students have never been measured explicitly regarding their problem-solving abilities. Equally important, the attitudes of the students who have been taking IMP classes have not been measured nor compared with students who are taking the traditional mathematics courses.

The purpose of this study is to measure high-school mathematical problem-solving performance and attitude toward mathematics by comparing first-year Algebra classes with first-year IMP classes at an urban high school in California.

Theoretical Rationale

The purpose of this study is to compare and correlate the mathematical problem-solving skills and attitudes of high-school students using two different teaching approaches: (a) the problem-centered learning approach and problem-solving skills and attitude.

Wheatley (1991) argued that much of current school practice is determined by textbooks where learning is seen as the slow accumulation of knowledge through practice. Wheatley emphasized the fact that the content of learning is broken down into small units and carefully sequenced for the learner by his or her teacher. In other words, teachers are expected to write behavioral objectives to tell learners what they will learn. Thus, the emphasis is on observable behavior rather than mental activity and competence. As a result of this behavioristic influence, school learning tends to be rule oriented. For example, Eisner (1980) found that most of the time students are applying or memorizing skills. Students spend most of their class time practicing skills. In mathematics, this means memorizing facts and practicing computational procedures (Tobin & Gallagher, 1989).

It is conjectured that better conditions for learning exist, however, when a person is given a problem for which no known procedure is available (Wheatley, 1991). In other words, learning mathematics requires a problematic situation. Moreover, Wheatley argued that in order to identify potential problematic situations, the teacher should focus on his or her students' understanding. Instead of trying to persuade students to see things their teacher's way, the teacher should understand the thought patterns of students, so he or she can frame tasks that might be considered as problematic to students.

Most educators consider mathematics to be the most factual of subjects and that students must first become proficient with a set of facts and skills before problem solving can take place (Wheatley, 1991). But, Wheatley argued that adding two numbers could be a problem to a first-grade student who has not developed a procedure for adding numbers. In fact, research studies revealed that first-grade students faced with the problem of adding two numbers will develop their own procedures for this task. By inventing these processes, students build meanings that provide the foundation for rapid advancement in mathematics learning (Cobb & Wheatley, 1988). Indeed, mathematics can be taught from a problem-centered perspective with considerable benefit to students at all stage levels.

The problem-centered model communicates several important messages to students (Wheatley, 1991). First, it builds "mathematical instincts" to construct meaning. This improves the students' attitude toward mathematics; they start realizing that they are capable of problem solving and do not have to wait for the teacher to show them the procedure of doing a certain problem or to give them the correct answer. Second, the problem-centered learning approach helps students change their belief that mathematical problems should always be completed in 5 minutes (Schoenfeld, 1985). Both D'Anderade (1981) and Schoenfeld (1988) argued that there is nothing wrong with these beliefs about mathematics; what needs to be changed is the curriculum that encourages such beliefs. Third, students come to believe that learning is a process of meaning-making rather than a game of pleasing the teacher and figuring out what he or she wants. All of these variables are part of an academic game played by students in a direct instruction environment. Rather than playing this academic game, problem-centered learning has the potential for students to become task-oriented rather than ego-involved (Nicholls, 1983), focusing on learning for its own sake.

Problem-centered learning has three components including (a) selecting and assigning tasks, (b) working in small groups, and (c) sharing results (Wheatley, 1991). In preparing for class, teachers select a task that is problematic to students. Then, students work on this task in small groups. During this time, the teacher attempts to facilitate collaborative work in the groups. Finally, students present and discuss their answers to the whole class. In these discussions, the teacher role is to facilitate and to encourage nonlectured dialogue.

Selecting tasks is the first component of problem-centered learning. although, tasks in conventional textbooks are designed using the explain-practice mode, tasks used in the problem-centered learning are problematic tasks that focus attention on the key concepts of the discipline that will guide students to construct effective ways of thinking about the subject. Wheatley (1991) provided a description of a such task. This task should be accessible to everyone at the start, invite students to make decisions, encourage students to use their own methods, promote discussion and communication, should be replete with patterns, lead somewhere, have an element of surprise, be enjoyable, and be extendable.

An example of a task that is used in the problem-centered learning practice is stated below.

> Five cubes, each three inches on a side, are to be wrapped as a single present. What is the size (area) of the paper needed to wrap the package? (Do not overlap the paper).

although, most of the classroom mathematics problems are tightly worded such that there is only one "logical" interpretation and thus one "correct" answer, this problem allows for a variety of solutions. This mathematical activity encourages students to make decisions and gives them the opportunity to frame their own problems and thus to explore mathematics.

Selecting tasks for a mathematics classroom requires the teacher to look at the world through his or her students' eyes and consider their thinking (Wheatley, 1991). In traditional mathematics, the issue for teachers is what procedures and knowledge students have learned in their classes. In the problem-centered learning practice, however, the concern for educators is what concepts students have constructed, the cognitive level at which they are operating, their belief, and intentions.

In order for tasks to be effective, they should require students to restructure their thinking and elaborate on what they already know (Wheatley, 1991). This approach avoids the error of teaching materials students already know and the error of presenting ideas beyond the students' level of comprehension.

Working in small groups is the second component of the problem-centered learning. Using small groups provides opportunities for students to explain and defend their views, a process that stimulates learning (Wheatley, 1991). Johnson and Johnson

(1985) argued that students can profit greatly by working together. Although students make sense of ideas for themselves, this does not happen in isolation from others (Bishop, 1985; Sigel, 1981). Piaget and Inhelder (1969) included socialization as one of the four factors in cognitive growth. A study by Haste (1987) indicated that participating in small-group problem solving will result in a measurable change over time in the structure of students' thinking. Another study by Doise and Mugny (1984) demonstrated that children working in pairs and in groups to solve logical problems produce more adequate solutions than when they are working alone. Working in small groups also will help students improve their attitude toward mathematics. For example, Grouws and Cramer (1989) found that the classrooms of effective teachers of mathematical problem solving were characterized by a support classroom environment where social norms encouraged students to be enthusiastic and to enjoy mathematical problem solving.

Wheatley (1991) argued that learning occurs in the social context of the classroom that is heavily influenced by interactions of students. More importantly, knowledge is constructed through interactions with others. As students exchange ideas, students develop shared meanings of these ideas, which allows them to communicate effectively with each other. Thus, the importance of the social setting in which learning takes place cannot be overemphasized.

In this step, students work together trying to understand the problem, find out a plan or a strategy that will help them solve the problem, carry out the plan to arrive to a solution to the given problem, and finally look back and check their results to determine if their solution to the problem makes sense. These steps are the same as Polya's four-phase of problem solving. Therefore, to describe what is happening in a mathematics class utilizing problem-centered learning approach, one needs to describe Polya's four-phase problem-solving model.

Presenting solutions is the third component of problem-centered learning. After working in small groups on tasks provided by their teacher, students come together as a whole class to present their solutions, inventions, and insights and to explain their methods to their classmates.

Class discussions initiate "conversations," which students learn to carry on within themselves. By continuing these discussions within themselves, students begin to act mathematically (Wheatley, 1991). More important, Wheatley emphasized that class discussions provide a forum for students to construct explanations of their reasoning. As students tell others how they thought through a problem, they elaborate and refine their thinking and deepen their understanding.

Wheatley's model extends the thinking of Polya's four-step model. The first step of Polya's model is understanding the problem. In attempt to understand the problem, students define the unknown, find out what is given, and if there is a figure connected with the problem, they should draw this figure and label the unknown and the data on it. Ideally, the students should not only understand the problem but also desire its solution. In comparison with the traditional mathematics class at which the role of the teacher is to explain the problem to his or her students and be sure that they understand it, in problem-centered learning approach students working in small groups not only attempt to understand the meaning of the problem themselves but also construct a solution for it.

In order to ensure basic understanding of the problem and prevent students from losing interest in it, the problem should be well chosen (not too easy and not too difficult), meaningful, and interesting. More important, the verbal statements of the problem should be understood.

The teacher can find out if the students understand the problem by having them restate the question in the problem in their own words and by making a list of only the relevant facts given in the problem. Understanding the problem requires students to point out the unknown, the data, and the condition that links the unknown to data.

The following problem, is an example of a traditional mathematics problem that is found in many geometry text books:

> Find the diagonal of a rectangular parallelepiped of which the length, the width, and the height are known.

The teacher can make this problem more interesting and concrete by relating it to the physical classroom structure, which is an example of a rectangular parallelepiped. The student's task is to find out the diagonal of the classroom. In order to help the student understand the problem, Polya (1957) suggested that the teacher might start a dialogue with his or her students similar to the following:

Teacher: What is the unknown?
Students: The diagonal of the classroom.
Teacher: What are the data?
Students: The length, width, and the height of the classroom.
Teacher: Which letter should denote the unknown?
Students: x.
Teacher: Which letter would you choose for the length, the width, and the height?
Students: l,w,and h.
Teacher: What is the condition, linking l,w,h, and x?
Students: x is the diagonal of the classroom of which l,w, and h are the length, width, and the height of the classroom.
Teacher: Is it a reasonable problem? In other words, is the condition sufficient to determine the diagonal of the classroom?
Students: Yes. Knowing the length, width, and height of the classroom, will enable us to determine the diagonal of the classroom.

Teacher: Is it a reasonable problem? In other words, is the condition sufficient to determine the diagonal of the classroom?

Students: Yes. Knowing the length, width, and height of the classroom, will enable us to determine the diagonal of the classroom (Polya, 1957, p. 8).

Thus, using a teacher-student dialogue may not only help students understand a given problem but also help the teacher understand how the students think about the problem.

The second step in Polya's model is "devising a plan." Devising a plan means finding out all the appropriate solution strategies that are useful to solve a given problem and knowing which calculations, computations, or constructions needed to be performed in order to obtain the unknown. Examples of these strategies are finding a pattern, guess and check, drawing tables and diagrams, and making lists. In order to come up with these strategies, students need to find the connection between the data and the unknown.

One way that may help students "devising a plan" to solve a certain problem is to help them find a familiar-related problem or a theorem that could be useful in solving the problem. Once students find a related or a similar problem that they solved before, they can apply the strategies that were used to solve this problem to solving the new problem. Finding a related problem, however, may be difficult for most students to do. The teacher can help his or her students find a related problem by starting a dialogue with them. For example, in order to help students find out a related problem to the above diagonal problem, Polya (1957) suggested that the teacher might start a dialogue with his or her students similar to the following dialogue:

Teacher: Can you think of a related problem?

Students: No clue!

Teacher: Can you find a related problem that has the same unknown?

Students: No clue!

Teacher: What is the unknown?

Students: The diagonal of a parallelepiped.
teacher: Can you think of a similar problem with the same unknown?
Students: No. We have not dealt with any problem that included the diagonal of a parallelepiped.
Teacher: Do you know any problem with similar unknown?
Students: No clue.
Teacher: The diagonal is a segment of a straight line. Have you ever solved a problem whose unknown was a segment of a line?
Students: Yes. we have solved many related problems.
Teacher: Can you give me an example.
Students: Find a side of a right triangle (Polya, 1957, pp. 10-11).

A second way that also may help students devising a plan is having them simplifying the problem. In other words, the teacher may have his or her students look for ways to make the problem easier. Students may be able to simplify the problem by exploiting symmetry or reducing the number of variables they might have to consider (Schoenfeld, 1985).

More important ways that may help students devising a plan are making them rewrite the problem using relevant mathematical notations, making sure quantities are clearly labeled, and express it in a concise and convenient mathematical form that can be manipulated easily.

Devising a plan is the most difficult step in the problem-solving process. Teaching students some of the above problem-solving strategies, however, will not only help students devise a plan for a given problem but also provide them with some techniques that may help them solve a new or unfamiliar problem.

The third step in Polya's model is "Carrying Out The Plan." In this step, students are expected to carry out and explain the strategies obtained in the second step. For example, if students, in the second step, decided to make a table as a strategy, then in the third step, they should draw the table, clearly label it, and provide adequate explanation to this strategy. This step requires checking each step in the plan and being able to prove that these steps are correct. In addition, students should be able to perform the steps and get the solution to the given problem. This is the step that most teachers and students are familiar with.

The last step in Polya's model is "Looking Back." In this step, students are expected to state in writing what their answer is, explain what it means, and label it correctly. The purpose of this step is to examine the solution obtained. This step requires that students check the results, determine other methods or plans to get to these results, and finally be able to use the obtained results or the methods used to solve other related problems. Checking the results means comparing them to observed numbers or to a common sense estimate of observable numbers. For example, if students found out that the diagonal of a classroom is 16,130 ft, they should realize that there is something wrong with their answer.

In summary, there are four major reasons for using Polya's model to teach problem solving: Polya's model provides essential steps for solving a mathematical problem, it provides students with an understanding of a problem and its solution, it engages students with activities that force them to reason about mathematics and communicate mathematical ideas, and it encourages students to connect mathematical ideas, procedures, and concepts.

In summary, Polya (1957) proposed that students need to understand not only the solution to a problem but also the motives and procedures of the solution. His model provides the essential steps that not only help students obtain the correct solution but also help them understand the steps needed to solve it, justify those steps, and check and prove that the solution is correct. In addition, Polya's model suggests teaching methods. For example, it indicates to teachers what questions they need to ask in order to help their students solve the problem without giving them the answer. Examples of such questions are "what is the unknown?" and "what are the data?" Furthermore, Polya's model can be applied to all sorts of problems and is not restricted by subject matter. For example, the problem could be algebraic, geometric mathematical or nonmathematical, theoretical, or practical, and still could be solved using Polya's model.

Polya's model emphases "doing" mathematics. It is the "doing"--the experimenting, abstracting, generalizing, and specializing-- that constitutes mathematics, and leads to learning it. Polya (1965) argued that learning should be active, not passive; merely reading books or listening to lectures or looking pictures without adding some action of student's own mind, the student will learn very little. Polya (1965) also argued that the best way to learn anything is to discover it by yourself. Many reports such as the Cockcroft Report (Committee of Inquiry into the Teaching of Mathematics in Schools, 1983), the Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics, 1989), and Everybody Counts (National Research Council, 1989) similarly recommend engaging students in purposeful activities that grow out of problem situations; requiring reasoning and creative thinking; gathering and applying information; discovering, inventing, and communicating ideas, and testing these ideas through critical reflection and argumentation.

Polya's model promotes connecting mathematical ideas and procedures. A mathematical idea, procedure, or fact is understood thoroughly if it is linked to existing networks with stronger or more numerous connections (Hiebert & Carpenter, 1992).

Understanding mathematical ideas involves recognizing relationships between pieces of information. For example, using Polya's model, students try not only to figure out what they are looking for (i.e., the unknown), but also how is the unknown connected to what is given in the problem. In addition, students attempt to find the relationship between problems; how problems differ from or are similar to each other, and how the solution to one problem may help them in solving another problem.

Research Questions

This study attempted to answer the following research questions:

1. To what extent does implementing the problem-centered learning approach enhance high-school students' mathematics performance compared with using the teacher-guided approach?

2. To what extent does implementing the problem-centered learning approach enhance high-school students' attitude toward mathematics compared with using the teacher-guided approach?

CHAPTER II

Review of the Literature

This chapter is divided into three parts. The first part focuses on mathematics education research that has examined the effect of instruction of heuristics on process problem-solving performance. The second part covers research studies on constructivism in mathematics and problem-solving learning. The final part consists of research examining the relationship between students' attitudes toward mathematics and mathematics achievement and problem-solving learning.

Heuristics

The term heuristics was defined by Schoenfeld (1985) as strategies and techniques for making progress on unfamiliar problems. Schoenfeld described heuristics as "rules of thumb for effective problem solving, including drawing figures, exploiting related problems, reformulating problems, and testing and verification procedures" (p. 15).

Heuristics were reintroduced to the literature by Polya (1945). According to Polya, Heuristics are "mental operations typically useful for the solution of problems" (p. 2). Polya (1945, 1957) laid the groundwork for the exploration in heuristics in his book How to Solve It. This book is a classic introduction to heuristics at an elementary level and a source on mathematical problem solving. In this book, Polya not only reintroduced the word "heuristic" to the literature but also introduced strategies and techniques that may help students in solving mathematical problems including: understanding the problem, devising a plan, carrying the plan, and looking back. In addition, Polya explained the process of solving problems by using specific examples taken largely from geometry. His aim was to teach a method that can be applied to the solutions of other problems. More important, Polya tried successfully to show both teachers and students how to strip away the irrelevancies that, in some ways, clutter their thinking and guide them toward a clear and productive mind. In his book, Polya included a "Short Dictionary of Heuristic" that supplies the reader with the history, techniques, and terminology of heuristic. In addition, the book is concluded with a section of 19 problems, hints, and solutions. Schoenfeld (1983), wrote that "this pioneering work and his other books (referring to "How To Solve It" and Polya's other books) are must reading for anyone interested in the way we think when we solve mathematical problems" (p. 98).

Since the publication of Polya's book on heuristics, heuristics have been the focus of most problem-solving research in mathematics education and the foundation for most development efforts in problem solving (Schoenfeld, 1985). This section of the literature review will cover research regarding the effect of using heuristics on problem-solving abilities and mathematical performance.

The review of literature reveals some contradictions regarding the effect of heuristics on students' mathematical performance. In other words, although some studies have indicated that heuristic strategies have not worked and consistently produced less than what was hoped for, other studies have shown that teaching a set of strategies to students can improve problem-solving abilities. This section reviews both types of research.

Schoenfeld (1985) argued that the literature of mathematics education is full of heuristic studies, but most of these studies have provided little concrete evidence that heuristics have the power that the experimenter hoped they would have. Schoenfeld pointed out that the faith in mathematical heuristics as useful problem-solving strategies has not been justified by results from the literature. Begle (1979) summarized the results of 75 empirical studies on problem-solving strategies by writing, " This brief review of what we know about mathematical problem solving is rather discouraging." (p. 146). For example, studies by Wilson (1967) and Smith (1973) indicated that general heuristics did not transfer well to a new situation.

One-hundred seventy-six college students were studied by Smith (1973) to determine the effect of heuristic training on problem-solving performance. Smith divided his students into two treatment groups. One group received only task-specific heuristic advise, whereas the second treatment group was given advice only on general heuristics. Students were given programmed booklets on finite geometry, Boolean algebra, and symbolic logic. The experiment was conducted over a 3-week period.

Results of the experiment showed that students in the task-heuristic group solved significantly more logic problems and completed the Boolean algebra and logic tests faster than the general heuristics group. Subjects who received general heuristic instruction did not solve more transfer problems than did the subjects who received task-specific heuristics. The conclusion of the experiment was that general heuristics appear not to have strong influence on transfer and that task-specific heuristics instruction is more effective than instruction in general heuristics in improving problem-solving performance.

An earlier and similar study conducted by Wilson (1967) investigated the problem-solving performance of 144 high-school volunteers on learning tasks following instruction in which the level of generality of problem-solving heuristics taught was the independent variable. Three levels of problem-solving heuristics were used: task specific heuristics, means-ends heuristics, and planning heuristics. The results favored the teaching of general heuristics for transfer tasks, whereas the problem solving performance on the training tasks was independent of the generality of the heuristics. Wilson also concluded that problem-solving performance was enhanced by the combination of different types of heuristics and that the planning heuristic was superior to both means-ends and the task-specific heuristic.

Reports by Kantowski (1977), Kilpatrick (1967), Lucas (1974), and Post and Brennan (1976) based on the examinations of problem-solving protocols, do indicate that the use of heuristics is related to scores on ability tests and to success on problem-solving tests; however, the results are far less dramatic than one might expect.

Kantowski (1977) conducted a study on the processes involved in mathematical problem solving. The purpose of the study was to gain information about the processes involved in solving complex, nonroutine problems. The primary interest in this study were processes indicating the use of heuristics and the relationship between their use and deductive processes observed during the solution of a problem.

Subjects in this study were 8 students selected from among the high-ability ninth-grade algebra students at a private school in a suburb of Atlanta. During this study, students went through four phases. First, the pretest phase; during this phase, students were asked to think aloud as they solved eight problems. Second, the readiness-instruction phase; this phase lasted 4 weeks and students were taught three lessons per week. During this phase, students obtained instructions on heuristics and how to use them in problem solving. A second test was administered after the readiness instruction. Third, during the third phase, students were given instructions in geometry using heuristic instructional techniques. The duration of this phase was 4 months. Fourth, the final phase was a posttest consisting of geometry and verbal problems and "prerequisite knowledge" tests including facts and concepts necessary for the solution of the posttest. During each phase, the subjects were asked to "think aloud" as they solved problems, and their protocols were recorded on cassette tapes for latter analysis. A process-product score was assigned to each problem. To calculate the score, one point was given for using a heuristic strategy. A median score was calculated for each of the subjects using their process-product scores. Percentages of problems in which the heuristic strategies were used were calculated for problems with scores above and below the median for each subject.

Results of the study indicated that more students with scores above the median used the goal-oriented heuristics than those who scored below the median. For instance, 59 to 95% of the solutions with scores above the median showed evidence of the use of goal-oriented heuristics, whereas at most 52% of the solutions with scores below the median showed indication of their use. Moreover, the use of heuristics was more evident in solutions with scores above the median. Thus, more use of goal-oriented heuristics was observed in successful problem solvers. Furthermore, the tendency to use goal-oriented heuristics increased as problem-solving ability developed. For instance, in test 1, percentages of solutions were from 17 to 50% with median percentage 25, whereas on test 2, the percentages were from 32 to 84% with median percentage of 47. In addition, comparing the results of the pre- and posttest exhibited evidence of the use of heuristics, for example, from 14 to 72% of the solutions on the pretest, with a median percentage of 36, and 14 to 100% of the solutions on the posttest, with a median percentage of 72. Another result in this study was that more regular patterns of analysis and synthesis are observed in successful problem solvers. For example, from 77% to 100%, with a median of 95, of the solutions with scores above the median disclosed these patterns, whereas only 18 to 41%, median percentage 23, of the protocols from solutions with scores below the median demonstrated this pattern sequence.

Although, the results of the study compared the successful (above the median) with the unsuccessful (below the median) problem solvers with regard to obtaining and using heuristics, they did not report on the effect of these heuristics on the mathematical problem-solving performance of the students as they obtained these strategies. One explanation to this is the lack of the control group in the experiment. Furthermore, some strategies, such as "looking back" and "evaluation of the solution," did not increase as problem-solving ability developed, and did not appear to be related to success in problem solving. These strategies were indeed emphasized in instruction and used in the group problem sessions but were not evident during the solution of problems in the tests.

Another study on the effect of teaching heuristics that utilized the "think-aloud" techniques on older students was conducted by Lucas (1974). This study was conducted to investigate heuristic usage and its effect on problem-solving performance and to analyze the influence of heuristic-oriented teaching on a group of first-year university calculus students. The subjects in this study were 30 university students from two calculus classes taught by the investigator. The students were divided into four groups to correspond with two experimental conditions (exposure to heuristic instruction versus no exposure) and two testing conditions (exposure to both pre- and posttests versus exposure to posttest only). Students were not assigned randomly to treatments because intact university classes were used. Treatments, however, were assigned randomly to classes.

The study was conducted in three phases lasting 13 weeks. Phase I was an initial diagnostic observation of 14 students. Phase I was executed to determine the preinstructional status of the students on heuristic usage and problem-solving performance. During this phase, every student was administered a 2-hour interview. During the interview, students were given two problems to practice then asked to take seven test problems. As students were working, they were asked to think aloud and were tape recorded.

Phase II lasted 8 weeks and was an instructional program. The emphasis of this program was a problem-solving strategy. Both the control and the treatment group received inquiry-style instruction from the investigator except with respect to problem solving. The control group was given only expository treatment of problem solutions with minimal attention to heuristic strategies. The treatment group, however, had the inquiry technique applied during the discussion of problems and received intensive training on heuristic strategies.

During phase III, all 30 students (17 experimental and 13 control) were interviewed under conditions identical with phase I. Problems used in phase III were changed but had the same level of difficulty as those of level I. The purpose of stage III was to collect data that would determine the postinstructional status of the students on heuristic usage and problem-solving performance.

The following results were obtained from this study. First, statistically significant differences favoring the instructional treatment were found on the following heuristic strategies: (a) using mnemonic notation ($p < .005$), (b) the method-result heuristic ($p < .05$), and (c) separating and summarizing data ($p < .02$). Second, there was a tendency for heuristic students to exhibit slightly (but not significantly) decreased solution time and significantly increased ($p < .08$) time spent looking back at a problem. The net effect, however, was no significant statistically difference between groups in total spent on the seven problems. Last, there were no significant differences between the experimental and control groups with respect to the number or kind of errors committed during the process of solving a problem. Similar results were reported by Kilpatrick (1967).

Post and Brennan (1976) investigated the effect of instructing students in a particular heuristic problem-solving process on their ability to solve problems. The subjects for this study were 94 tenth-grade students from a private high school in Minnesota. Students were enrolled in a traditional geometry class. Students were assigned randomly to either a control or experimental group. This study utilized group-paced learning and emphasized the use of a General Heuristic Problem-Solving Procedure (GHPSP). GHPSP is a problem-solving procedure that includes parts of Polya's heuristic process, especially some of his questioning subparts. This procedure has four general phases. The first phase is understanding the problem. In this phase, students are asked to read the problem, state the problem in their own words, and draw a diagram to aid in clarification. Some of the Polya's (1957) questioning techniques are used in this phase. Examples of such questions include the followings " What is the unknown?", "What are the data?", and "What is the condition?". The second phase is "plan of attackanalysis." In this phase, students are required to gather data, recall missing data, eliminate irrelevant data, and decide on needed approach to solve the problem. The third phase is the "productive phase." In this phase, students apply the approach obtained in phase II to obtain a solution to the given problem. The last phase in this procedure is the "validating phase." In this phase, students are asked to look back and check their results.

In this experiment, the treatment group was provided with the teacher-directed group-paced learning package where emphasis was placed on solving problems using the GHPSP. The control group received no formal instruction in problem solving. Instead, the control group received the normal instruction provided in sophomore geometry.

The experimental factors were treatment and ability. Ability was divided into two parts: high versus low ability for both the control and experimental groups. Results of the study were as follows. No statistically significant difference was found on problem-solving posttest scores between the experimental and control group (experimental group mean was 13.28 and standard deviation was 2.73 with a class size of 25 vs. control group mean of 12.43 and standard deviation was 3.23 with a class size of 23), with an effect size of .29. No statistically significant difference was found on posttest problem-solving between the low and the high group (experimental high-ability group mean was 13.28, SD of 2.73 vs. experimental low-ability group mean of 9.29, SD of 3.22). There was no statistically significant interaction between treatment and ability level.

In general, instructing students in a general heuristic problem-solving procedure (GHPSP) had no marked effect on their problem-solving ability as measured by problem-solving test used in this study. The experimental means were higher but not to a statistical significant degree. This result also was observed in an earlier study (Post, 1968). Post studied 10 seventh-grade students. He concluded that special study of a structure of problem-solving process appeared not to enhance the problem-solving ability of seventh-grade students; moreover, he found that instruction in problem solving appeared not to statistically significantly improve problem-solving ability of students regardless of IQ.

Treatment comparison studies (e.g., Goldberg, 1974; Loomer, 1980) have yielded promising but contradicting results regarding the effect of heuristic training on problem-solving performance of students.

To summarize, these studies have shown that heuristics have proven far more complex and far less obtainable than had been expected. Schoenfeld (1985) argued that the reason for these discouraging results is that, in most of these studies, the characterization of heuristic strategies was not sufficiently prescriptive. In other words, not enough detail was provided for the characterization to serve as guides to the problem-solving process. Furthermore, the implementation of heuristic strategies is far more complex than at first appears. For example, carrying out a strategy such as "exploiting an easier, related problem" involves six or seven separate major phases, each of which is a potential stage of difficulty.

Other studies, however, have shown that teaching a set of strategies to students can actually improve their problem-solving abilities, if heuristics are taught appropriately. In the following section, the research will be reviewed at two different levels including research studies involving elementary- and middle school students and adult students who are in high school or college.

At the elementary- and middle- school level, research studies (Charles & Lester, 1984; Duckworth, 1964; Lee, 1982; Morgan-Brown, 1990; Piaget, 1964; Polya, 1957;

Van Akkeren, 1995) have indicated the following four conclusions. First, teaching a set of strategies to students can improve their problem-solving abilities. Second, in order for Polya's strategies to be implemented successfully with younger students, these strategies have to be adapted in language and complexity. Third, students need to be actively involved while they are learning how to solve problems. Last, children should be provided with situations where they make decisions, observe what is happening, and be actively involved.

Charles and Lester (1984) conducted a study to evaluate a Mathematical Problem Solving (MPS) Program that utilizes Polya's (1957) four-phase model of problem solving. The MPS program promoted the learning of problem-solving strategies, emphasized solving problems (i.e., lessons on skills such as making tables), and encouraged an active role for the teacher. Furthermore, MPS is a curriculum research and development project sponsored by the West Virginia Department of Education under Title IV-C of the Elementary-Secondary Education Act. The program consisted of (a) instructional material for problem solving; (b) guidelines concerning ways to create a classroom atmosphere conducive to problem solving, to group students for instruction, and to evaluate students' performance; and (c) a teaching strategy for problem solving.

To evaluate the MPS program, Charles and Lester (1984) identified 36 schools in West Virginia with similar levels of achievement on the Comprehensive Test of Basic Skills (CTBS, 1973) and asked 23 fifth-grade and 23 seventh-grade teachers to participate in the problem-solving project. At grade 5, there were 451 students, and, at grade 7, there were 485 students. Neither the teachers nor the students in the study had had any prior special training related to problem solving.

Twelve treatments and 11 control classes were selected at grade 5, and 10 treatment and 13 control classes were selected at grade 7. On 3 consecutive days, every participant in the study took the pretest. After 23 weeks, each student took a different form of the pretest as the posttest. The teachers of the treatment classes received 3 hours of training on the use of the problem-solving program prior to the pretest. Each class had a period of 45 to 55 minutes each day for mathematics. The treatment class had both the regular mathematics program and the MPS during the same period. The control class had only the regular mathematics program. Both the control and the treatment group covered the same number of pages in the regular textbook at the end of the study. Each teacher of a treatment class was interviewed individually for approximately one hour within a 2-week period following the administration of the posttest.

The results of this study showed that the instruction given to the students, at both the fifth and seventh- grade levels, in the MPS program had benefited their problem-solving performance in four ways. First, the MPS improved students' abilities to understand the problem and to plan solution strategies faster than it improved their abilities to get correct results. Second, MPS improved students' willingness to engage in problem solving. Third, students gained confidence in their ability to succeed in problem solving. Finally, the program had a positive effect on the attitude of teachers. All teachers of the treatment classes became increasingly more positive toward both the important of problem solving and their ability to teach it.

This study distinguishes itself from many other studies of problem-solving instruction in three ways. First, it investigated long-term (23-week) changes in performance. Second, differences in the nature of performance changes at two grade levels, namely grades 5 and 7, were considered. Third, in addition to quantitative measures of performance, teachers' perceptions and opinions about the program were solicited and served as an integral part of evaluation data.

Lee (1982) studied the effects of using heuristics in the classroom. The purpose of his study was to determine if fourth graders who are concrete operational can acquire heuristics and use them effectively to become better problem solvers. The subjects in this study were 16 fourth-grade students from a rural elementary school, 8 boys and 8 girls. Each of the students was classified as either a II-A child or II-B child. Child II-A is defined as a child who cannot make accurate serial orderings of the effects of weights on a pendulum. Child II-B is a child who is able to make accurate serial orderings of the effect of weights but is not able to isolate the variables affecting the pendulum's frequency of oscillation such as length of string, height of releasing position, and weight of object. Each of the two groups was randomly divided into two groups of 4 students each, one instruction and one no instruction group. The instruction group had a preinstruction interview, the instruction, and a postinstruction interview. The instruction group had 20 problem-solving sessions of 45 minutes each over 9 weeks; while the no-instruction group attended their regular classes and did not receive any instruction.

In Lee 's (1982) study, Talyzina's (1970) clinical methodology was adapted in collecting and analyzing the data. The data included each student's written responses on worksheets during the interviews, audiotapes of the dialogue between the investigator and each student during the interviews, and the investigator's log for the interviews. The investigator's log included any of the student's actions and reactions that might not be shown on the worksheets or tapes. Results of this study have shown that after 20 heuristic problem-solving instruction sessions, every student in the treatment was able to select an appropriate heuristic and use it effectively. In addition, 73% of the postinstruction problems and 79% of the delayed postinstruction problems were solved successfully by the treatment students compared with 6% of the postinstruction problems solved successfully by the control students. More important, the results of this study indicate that children are capable of using heuristics when attempting to solve problems and be successful in solving these problems.

Morgan-Brown (1990) applied modeling of Polya's four-phase approach to problem solving in mathematics with sixth-grade students in an urban middle-school. The study employed a repeated-measures experimental format to compare a group who received instruction using a "thinking aloud" modeling approach with a procedural group and a control group. The modeling group had a three-part treatment cycle. In the first stage of the treatment, the instructor modeled Polya's four-phase approach to solving problems. In the second stage, students worked in small groups to practice the problem-solving process as the instructor provided guide questions to direct their thinking. In the last stage, students practiced problem solving independently with some guidance by the instructor. The procedural group had no formal instructions but received guided practice from the instructor. The control group did not have any problem-solving instruction or practice.

Findings showed that the modeling group outperformed both the procedural and control groups immediately after treatment and again 4 weeks later. The data, revealed large effect sizes for both the modeling ($d = 2.90$) and procedural groups ($d = 2.75$) when compared with the control group.

A more recent and similar study, but at a different grade level was conducted by Van Akkeren (1995). Van Akkeren also examined the effect of cognitive modeling on fourth-grade students. The purpose of the study was to determine experimentally the effect of cognitive modeling of Polya's four-phase approach on fourth-graders' performance and self-efficacy for problem solving in mathematics. The study employed a repeated-measure experimental design. The subjects for this study were 84 fourth-grade students, 42 girls and 42 boys, in a public elementary school in the San Francisco area of California. The majority of the students (e.g., 93%) were White and upper-middle class students. Two instruments were used to measure dependent variables including a researcher made problem-solving test to measure the students' problem-solving performance in mathematics and a Likert-scale survey used to measure students' self-efficacy. The experiment took place on five consecutive school days. Treatment occurred for 55 minutes on each of the first 4 days. Performance and self-efficacy were measured on the fifth and sixth days during a 60-minute period. The students from three fourth-grade classrooms were assigned randomly to one of three groups: cognitive modeling, exemplar modeling, and guided practice (control).

Both the cognitive and exemplar groups received instruction that focused on Polya's four-phase problem-solving approach. While the cognitive group received instruction on how, what and why something should be done to solve the problem, the exemplar group only received instruction on what, and how things need to be done to solve a certain mathematical problem.

Results of the study indicated that both the cognitive and exemplar group outperformed the control group on the problem-solving test. The cognitive group mean was 4.23, the exemplar group mean was 4.45, whereas the control group mean was only 2.38 with effect size of 1.26 and 1.34, comparing the treatments to the control, respectively. In addition, both the cognitive and the exemplar group outperformed the control group on the problem-solving test that was given 2 weeks after the end of treatment. The cognitive group mean on that test was 4.26, the exemplar group mean was 4.46, whereas the control group mean was 2.44, with effect size differences 1.2 and 1.25, respectively.

Guernon (1989) went one step further by studying the effect of heuristics with an emphasis on metacognitive control. The purpose of his study was to consider whether the problem-solving ability of average eighth-grade students would be enhanced if these students were taught heuristics with an emphasis on what Schoenfeld (1985) referred to as metacognitive control. In Guernon's study, control was defined as the ability of students to monitor when and how certain heuristics would facilitate the solving of a problem. Subjects in this study were 55 eighth-grade students. The students were assigned to three treatment groups: TR1 was taught specific problem-solving heuristics and when and how to use them, TR2 was given various types of problems but were not made aware of the specific heuristics that might facilitate their solution, and CL was not given any problems and served as the control for the experiment. The students were taught in their regular classroom for a period of 16 weeks, and the problem solving was made part of the regular general-mathematics curriculum, with the investigator serving as the students' regular teacher.

All students were administered a pretest and posttest consisting of matched pairs of problems. Results of this study showed that students who were taught specific problem-solving heuristics and when and how to use them (TR1) significantly outperformed both TR2 (students who did not learn heuristic strategies) and the control group ($p < .01$). The TR2 group outperformed the control group ($p < .05$).

At a secondary level, Beach (1985) instructed two groups of high-ability eight- and ninth-grade students: One group was given instruction in a specific heuristics, whereas the control group was instructed to solve problems using an intuitive global approach. Beach's study concluded that students given instruction organized around heuristics scored statistically significantly higher on the test assessing "looking back" and had a greater propensity to use "making a table" and "checking" than did the control group.

Another study, at a secondary level, Ghunaym (1985) gave 4 weeks of instruction to 88 advanced high-school students in the heuristics of pattern discovery, trail and error, working backwards, contradiction, and substitution and in the use of diagrams. This group of students outperformed the control group, which had no such instruction, in a test of nonroutine problems.

At the college level, Schoenfeld (1979) conducted a small-scale research study to determine whether students who received explicit training in the use of five heuristic strategies would be able to use them in solving problems. The subjects in this study were 7 upper-division science and mathematics majors at the University of California, Berkeley. Four of the seven students were randomly chosen for the experimental treatment. Each of the 7 students took a pretest, consisting of five problems. They worked out loud on each problem for 20 minutes. The "data" produced by each student consisted of written work on the problems plus the transcripts of his or her think-aloud protocol.

The control group worked 20 practice problems, then were given written solutions to the problems, and then they listened to tape-recorded explanations of those problems. The experimental group was given a list of the strategies being studied and that list was kept in front of them at all times during the experiment.

Two comparisons of pretest-to-posttest gains, with regard to two different scoring procedures, indicated that the experimental group significantly outperformed the control group. However the results suggested that problem-solving practice, by itself, is not enough. In addition to practice, students need training in using heuristic strategies to improve their problem-solving skills. Second, students can master certain heuristic strategies well enough to use them only on related problems. Last, explicit heuristic instruction does make a difference with regard to problem-solving performance.

Another study conducted at the college level measured problem-solving performance and instruction (Schoenfeld, 1982). The purpose of this study was to examine the effects of a college-level course on problem-solving processes. The subjects were 19 undergraduate students at the beginning and end of the 1980 winter term at Hamilton College in New York. In order to examine the effects of the course, three pairs of tests were developed. Measure 1 consisted of a pair of matched tests (pretest and posttest). These tests were matched according to solution methods. Measure 2 is a qualitative companion to measure 1, examining students' subjective assessments of their problem-solving behavior. It records students' perceptions of how well they planned and organized their work on the problems in Measure 1 and their perceptions of the difficulty of those problems. Measure 3 is a test of heuristic transfer. It consists of a pair of tests (pretest and posttest) that were matching according to solutions methods. These two tests contain subsets of problems that are (a) closely related, (b) somewhat related, and (c) completely unrelated to the problem-solving instructions. Performance on these three categories of problems indicates the degree to which students have generalized their newly learned skills and the degree to which they can transfer them to new situations.

The treatment in this study lasted a month. Both the experimental and control groups met for 18 days. There were 2 1/2 hours of class meeting each day, in addition to 4 to 5 hours of daily-homework assignments. The experimental group consisted of 11 students enrolled in a course called "Techniques of Problem Solving." This course can best be described as a workshop course in problem solving that emphasizes the heuristic techniques. The control group consisted of 8 students with background similar to those of the experimental group. These students were enrolled in a structured-programming course. This course was given concurrently with the problem-solving course and made the same work demands on its students. The control-group students did not study the mathematics problems studied in the treatment course. Instead, the control-group course was designed to teach a structured, orderly way to approach problems. The control group served two purposes. First, it provided some validation of the measures themselves, as a check that the pretests and posttests were of comparable difficulty. The data indicate that the control group did no better on the posttests than on the pretests, so that improvements in the experimental group's are not attributable to difference in test difficulty. Second, it provides a baseline of performance against which the treatment group's scores could be compared.

Results of the study indicated the following: there was statistically significant improvement of the treatment group on problem-solving performance. On the posttest, the experimental group was able to generate one and a half times as many relevant problem-solving strategies than on the pretest. Almost 10 times as many complete problem solutions were obtained on the posttest as on the pretest; average scores on the "best approach" grading increased from 21 to 72%, whereas those of the control group increased from 14 to 24%. Measure 1 provided evidence that students in a problem-solving course can learn to employ a variety of heuristic strategies.

On the measure 2 posttest, the experimental group had a good idea of how to start the problems twice as frequently as they did on the pretest. Similarly, the percentage of solution attempts they rated as "somewhat structured" or better increased from 36 to 70%.

Measure 3 provided evidence not only of heuristic mastery but also of transfer. On the posttest, the experimental group generated two and a half times as many relevant solution suggestions for the problems "closely related" to the course as they did on the pretest (control group performance remained constant from pretest to posttest). On problems "somewhat related" to the course, the number of relevant suggestions jumped from 15 to 38, with some pursuit of the solutions. There was also improvement on problems unrelated to the course (relevant suggestions increased from 13 to 18) and a marginal difference in performance. This result indicates a substantial degree of heuristic transfer tails off as the problems become less and less familiar. Measure 3 provided clear evidence that students can master heuristic strategies and use them in somewhat new situations.

In summary, research studies regarding heuristic strategies suggested a few conclusions. First, the early studies in heuristics provided little concrete evidence that heuristics are powerful tools in teaching problem-solving skills. Second, the more recent studies have indicated that heuristic strategies can be learned and used and, consequently, enhance problem-solving performance. Third, "think-aloud" techniques are very common in heuristic studies. Fourth, most of the studies surveyed were limited to small samples or were performed with groups whose members were not typical (high-ability students or volunteers, for example). Fifth, at the elementary- and middle-school level heuristic strategies can be learned, but they have to be adapted at the language and complexity. Sixth, most experiments that were performed with precollege students were not performed in natural settings. For example, many studies were done at colleges or in the summer when students were not engaged in their regular curriculum. Seventh, at the secondary and college level, problem-solving practice by itself is not enough to improve problem-solving performance and explicit training is required. Eighth, improvement in problem solving is due to learning to use certain problem-solving heuristics with some efficiency and not simply to having worked a lot of related problems. Finally, and most important, students can master heuristic strategies and transfer them to similar situations.

Constructivism

Constructivism is an instructional approach that views mathematics as a personal activity with opportunities for each student to construct his or her own mathematics and views problem solving as a problem-centered rather than teacher-centered (Wheatley, 1991). Wheatley argued that Practicing constructivism means providing students with opportunities to solve problems/do mathematics in order to learn and give meaning to their experiences. In this learning environment, students focus on heuristics rather than answers and expect all solutions and ideas to make sence. Constructivism has become one mode of instruction as a result of emphasis shifted, during the cognitive revolution of the late 1950s, from performance to competence (Bruner, 1986). In the constructivist approach to learning, the instructor, rather than using the explain-practice mode of instruction, establishes settings for meanings and sets activities for students that force reconstructing of ideas at a higher level, Thus, creating a rich learning environment for students' meaning-making.

According to Wheatley (1991), the theory of constructivism rests on two main principles. Principle one states that knowledge is not passively received, but is actively built up by the subject. Simply put, in order for students to gain new ideas and knowledge, they must construct their own meanings. Principle two states that students know their world only through their experiences. Thus, students do not find truth but construct viable explanations of their experiences.

This section of chapter two is divided into three parts. First, the influence of constructivisit thought from Piaget's cognitive-development psychology is presented. Second, the preconstructivist revolution in research in mathematics education beginning in 1970 and proceeding on up to 1980 is covered. Last, studies that show the influence of constructivist thought on the mathematics-reform movement that is currently underway in schools are detailed.

There are several documents that mark the beginning of the influence of constructivist thought on mathematics educators including important books, journal articles, conferences, and events. For instance, an initial document was a report **from** the Woods Hole Conference entitled "*The Process of Education*," by J. S. Bruner (1960). Bruner distinguished between the structures of mathematics (this was the emphasis of mathematics educators in the 1960) and Piaget's genetic structures. This distinction was viewed as an endogenic (mind centered) versus exogenic (world centered) view (Gergen, 1994; Konold & Johnson, 1991). Furthermore, the structure of mathematics was thought to be attained by capacities for reason, logic, or conceptual processing. Mathematical structures were regarded as having a mind-independent existence, and the function of rationality was to come to know these fundamental structures (Steffe & Kieren, 1994). Bruner (1960) defined the capacities for reason and logic of young children within the framework of Piaget's cognitive-development psychology. Based on Piaget's genetic structures, the think-by was that concrete operational children were ready to learn and indeed could learn fundamental structure of mathematics (Steffe & Kieren, 1994). This idea was the foundation for Bruner's concept of readiness to learn the fundamental structures of mathematics: "any subject can be taught effectively in some intellectually honest form to any child at any stage of development" (p. 33).

Bruner' concept of readiness to learn seemed to be quite sweeping at the time. A reason why it may have seemed to make "readiness to learn" a nonissue is that genetic structures were in the main ignored by the developers of mathematics programs. In his report on the School Mathematics Study Group (SMSG) at the Conference on Cognitive Studies and Curriculum Development, Kilpatrick (1964) caught the spirit of the times:

> "In a sense, the mathematicians who have guided the recent curriculum reforms have been waiting to be shown that psychology theories of learning and intelligence have something relevant to say about how mathematics shall be taught in the schools. These reformers (and I speak now not only of SMSG) have been so successful in teaching relatively complex ideas to young children, and thus doing considerable violence to some old notations about readiness, that they have become highly optimistic about what mathematics can and should be thought in the early grades. (p. 129)"

Kilpatrick provided an ad hoc analysis of these features of SMSG that were most harmonious with the results of Piaget's studies, but the common attitude of some curriculum developers, at that time, was that Piaget was an observer rather than a teacher (Goals for School Mathematics, 1963). The thinking was that if Piaget had observed the mathematical thought of children who participated in the modern mathematics programs, he would have realized the elasticity of their cognitive processes.

Piagetian studies. Besides "The Process of Education," the "Piagetian studies" were studies conducted mainly to investigate the influence of the constructivisit thought on mathematics education. Henry van Engen, while working at the Research Center for Cognitive Learning at the University of Wisconsin during the last half of the 1960s (Van Engen, 1971), provided leadership for a series of studies in mathematics education that became known as the "Piagetian studies" (e.g., Adi, 1978; Branca & Kilpatrick, 1972; Carpenter, 1975; Days; Hiebert, Carpenter, & Moser, 1982; Johnson, 1974; Kieren, 1971; Martin, 1976; Mpiangu & Gentile, 1975; Silver, 1976; Steffe, 1970; Taloumis, 1979).

The Piagetian studies are divided into three types: first, studies that were devoted to investigating the readiness of young children to learn mathematics; second, studies that attempted to explain children's mathematical behavior using mathematical structure; third, studies that explored the role of play and manipulatives in mathematics teaching.

The first type of the Piagetian studies was the experiments conducted to investigate the readiness of young children to learn mathematics. There are two basic types of readiness studies--correlational and training. The hypothesis of the readiness studies was that students' abilities for reason or logic could be increased through intensive learning experiences. If the hypothesis was not disconfirmed, then the elasticity of the limits in the cognitive development theory would be falsified as a theory of readiness to learn mathematics.

The first type of readiness research is the training study. Silver (1976) argued that many researchers in psychology and mathematics education have attempted to understand concrete operational behavior by investigating the effect of training experiences on conservation acquisition. The roles of reversibility, cognitive conflict, modeling procedure, verbal rule instruction, perceptual screening in conservation acquisition, and grouping structures have all been investigated.

Adi (1978) investigated the roles of reversibility of thought in equation solving. The purpose of this study was to investigate the relation between the developmental levels of students and their performance on equation solving when different reversible processes are applied. Subjects in this study were 75 college students (preservice elementary school teachers), 6 males and 69 females, and their ages ranged between 18 and 22 years. Subjects were classified according to their performance on a paper-and-pencil Piagetian-related task of keeping a beam balance in equilibrium. Three distinct groups were identified by the task according to their performance on (a) early concrete operations (IIA), (b) late concrete operations, and (c) early formal operations. All students were given a pretest on equation solving. The pretest consisted of a set of five questions. At the end of the treatment, a posttest on equation solving was given to all the students. This test consisted of a set of 12 questions. The test was constructed to measure the student's ability to solve equations by applying (a) reversal or cover-up techniques (inversions) and (b) formal techniques (compensations). The administration of the posttest required 40 minutes. The instructional treatment consisted of five 50-minute sessions on equation solving. Both skills and processes were stressed. All three classes were taught by the investigator. Results of this study indicated the following. First, results of the pretest showed a low performance on equation solving. Sixty-eight out of the 75 students had a score of zero. Second, the results showed that there was a statistically significant positive relationship between the developmental levels of the learners and their performance on equation solving when different reversible processes are applied (inversions and compensations). The computed coefficient of contingency was statistically significant at the .05 level (C= .23).

The hypothesis of this study was that capacities for reason or logic could be increased through intensive learning experience. The results confirmed this hypothesis.

Piagetian Grouping Structures also was investigated in the literature. For example, Silver (1976) pointed out that research information related to the grouping structures has been obtained through the use of two different general strategies. The first strategy has been an investigation of the hierarchical nature of cognitive growth involving an examination of the stages passed through in attaining certain concepts. Examples of research focusing on this strategies are the investigations of Kofsky (1966) into classificatory development and a study by Wohlwill (1960) that researched the development of the number concept. The second general strategy has been the investigation of the integrative nature of cognitive growth involving an examination of the convergence among distinct abilities. The research by Dodwell (1962) into children's understanding of cardinal number and the logic of classes is an example of this second strategy.

A study by Silver (1976) investigated a third general strategy of the grouping structures. The purpose of this research was to investigate the effects of training experiences involving behaviors in one grouping (primary addition of classes) on certain other behaviors (e.g., class inclusion) within the same grouping and also on certain behaviors in other groupings (number and substance conservation in the grouping of multiplication of relations; transitivity in the grouping of addition of asymmetrical relations).

The subjects in this study were 55 lower-class and lower-middle-class children attending first grade at a parochial school in New York City. Materials used during training were 18 "Logic" blocks varying along three dimensions--size, shape, and color. The experiment consisted of three phases: pretesting, training, and posttesting. In pretesting, students were given standard tests for number and substance conservation, class inclusion, and transitivity, consisting of three items for each. After pretesting, the subjects were matched for age and total number of correct judgments, and they were randomly assigned to the training and control groups.

There were two training sessions, each divided into three phases. For each phase of training, a criterion of four consecutive correct responses was used to allow movement to the next phase. The members of the control group had two sessions, during which they made configurations with the blocks and discussed the figures with the experimenter. There was no discussion of attributes during these sessions.

There were two posttests. Posttests I was given one to 3 days following training and was identical to the Pretest. Posttest II was given 3 weeks after training.

The results of this study were as follows: First, 8 students in the training group attained operativity (operative indicates a score of 4 or higher for all posttests) for class inclusion, and 5 subjects attained operativity for number conservation; no subjects showed the reverse trend. The category changes were statistically significant both for class inclusion and for conservation. Training significantly improved scores for class inclusion and for number conservation. For class inclusion, 12 students improved and none regressed; for number conservation, 9 subjects improved and none regressed (sign test, $p < .01$). There were no changes for the control group.

Second, training and control students were matched on the basis of pretest class-inclusion scores, and the trained group scored statistically significantly higher than control mates (Wilcoxon Matched Pairs Test, $p < .005$). A similar analysis was made for number conservation scores. Trained students scored statistically significantly higher than their control mates (Wilcoxon Matched Pairs Test, $p < .005$). Third, training had minimal effects on substance conservation and transivity performance. To summarize, the result of this study has shown that class inclusion could be trained. This finding was supportive of the results found by Kohnstamm (1963).

The second type of readiness research is the correlational study.
These studies correlate different variables with problem-solving performance. The most important variables that affect problem-solving performance are subject and task variables (Days, Wheatley, & Kulm, 1979). Subject variables, such as age, cognitive level, mathematical experience, and gender, may have a significant affect on the processes students use when attempting to solve problems as well as on their ability to obtain correct solutions to the problems. Task variables, such as problem structure, problem context, problem length, magnitude of the numbers, and placement of the question, may affect also problem-solving performance. Thus, the major goals of the correlational studies were, first, to collect data about the relationship between subject variables, task variables, and problem-solving performance in order to provide students

with profitable instruction in mathematical problem solving. Second, study the interaction between subject and text variables to provide information for teachers to use in selecting and sequencing problems in their problem-solving instruction.

A correraltional study was conducted by Days, Wheatley, and Kulm (1979). The purpose of this study was threefold. First, the study attempted to examine the effect problem structure has on the processes and strategies used by concrete- and formal-operational subjects. Second, the study attempted to examine the interaction between problem structure (simple, complex) and cognitive level (concrete, formal) for process use. Finally, an attempt was made to ascertain if both concrete- and formal-operational students found the complex structure problems to be more difficult than the simple structure problems. Subjects in this study were 58 eighth-grade general mathematics students enrolled in 3 junior high-schools in a midwestern city. Each of the 58 students was scheduled for an individual interview (from 35 to 110 minutes in length). In the interview, each student was given an 11-page booklet consisting of instructions and two practice problems, followed by eight experimental problems. Students were instructed to think aloud as they solved the 10 problems. The interviews were audiotape recorded, and a written record was kept of key responses. The protocols were used to determine the processes and strategies employed by the students.

The following results were obtained from the study. First, according to Piaget's theory of cognitive development, the problem-solving processes available to formal students may not be available to concrete students. The results of this study support that claim. The concrete- and formal-operational students differed in the use of production and evaluation processes. For example, the concrete subjects' mean score on the production process was 1.83 with standard deviation equals 1.85. Whereas, the formal subjects' mean score was 4.83 with standard deviation equals .71. In addition, the production and evaluation scores of the concrete and formal subjects differed more widely on the complex structure problems than on the simple structure problems. Days, Wheatley, and Kulm (1979) argued that the effective use of production and evaluation processes on the complex structure problems may have required formal thought, making the effective use of these processes impossible for the students at the concrete operations stage.

Second, problem structure played a greater role in determining process use for formal operations students than for concrete operation students. Also, problem structure had a greater effect on problem difficulty in the formal operations group than in concrete operation group.

The second type of research of Piagetian studies is research conducted to serve as mathematical analysis of Piaget's genetic structure. The major purpose of these studies was to demonstrate logically that basic mathematical structures would serve as well as Piagetian genetic structures as models of the mathematical knowledge of children. The logical analyses generally were followed by an attempt to use the mathematical structures to explain children's mathematical behavior. These studies investigated Piaget's (1952) theory that several logical reasoning abilities may be needed to achieve an operational understanding to certain mathematical concepts.

Hiebert, Carpenter, and Moser (1982) tested Piaget's cognitive development theory on children's solution to verbal arithmetic problems. Specifically, the purpose of this study was designed to investigate the relationship between several Piagetian abilities information processing capacity and first-grade children's performance on verbal addition and subtraction problems. Subjects in this study were 149 first-grade children from three elementary schools. All schools used a modified version of Developing Mathematics Processes (Romberg, Harvey, Moser, & Montgomery, 1974) for their instructional program. At the time of testing, the children had received lessons on solving different kinds of verbal arithmetic problems. Each child was tested three times in an individual interview setting. The first interview consisted of 12 arithmetic problems with smaller numbers, the second interview consisted of 12 large-number arithmetic problems, and the third interview contained the cognitive ability tasks. Each interview lasted about 15 minutes. The cognitive tasks were presented in the following order: length transitivity (no Mueller-Lyer illusion), class inclusion (blocks), backward digit span, number conservation (one row spread), length transitivity (Mueller-Lyer illusion), class inclusion (fruit), and number conservation (one row grouped).

The results of this study show a small but consistent relationship between possession of a cognitive ability and solving an arithmetic problem. For example, the mean number of correct responses on join addition for students who developed one cognitive ability (i.e., number conservation) and zero development level was 2.00 with standard deviation equals 1.37. Whereas, the mean score for those who developed number conservation as well as class inclusion was greater (i.e., 2.44) with standard deviation equals 1.33. Students who had developed a particular cognitive ability performed better than those who had not on all problem types and all problem condition. For example, the mean number of correct responses for students who developed transitive reasoning (developmental level equals 2) on join addition was 2.80 with standard deviation equals 1.15. Whereas, the mean score for students who did not develop this cognitive ability (developmental level equals 0) was 2.50 with standard deviation equals 1.26.

The third type of Piagetian study is research conducted to explore the role of play and manipulatives in mathematics teaching. Manipulative activities are important in learning and understanding mathematical concepts and procedures. Dienes (1967) suggested that a mathematical concept is best developed through the use of multiple concrete and gamelike embodiments. Further, Dienes suggested that the use of such varied materials helps the child to learn to look for patterns and relationships. Biggs (1965) claimed superiority for a multimodel environment over a unimodel environment in promoting learning. Piaget (1967) discussed the role of play among preschool children and indicated that one role for play is in aiding assimilation or extending the class of stimuli that a child can handle with currently available schema. Further, play allows responses in fantasy that the child cannot make in reality and provides for the exercise of learned skills that might otherwise fall into disuse.

Much of the related research has dealt with Cuisenaire rods. The results of numerous studies (Hollis, 1965; Nasca, 1966; Nelson, 1964) have supported Gattegno's (1960) contention that Cuisenaire rods contribute to learning advanced computational skills especially in elementary school.

The functional role of play in learning has been seen as that of increasing the child's response repertoire and of encouraging him or her to seek information (Sutton-Smith, 1967). These findings are supported in part by those of Vance (1969). Vance found that junior-high-school students who engaged in 10 manipulative mathematics laboratory experiences were more highly productive on a free-response test calling for mathematical uses of playing cards than were comparable students without any laboratory experience.

Vance (1969) found other values in the manipulative experience for junior-high students. These students used the object to test hypotheses about mathematical ideas and appeared to like working with manipulative materials. This latter idea finds support in a small interview study reported by Davis (1967). Vance also reported that a guided discovery demonstration was at least as effective as manipulative experience in fostering a higher feeling of independence and more of an experimental attitude toward mathematics than did the guided discovery demonstration.

In summary, the Piagetian Studies were studies conducted to investigate the influence of constructivism thought on mathematics education. The Piagetian Studies are divided into three parts. First, studies devoted to investigating the readiness of young children to learn mathematics and were divided into two parts including readiness and correlational studies. The readiness studies investigated the following hypothesis: "students' abilities for reason or logic could be increased through intensive learning experience." The correlational studies correlated different variables such age and cognitive level with problem-solving performance. Second, studies attempted to explain children's mathematical behavior using the mathematical structure. Third, studies explored the role of play and manipulative in mathematics teaching.

Current constructivist theory. The preconstructivist revolution was marked by a reformulation of the understanding of mathematics educators of Piaget's genetic structures. Mathematics researchers finally came to understand Piaget's genetic structures as models that he made to explain his observations of children's ways and means of operating rather than a hypothetical-deductive system. Furthermore, this revolution was a result of educator's struggle to use Piagetian theory in mathematics education. This struggle led researchers to the conclusion that they need to make their own models to serve their own educational purposes rather than using and relaying on Piaget's theory. The long-lasting effects of this observation can be seen in contemporary constructivist research in which the researchers seek to observe and describe mechanisms that children and indeed students of any age use as they, individually or interactively, build up mathematical knowledge in a particular learning space (Pirie & Kieren, 1994; Pothier & Sawada, 1983; Steffe & Wiegel, 1994; Thompson, 1994).

An example of the "contemporary constructivist research" is a study conducted by Pothier and Sawada (1983). The purpose of this study was to trace the emergence and differentiation of the process of partitioning as revealed in children's attempts to subdivide a continuous pie into equal parts. Subjects in this study were 8 kindergarten, 8 grade one, 12 grade two, and 15 grade three children. Each subject was asked to demonstrate how he or she would cut a cake into equal pieces. Specifically, students were requested to partition the cake for 2 people. In most cases, the subsequent partitions were for 4, 3, and 5 people. The older children were asked to make more partitions than the younger ones. In addition to video and audio recording, any distinctive behavior observed or impressions made were recorded in written form during the sessions.

Results of this study revealed that a child first learns to partition in two; then, with the acquisition and eventual mastery of having the algorithm, in powers of 2; then, with the use of geometric motions, in even numbers. Partitioning in odd numbers follows the learning of a first move other than a median cut. With the discovery of the new first move, children are able to partition in thirds, fifths, and other odd numbers. The algorithm involves counting, and equality of parts is usually achieved by rotational (for circular shapes) and translational (for rectangular shapes) moves. These results are in substantial agreement with the results of Piaget, Inhelder, and Szeminka (1960).

Steffe and Kieren (1994) argued that the separation between the practice of teaching and the practice of research was the major factor that paved the way for the emergence of constructivism in mathematics education. Moreover, Erlwanger (1973) was able to demonstrate how Benny (a child who participated in the program Individualized Prescribed Instruction (IPI) produced by the Pittsburgh Research and Development Center) did not have any understanding or any "common sense" of what it meant by "good mathematics." This was a crucial aspect of Erlwanger's work, because by demonstrating what a "common sense" view of mathematics should not be, Erlwanger was able to falsify the behaviorist movement in mathematics education at that very place where behaviorism has its greatest appeal-at the level of common sense (Steffe & Kieren, 1994). In his study, Erlwanger focused on the mathematical thinking of an individual child, interpreted that thinking in a constuctivist framework, and was able to demonstrate an understanding of mathematics education different from empiricism. Moreover, Erlwanger tried to show in detail how Benny "made sense" of his experiences in IPI. Erlwanger's work was the first to focus on both the dynamics of an individual, as interpreted from the actions and words of Benny, and on the interactional dynamics between Benny and the ways in which the IPI environment occasioned his actions. In this, the IPI environment was changed by Benny through his actions and through his interactions with others in this environment. Kieren and Steffe (1994) pointed out that both types of dynamics became the hall-mark of later constructivist research in mathematics education.

Cobb and Steffe (1983) argued that the constructivist researcher needed to be a teacher as well as a model builder, which pushed research methodology beyond the clinical interview. In a teaching experiment, it is the mathematical actions and abstractions of children that are the source of understanding for the teacher-researcher. Second, it is the children's way of making sense that determines their own knowledge. Third, using "conservation" and mathematics performance as variables does not provide a way of seeing how children build up mathematical knowledge. Fourth, children with different developmental backgrounds may well be able to get the same answers on an arithmetical task, but the ways in which they do so might differ significantly.

Constructivism and mathematical performance. One major type of constructivist research are studies dealing with cooperative learning settings and their impact on mathematical learning and performance. For instance, a study conducted by Yackel, Cobb, and Wood (1991) examined the use of small-group problem solving as a primary instructional strategy. This study was a 3-year research and development project that investigated the mathematical learning of 20 second-grade students from a public school as they attempted to complete educational activities compatible with constructivist learning theory in general

von Glaserfeld (1984). In this study, children were paired for small-group problem solving until both the teacher and students were accustomed to working in groups.

Each class session had three components. First, the teacher introduced the activity to the class. This activity was limited to establishing the symbols and conventions used to present the tasks and occasionally to having the class work out an example together. It was never the teacher's intent to show the students a procedure for completing the activities or to explain how to do them. After the introduction, students worked in small groups to solve a certain problem assigned by the teacher. Finally, students were engaged in a whole-class discussion for about 20 to 25 minutes.

In this study, the role of the teacher was to initiate and guide the following mutual construction of social norms. First, students cooperate to solve problems. Second, meaningful activity is valued over correct answers. Third, persistence on a personally challenging problem is more important than completing a large number of activities. Last, partners should reach consensus as they work on the activities.

Results of this study indicated the following. First of all, students in this study developed both social autonomy, taking responsibility for their conduct, and intellectual autonomy, taking responsibility for their own learning. Second, students were high on task-orientation and low on ego-orientation with respect to mathematics, when compared with other second graders. Third, students rated higher than the other children on their belief that cooperation is important in learning mathematics and rated lower than when on work avoidance and the belief that being good at mathematics is due to superior ability.

Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti, and Perlwitz (1991) conducted a similar study using problem-centered learning approach as a model of instruction. The purpose of this study was to assess a project that utilizes a problem-centered approach in a second-grade class. Subjects in this study were students attending three schools that contained both project and nonproject classes. The number of children in both project and nonproject classes ranged from 15 to 20 students. Total number of students were 187 project students and 151 nonproject students. The project lasted one year in which instruction was generally compatible with a constructivist theories of knowledge and recent recommendations of the National Council of Teachers of Mathematics (NCTM, 1989).

Instruments designed to assess student's mathematical achievement, computational proficiency, and their beliefs and motivation about mathematics were administered to project and nonproject students at the end of the school year. In addition, a questionnaire on pedagogical beliefs was given to each teacher.

Analyzing the data indicated the following results. First, project students developed a higher level of reasoning in arithmetic than nonproject students, but the project students were less familiar with the idiosyncratic textbook conventions of traditional elementary-school students. Second, the project students' ability to perform computational tasks was similar to nonproject students on all arithmetic instruments. Third, project students were statistically significantly less ego-involved than the nonproject students.

Constructivism research is also involved with studying the impact of computer on learning mathematics. For example, several researchers have provided evidence that working with a Logo environment (Papert, 1980) provides a rich and accessible framework for doing mathematics (Hoyles, Sutherland, & Evans, 1985; Noss, 1985; Papert, Watt, diSessa, & Weir, 1979).

As an example of using Logo as a primary mode of instruction is a study conducted by Noss (1987). The purpose of this experiment was to investigate some elements of geometrical concepts that children learn through Logo programming. Specifically, this study aimed at studying the effects of Logo experience on children's understanding of two geometrical concepts: length and angle. Subjects in this study were 118 pupils aged between 8 and 11 years. These students were distributed among five classrooms: one Grade 3, one Grade 4, and three grade 5. The treatment lasted for one school year.

In this study, each class was equipped with one computer, a printer, and a floor turtle. The teachers of each class taught the pupils throughout the school day and accommodated the Logo work within the curricular activities of each classroom. The students programmed in pairs for about 75 minutes per week and chose their own projects within loosely structured classroom environment. The approach adopted for the geometry study was to compare the Logo students' performance on specific geometrical tasks with that of students who had not studied Logo. A test was constructed to evaluate the students' understanding of components of the concepts of length and angle. This test was administered to both treatment and nontreatment students.

The following results were obtained. First of all, with regard to "*length conservation*" , the data indicated that the performance of the Logo students was better than the non-logo students (39% compared to 29%). Second, results from the "*length measurements*" indicated a modest and nonsignificant trend in favor of the Logo group (44% as against 37%). Third, analyzing data with respect to "*angle conservation*" indicated a significant effect ($p < .05$) in favor of the Logo groups emerged (81% of the

Logo students responded correctly compared to 66% of the non-Logo students). Fourth, with regard to "*angle measurement*" results indicated that there was a statistically significant main effect in favor of the Logo student: 69% of the Logo students responded correctly, compared to 42% of the comparison students. To sum up, the findings of this study indicate a trend toward a positive effect of the Logo work on two components of the concepts of length: (a) length conservation and (b) the concept of a unit of measurement.

In this study, the Logo turtle appeared to offer an opportunity for children to base their mathematical activities on their existing conceptions and intuitions and at the same time it provided a context in which formalization appears both natural and meaningful. The essence of the turtle geometry microworld was that the learner's attention was precisely focused on the important aspects of the environment. For the turtle, these commands were the ideas of length and angle, or the commands FORWARD and RIGHT (Noss, 1987).

A study utilizing a clinical teaching experiment was conducted by Behr, Wachsmuth, and Lesh (1984) to investigate students' understanding of the order and equivalence of rational numbers. Subjects in this study were 12 fourth-grade students. The treatment consisted of 13 lessons, and lasted 18 weeks. Subjects received instructions on five topics: naming fractions, equivalent fractions, comparing fractions, adding fractions with the same denominators, and multiplying fractions.

Each subject who participated in this study was interviewed individually every 8 days during the 18-week instructional period. Each interview was audiotaped or videotaped and later transcribed. The children's responses were coded in a matrix. The matrix were examined for patterns in the children thinking.

Results of this study indicated that there were four common features across the fractions classes in the strategies used: (a) thinking that demonstrates attention to both numerator and the denominator of each fraction, (b) thinking that depends on manipulative aids, (c) thinking that compares the fractions to a third fraction or whole number as a reference and (d) thinking influenced by one's knowledge of whole numbers.

Second, the results indicated that most children by late in the fourth grade are able to develop adequate thinking to deal with questions of the order and equivalence of fractions.

Trowell and Wheatley (1995) conducted a study at college level that utilizes the problem-centered learning model proposed by Wheatley (1991). The focus of this study was a problem-solving course for undergraduate mathematics education majors in which problem-centered learning was used as an instructional strategy. The purpose of this study was to examine the negotiation of social norms and how learning environments which promote meaning making are fostered (Trowell, 1994). In this study, negotiation is defined as a mode of communication in which individuals' attempt to make sense of each others' statements.

Subjects in this study were enrolled in a class titled "Problem Solving in Mathematics." This class was different from a conventional mathematics course where the instructor lectures. Instead, students solved problems without any prescribed methods having been shown and then they presented their potential solutions to the class for discussion. Furthermore, the students became responsible for determining the viability of their solutions rather than depending on the teacher to tell them whether they were right or wrong. The goal of the course was for students to learn to solve nonroutine problems, drawing on algebra, geometry, number theory, calculus, and probability. In this class, mathematics was viewed as a personal activity with opportunity for each person to construct their own mathematics (Wheatley, 1991).

Each of the class sessions was video recorded. Immediately following each class session, the instructor shared his ideas, reflections, and thoughts concerning the class in video recorded sessions. Four students from the class were also chosen to be interviewed regarding their experiences in the problem solving class.

Three important results were obtained from this study. First of all, it was found out that the instructor should listen carefully to students in attempt to infer their mathematical constructions and beliefs. Second, a class has the potential to form an intellectual community. In other words, students in this class began to act autonomously in the learning environment. For example, students could share and extend their ideas freely and spontaneously. Students worked in an environment of collaboration rather than competition. Students questioned each other with an attitude of curiosity as they attempted to make sense of mathematical tasks being described by others. Third, Trowell and Wheatley claimed that at the end of this course, most students were able to solve more difficult and complex mathematics problems than the beginning of the treatment. No numbers were given, however, to support this claim.

At the secondary-school level, a problem-centered learning curriculum that utilizes the constructivism approach is the Interactive Mathematics Program (IMP). IMP's 4-year program of mathematics problem solving that replaces the traditional

Algebra I, Geometry, Algebra II/ Trigonometry, and Calculus sequence. IMP integrates algebra, geometry, and trigonometry with probability and statistics and uses technology in order to enhance students' understanding.

IMP curriculum was designed based on the two principles of constructivism. Recall, these principles are knowledge is not passively received but actively is built on the subject, and Students know their world only through their experiences.

In IMP, students gain their mathematical ideas and knowledge by actively working in small groups to find solutions and answers to new problems given by their teacher. Then, they communicate their findings to the class. Doing the mathematics and communicating the mathematical ideas enable IMP students to construct not only their knowledge, but also the viable explanations of their experiences.

Several studies were conducted to evaluate the IMP program and students and focused on non-referenced, standardized tests. For example, a comparison study conducted to compare IMP students' performance on the SAT test with Algebra students in a selective college preparatory school (Webb et al., 1993). Results of this study have shown that IMP students taking the SAT test had a higher mean mathematics score (545 compared to 531) than Algebra students, even though a higher portion of IMP students took the test (83% compared with 72%) and both IMP and Algebra students had comparable pretest results. Furthermore, of students taking the test, IMP students also had a higher percentage doing "very well" (600 or higher) than the Algebra students.

An evaluation study conducted by the Wisconsin Center for Educational Research (Webb et al., 1993). Subjects in this study were 1,121 (53 % female and 47 % male) students in three high schools. Results of this study indicated that only 60% of U.S. high school students graduating in 1993 completed 3 years of college-preparatory mathematics. In contrast, 90% of 1993 IMP graduates completed three years of the IMP college preparatory sequence. This result was statistically significant at the .01 level.

More research is needed to determine the impact of problem-centered curricula that utilizes the constructivism model of learning such as IMP on mathematics problem-solving performance. Research needs to be conducted to find out if IMP is an effective environment for teaching problem solving. The review of literature indicated that there are not sufficient studies of a successful high-school mathematical curricula for teaching problem-solving skills. Although it seems reasonable that IMP may accomplish this goal, students who are taught with IMP curriculum have never been measured explicitly regarding their problem-solving abilities.

Attitudes

Learning mathematics is a cognitive endeavor. Yet, in mathematics, students' attitude plays a significant role in learning. For instance, Reyes (1984) pointed out several ways affective variables are related to mathematics learning. Reyes argued that it is likely that a student who feels very positive about mathematics will achieve at a higher level than a student who has a negative attitude toward mathematics. Reyes also argued that a high achiever will enjoy mathematics more than a student who does poorly in mathematics. More important, Macleod (1992) discussed that affective issues play a central role in mathematics learning and instruction for both teachers and students. Macleod emphasized that if research on learning and instruction is to maximize its impact on students and teachers, affective issues need to occupy a more central position in the minds of researchers. Macleod firmly believe that all research in mathematics education can be strengthened if researchers will integrate affective issues into studies of cognition and instruction.

Research on attitudes has a relatively long history (Macleod, 1992). Many of the research studies use attitudes as a general term that includes beliefs about mathematics and self (Simon, 1982). Moreover, in the educational and psychological literature, the affective domain is characterized in different ways. Attitude, in this study, is defined as a mental position or feeling with regard to mathematics as a field of study. Dislike of geometric proof, enjoyment of problem solving, and preference for discovery learning are examples of how attitude is used in mathematics education.

Macleod (1992) pointed out that attitudes toward mathematics appear to develop in two different ways. First, attitudes may results from the automatizing of a repeated emotional reaction to mathematics. For example, if **a** student has repeated negative experience with solving equations, the emotional impact will usually lessen in intensity over time. Eventually the emotional reaction to solving equations will become more automatic, there will be less physiological arousal, and the response will become a stable one that can be probably be measured through use of a questionnaire. The second source of attitude is the assignment of an already existing attitude to a new but related task. A student who has a negative attitude toward geometric proof may attach that same attitude to proofs in algebra. To phrase this process in cognitive terminology, the attitude from one schema is attached to a second related schema (Abelson, 1976; Marshall, 1989).

Macleod (1992) argued that there have been a large number of studies of attitudes toward mathematics over the years. There is a shortage, however, **of** research studies that emphasiz the impact of the reform, nontradional mathematical curriculum, such as IMP on the attitude of high-school students. This section covers review of literature that study how students' attitudes effect their mathematics achievement and problem-solving learning in both traditional and reform-mathematics classrooms. More specifically, this section is divided into 3 parts. First, research studies that reveal students' attitudes and beliefs toward mathematics are examined. Second, research studies that showed the effect of attitude on mathematical problem-solving performance are detailed. Last, research studies that examine the effect of the curriculum and instructional method on both mathematical achievement and attitude toward mathematics are reviewed.

The first part of this section reviews research studies concerning students attitudes and beliefs towards mathematics. An example of a research regarding students attitudes and beliefs about mathematics is a study conducted by Schoenfeld (1989). The purpose of this study was to examine the ways the students' conceptions of mathematics shape the ways that they engage in mathematical activities. The study emphasized the relationship between students' understandings about the nature of deductive proof in plane geometry and other geometric endeavors. Subjects of this study were 230 students (112 female, 118 male) enrolled in high school mathematics courses. A questionnaire containing 70 closed and 11 open questions was developed for the study. Sections of the questionnaire dealt with the students's attributions of success or failure; their comparative perceptions of mathematics; their view of mathematics as a discipline; and their attitude toward mathematics. The questionnaire was distributed to the students by their teachers during the last two weeks of the school year. The questionnaire took 20-25 minutes to complete. The mean and standard deviation for each question were calculated.

Results of this study indicated the following. First, the students consider mathematics to be an objective, and objectively graded, discipline that can be mastered. They believe that it is work and not good luck that accounts for good grades, and they place much more emphasis on work than on inherent talent. If the students do poorly, they believe it to be their own fault. Students believe that teachers attitudes toward their students are not considered to be factor in grading. In addition, students believe that they were expected to master the subject matter, by memorization, in bite-size bits and pieces. For example, to stress the importance of memorization, one student wrote as an answer to one of the questions, " You must know certain rules, which are a part of all mathematics. Without knowing these rules, you cannot successfully solve a problem." Another student wrote the following, " Memorizing is very important, and in geometry, especially for the final exam, because I am required to write proofs from memory. Memorization of equations and formulas are essential in mathematics. If you memorize those then you plug in your variable and solve what you' re solving for."

Second, the students enjoyed their successes and liked solving problems and felt it was beneficial. Students were aware of the connections between doing deductive mathematics and constructing things (e.g., angles, segments, etc.). For example, students wrote that constructions have little to do with other things in geometry like proofs and theorems.

Corbitt (1984) conducted a study to examine students' beliefs and feelings toward mathematics. The purpose of this study was to characterize the students' beliefs and feelings about mathematics. In specific, this study was focused on student's liking for mathematics and how important they perceived the subject to be. The subjects involved in this study were fifty eighth-grade students in a middle school. Two instruments were used in this study including a questionnaire and individual interviews. Students rated the relative importance of five school subjects including mathematics, science, social studies, English, and physical education (PE) and their preferences among these subjects on a written form, then answered follow-up questions during individual interviews. The purpose of these interviews was to allow students to elaborate on their feelings. Forty-eight students ranked mathematics as the most important subject. Thirty-two students cited the everyday usefulness of mathematics as justifying importance of mathematics. Students ranked PE as their most favorite subject and ranked mathematics right after PE. Most students gave two reasons for liking mathematics. First, students like mathematics if they are good at it. Second, the mathematics teacher influences whether or not mathematics is liked. Other results in this study showed that 40% of the students found mathematics enjoyable while 0% found it unenjoyable, 30% of the subjects found mathematics interesting while only 4% of these students found it boring, and only 2% found it fun. 98% of the subjects enjoyed playing mathematical games and 53% of them found these games important. The majority of students disliked memorizing rules and formulas and ranked solving mathematical puzzles and playing mathematical games as something important.

A study by Stodolsky, Salk, and Glaessner (1991) studied the students' views about mathematics. The purpose of this study was to describe an interrelated set of attitudes, perceptions, and dispositions that students hold about mathematics and social studies. The subjects in this study were 60 fifth-grade students; 28 boys and 32 girls. These students come from six schools located in the metropolitan area of a large midwestern city and from 11 classrooms in four public and two private schools. Students were all fluent in English and their socioeconomic backgrounds ranged from upper-working class to upper middle class. Students were interviewed over a 2-year period. Each interview took between 30 to 40 minutes and conducted by one of two graduate students, each with experience as a classroom teacher and as interviewer. All interview questions were asked regarding one school subject and then repeated for the other. In alternating order students were interviewed about mathematics or social studies. The interviews were tape-recorded and transcribed. Each interview was coded by at least two researchers. Across all items, a coding reliability of 91% was established on four randomly selected protocols.

Results of this study indicated that students' views regarding mathematics and reading are similar in rank and are liked better than science and social studies. Mathematics is placed as the most important subject among school subjects. In term of easiness, students rated mathematics along with social studies and science as the most difficult subjects. In addition, the majority of the students defined mathematics in terms of the basic arithmetic operations and as dealing with numbers. 30 percent of students added fractions and decimals along with basic arithmetic operations as they defined mathematics. A smaller number of responses defined mathematics as measuring, doing problems, geometry, counting, and telling time.

The second part to be reviewed focuses on the effect of students' attitudes and mathematics anxiety on their problem-solving achievement and performance. Mathematics anxiety and its relationship to achievement is emphasized because mathematics anxiety has been found to be related to mathematics achievement. Studies have shown that high achievement in mathematics is related to low anxiety for students from grade school through college (Aiken, 1970, 1976; Callahan & Glennon, 1975; Crosswhite, 1972; Hendel, 1977; Richardson & Suinn, 1972; Szetela, 1971).

Aiken (1970) argued that the assessment of attitudes toward mathematics would be of less concern if attitudes were not thought to affect performance in some way. Neale (1969) pointed out that the relationship between attitudes and performance is the consequence of a reciprocal influence, in that attitudes affect achievement and achievement in turn affects attitude.

Anttonen (1968) reported moderate correlations of mathematics attitude scores with mathematics grade-point averages and standardized test scores in eleventh and twelfth graders. Achievement was also greater for students whose attitudes had remained favorable or had become favorable since elementary school. Brown and Abell (1965) clearly demonstrated that the correlation between pupil attitude toward a subject and achievement in that subject was higher for arithmetic than for spelling, reading, or language.

Degnan (1967) compared the attitudes and general anxiety levels of 22 eighth-grade students designated as low achievers in mathematics with those of 22 eighth-grade students designated as high achievers in mathematics. Dutton's (1962) scale was the measure of attitudes; The Children's Manifest Anxiety Scale (Castaneda et al., 1956) was the measure of general anxiety. Results of this study indicated the following. First, it was found that the achievers were generally more anxious than the underachievers. The achievers, however, had more positive attitudes toward mathematics than the underachievers. Second, when students were asked to list their major subjects in order of preference, the achievers gave mathematics a significantly higher ranking than the underachievers.

Reynolds and Walberg (1992) attempted to study the effect of nine factors on mathematics achievement and attitude. These factors include student ability, motivation, age (aptitude), quantity and quality of instructions, classroom climate, home environment, peer group, and exposure to media outside of school including television. Subjects in this study were 3,116 seventh-grade public school students. Forty-eight percent of the participants were girls, and 52% were boys. The majority of the students were white (62%), followed by Hispanic (17%) and then black (13%).

Data were collected 3 times. First, at the beginning of grade 7. This wave of data included 3 sets of mathematics items on skills and knowledge, routine application, and problem solving. These items were multiple-choice items that measured measurements, numbers and operations, functions and algebraic expressions, and geometry. Also collected self-reported student background data, and information about motivation, peer environment, and mathematics attitude.

The second wave of data, collected at the end of grade 7 in student and teacher surveys and parent interviews, provided information on students on students' home resources, exposure to out-of-school reading, and classroom and instructional factors. Telephone interviews were conducted to obtain data on parents and their children, including educational attainment and parents' expectations for their children's school success. Students furnished data on home resources. The last set of data was collected at the beginning of Grade 8. This set of data repeated much of the first wave including the achievement test. Two-stage testing, however, was implemented in which three tests, tailored to students' performance level, were administered. It took about a year (from fall 1987 to fall 1988) to collect all the data.

Results of this study indicated the following. First, there was a strong relation ($r = .727$) between prior achievement and later achievement as well as a moderate relation ($r = .501$) between prior attitude and later attitude. Correlates with mathematics attitude were in the .30 range, including instructional quality and motivation.

Second, home environment had relatively low correlations with Grade 7 and Grade 8 attitude. Third, achievement and attitude had a relatively low correspondence ($r < .20$), and this relation was stable from grade 7 to grade 8. In addition, prior attitude did not influence attitude one year later. Fourth, two factors, prior attitude and instructional quality as perceived by students, were the only significant direct effects on Grade 8 mathematics attitude. The influence of instructional quality indicates that perceptions of instructional quality may play a role in improving attitude in mathematics. Instructional time, however, had no influence on attitude. This result suggests that attitude may be more related to features of the classroom context than to coverage.

Hembree (1990) did a meta-analysis about the nature, effects, and relief of mathematics anxiety. More specifically, Hembree's study attempts to answer the following questions:

1. Is there a causal direction in the relationship between mathematics anxiety and mathematics performance?

2. Does test anxiety subsume mathematics anxiety?

3. Are behaviors related to mathematics anxiety more pronounced in females than males?

This meta-analysis integrated the results of 151 studies by using meta-analysis to understand mathematical anxiety. The studies include 49 journal articles, 23 ERIC documents, 75 doctoral dissertations, and 4 reports in other sources. One hundred and twenty two studies conducted at the college level, 74 studies conducted at the high-school level, and 19 studies conducted at the elementary school level. Because studies differed across a board range of properties and features such as school grade level, ability levels, and quality of research designs, their findings seemed likely to vary. Meta-analysis , however, draws strength from its capacity to identify interactions and relationships among properties of studies and their outcomes. In this analysis, study properties such as grade level (k-12, postsecondary) and ability level (low, average, or high) were coded as independent variables, with outcomes treated as dependent variables. Then, interactions were explored at the time of data analysis. In this study, independent variables included: grade level (K-12, postsecondary), ability level (low or high), socioeconomic status (low, middle, upper), ethnicity, instrument used for mathematics anxiety, length of treatment, and research design quality (1 = poor to 3 = excellent). Dependent variables were performance correlates, attitude correlates, and avoidance behavior.

The following results were obtained. First of all, higher mathematics anxiety was slightly related to lower IQ levels. For example, at the sixth-grade level, a negative correlation was reported between IQ test and mathematics anxiety ($r = -.17$). Correlations between mathematics anxiety and aptitude/ achievement measures were inverse across grade levels, so higher mathematics anxiety consistently related to lower mathematics performance. For example, a negative correlation was recorded at eleventh grade between achievement and mathematical anxiety ($r = -.34$). Grades in mathematics courses seemed depressed in relation to anxiety by about the same proportion as the students' test scores.

For instance, the mean correlation of mathematics anxiety and grade in mathematics course at high school was found to be - .30.

Second, positive attitudes toward mathematics consistently related to lower mathematics anxiety, with strong inverse relations observed for an enjoyment of mathematics and self-confidence in the subject. For instance, the mean correlation for the mathematics anxiety and enjoyment of mathematics, at grades 5 through 12, was equal to -.75 and statistically significant. Relationships seemed weaker at postsecondary levels. Small correlations were found between mathematics anxiety and desire for success. High-anxious students viewed parents and teachers attitudes as somewhat negative toward mathematics. These relationships were smaller at postsecondary levels.

Third, high-anxious students took fewer high school mathematics courses and showed less intention in high school and college to take more mathematics. A statistically significant gender difference appeared in junior and senior high school. Males with higher levels of mathematics anxiety appeared less likely than high-anxious female to take more mathematics.

Green (1990) conducted a study to examine test and mathematics anxiety and their relationships to achievement in a remedial mathematics class. Subjects in this study were 132 undergraduate students. The setting for the study was the mathematics laboratory of the Center for Academic Reinforcement (CAR) program. The mathematics laboratory services the needs of students with inadequate mastery levels of mathematics skills for college courses. In this study, the classes are taught using combinations of lecture/discussion/practice. Students are enrolled to the program based on their score on the Scholastic Aptitude Test (SAT) and/or the recommendation of the student's high school counselor.

The data for this study were collected by two published anxiety research instruments (Test Anxiety Scale and Mathematics Anxiety Scale) and six CAR-Mathematics department examinations. The students were divided into three groups. The first group of students, the specified-comment group, received their graded test papers marked with a numerical score, a letter grade, and additional teacher comments (which were designed in advance) for each of the possible letter grades (i.e., A, B, C, D, and F). Examples of these comments include "A"-- excellent! keep it up, or "D" Let's bring this up. The second group, the free-comment group, received their test papers marked with the numerical score and the letter grade, along with whatever encouraging comments the teacher believed appropriate for the students' circumstances. For example, teachers used the following comment if the student received an "F" grade on his or her test " Make an appointment to see me". The third group is the "no comments" group. This group received their test papers with only the numerical score and letter grade marked.

All students were given a pre- and posttest. The pretest measured test anxiety. The same test was given at the end of the semester along with a mathematics anxiety test. Mathematics performance, the dependent variable, was operationalized as the course grade. The other dependent variable was test anxiety. The independent variables were mathematics anxiety, test anxiety, and teacher feedback.

Results of this study indicated the following. First of all, test anxiety has a greater effect on the mathematics achievement of remedial students than either mathematics anxiety or teacher comments. The free comments treatment was superior to the specified and no comments treatment in facilitating student performance. The specified comments treatment was second best in facilitating students' test performance. Third, teacher feedback in the form of free comments and specified comments are more facilitative of test performance than no comments. Teachers comments are very effective instructional tools when applied in a systematic manner.

Bessant (1995) investigated the relatioships between of various types of mathematics anxiety and attitudes towards mathematics, learning preferences, study motives, and strategies. Subjects in this study were 173 university students enrolled in one of three introductory statistics courses. All data collected in the first two weeks of the semester using a questionnaire.

Results of this study indicated the following. First of all, mathematics anxiety is associated with reading, studying, thinking about, and using a wide range of mathematical skills. Second, there is a negative relationship between Mathematics enjoyment and anxiety. Similarly, enjoyment of problem solving is negatively related to problem-solving anxiety. Positive orientation to mathematics, however, can reduce reactivity to anxiety-producing stimuli, or vice versa, low levels of anxiety can facilitate the development of attitudes favoring mathematics. Conversely, guided learning is positively correlated with general anxiety, but more so among moderate and high math-anxious learners. Third, favorable attitudes toward the technical applications of mathematics do not interact in a simple linear fashion with anxiety. Fourth, in some circumstances, high anxiety levels can outweigh the facilitating effects of mathematics enjoyment and valuation.

The last part of this section reviews the impact of the curriculum and instructional method on achievement and attitude. Few studies were conducted to compare the traditional curriculum that encouraged rote memory to the nontraditional mathematics curriculum that emphasized meaningful and discovery learning with regard to the effect of the students' attitude toward mathematics on their mathematics achievement. In a discussion of a variety of unpleasant experiences in the grades that cause students to avoid high-school mathematics, Bernstein (1964) apparently concerned with Wilson's conclusion that mathematicians and teachers are almost universally agreed that rote learning procedures are a major factor in producing negative attitudes toward mathematics. Collier (1959) maintained that teachers should emphasize computational speed less and place more stress on developing mathematical understanding and logical reasoning ability.

Clark (1961) suggested that reliance on rote memory rather than logical reasoning is a consequence of the assignment of formal arithmetic at too early grade. In his opinion: " Children are often confronted in school with situations which few adults would tolerate. Day in and day out there is a repetition of meaningless expressions, terms, and symbols. Eventually, many children come to dislike arithmetic. Lack of understanding and skills is associated with personality maladjustment and delinquent behavior, including truancy and incorrigibility (p. 2)."

In a study of fourth-grade pupils in a Gerogia school, Lyda and Morse (1963) noted positive changes in attitudes toward arithmetic and significant gains in arithmetic computation and reasoning when a "meaningful method" of teaching the subject was employed. The method emphasized the mathematical aim of arithmetic: stressing the concept of number, understanding of the numeration system, place value, the use of fundamental operations, the rationale of computational forms, and the relationships which make arithmetic a system of thinking.

Another way that has been suggested for making arithmetic more meaningful, or at least more interesting, is televised instruction. Kaprelian (1961) administered a questionnaire to 65 fourth-grade pupils to obtain their reactions to the television program " Patterns in Arithmetic." Over 90% of the pupils approved of the program to some extent, and over 75% said that they liked arithmetic better after viewing the new arithmetic television program. Finally, 75% of the pupils stated that their attitudes toward arithmetic had changed because the television program helped them to understand the subject.

Ellington (1962) found out high school students in college preparatory classes had somewhat more positive attitudes towards mathematics than students in terminal or general selection classes.

An example of a study that look at the relationship between anxiety, teaching method, and their interaction to mathematics achievement is a study conducted by Clute (1984). This study hypothesized that college students with low mathematics anxiety would perform higher on a mathematics achievement test when taught using a discovery approach, whereas students with high anxiety would find an expository approach more conducive to learning.

Subjects in this study consisted of 81 college students (38 males and 43 females) enrolled in a mathematics survey course. Three instruments were used in this study. The first instrument used was the Mathematics Anxiety Rating Scale (MARS). This instrument was used to determine the level of anxiety. MARS included 98 items presented an anxiety-arousing situation; the student decided the degree of anxiety aroused using five categories ranging from very much to not at all. The second instrument used in this study was the University of California and the California State University Mathematics Test. This test assessed student's ability to handle algebraic computations of the type found in high school mathematics courses. This test was used as a pretest. Third, The Mathematics Achievement Test. This test was a multiple-choice test that was developed by the researcher to measure how well the students acquired the course content.

Based on the MARS scores, students were randomly assigned to one of two treatment groups, *direct instruction discovery* or *direct instruction expository*. The direct instruction discovery approach is described as follows. The class started by the teacher asking students simple questions. Then the students were given a problem to work on. The teacher acknowledged correct answers and responded to incorrect answers by asking related questions, which indicated to the students that an answer was needed. The teacher worked the questions closer and closer to the solution of the problem until the students suddenly " discovered the answer." The direct instruction expository method is described as follows. First, the teacher introduced the lesson, then presented the concept of the lesson followed by some examples. The last portion of the class was an assignment for the students to work on as homework.

All students were given a pretest. Then were assigned to one of the two treatment classes in this study. Each treatment lasted for 3 hours a week for 10 weeks. At the end of the treatment, a 3-hour final examination was administered to all students.

The results of this study indicated the following. Students with a high level of mathematics anxiety had significantly lower achievement than students with a low level of anxiety. For example, the discovery group with a high anxiety level had a mean score of 225.38 (out of 400 points) with standard deviation equals to 59.09, whereas the mean score for the discovery group with low anxiety was 352.50 with standard deviation equals to 27.31. A significant interaction between method of instruction and level of anxiety suggested that students with high anxiety benefited more from the expository approach, whereas students with low anxiety benefited more from the discovery approach. This result reinforces suggestions concerning the importance of considering anxiety level in planning the program to be used in teaching mathematics.

Madsen and Lanier (1992) conducted a study to determine the effect of conceptually oriented instruction on students' computational competencies and their attitudes towards mathematics. Subjects in this study were students who were enrolled in 4-general mathematics classes. Each class consisted of 24 to 30 ninth-grade students. This study was part of a four-year project which was implemented to improve the curriculum and instruction in general mathematics classes. The focus of this project was to teach mathematics for conceptual understanding.

Students were divided randomly into two groups. The treatment group consisted of two classes. In each of the two classes, students learned concepts and meanings of the operations through problem solving, activity-based skills, and cooperative-learning assignments. The teacher in this class did not include drill-and-practice exercises in the curriculum.

The second group participated in this study was the control group, also consisting of two classes. The mathematics curriculum in these two classes consisted of arithmetic review. Every day students practiced computational procedures from the textbook or a mimeographed worksheet.

All students took a pretest and posttest. The Computational Skills Test: Grades 7-9 (Shaw-Hide: Individualized Computation Skills Program, 1972) was used to measure students' computational competencies. This test consisted of 60 computational problems. Students were required to apply arithmetic procedures to solve problems involving whole numbers, fractions, and decimals.

The following results were obtained. First of all, although the mean number of correct items on the pretest was lower for the experimental group than for the control group, the posttest scores were higher for the experimental group. The experimental group students raised their posttest mean by 15.6 in one class, and 13.4 in the other class. The posttest means in the control group classes were raised by only 1.8 and 3.6. In addition, the average grade equivalence in the experimental class increased from a grade level of 6.5 to 9.1. This result was a gain of over two and a half years in computational ability during one year. Students in the control group increased their average grade level equivalence by less than half a year.

Second, observations of the treatment classes indicated that the students' attitudes towards mathematics had changed. The students became more confident in their ability to be successful in mathematics and more willing to try new approaches to learning mathematics by the end of the year. They explored mathematical ideas using manipulatives, drew pictures, and wrote conjectures in mathematics.

Rieck (1995) compared a traditional algebra class with an innovative mathematics course in college mathematics which was designed to place mathematics into the context of everyday life and to use concepts from several disciplines within mathematics including algebra, geometry, statistics, probability, and data analysis with regard to attitudes towards mathematics and mathematics achievement. In addition, Rieck's study compared the small- size classes with large-size classes. Subjects in this study were 140-college students who were enrolled in the first college level mathematics course.

To measure student attitudes towards mathematics, and the change in those attitudes, a Likert-type scale consisting of twenty-three items was developed by the researcher. This instrument was administered to all students involved in the study on the first day of class after the drop/add period, and again shortly before the final exam.

To measure cognitive growth a pre-test was administered during the second class session after the drop/add period. The same test was again administered shortly before the final examination.

There were four different groups in this study. First of all, control group "A" included students enrolled in the traditional algebra class. Second, control group "B" included those who enrolled in large classes. Third, the experimental group "C" included all students enrolled in the innovative mathematics course. Fourth, experimental group "D" included all students enrolled in small classes.

Results of this study indicated the following. First of all, there was no statistically significant difference in growth in mathematics understanding between the control and experimental groups (mean for group A = 21.179 vs. mean for group C = 25.814). Second, there was no significant change in attitude for students enrolled in the algebra class group "A", whereas those enrolled in the innovative math class (group "C") showed a significant positive change in attitude. In addition, there is statistically significant difference in positive change in attitude toward mathematics, in favor of group "C."

In summary, research studies regarding students' attitudes towards mathematics indicate the following conclusions. First of all, mathematics anxiety and attitudes toward mathematics have been found to be related to mathematical achievement in the following manner. Studies have shown that high achievement in mathematics is related to low anxiety for students from grade school to college. In addition, positive attitudes toward mathematics consistently related to low mathematics anxiety. Furthermore, relationship between attitudes and performance is the sequence of a reciprocal influence, in that attitudes affect achievement and achievement affects attitudes. Also, high mathematical achievers have more positive attitudes toward mathematics than the underachievers.

Second, one of major factors that leads high-school students to avoid mathematics and develop negative attitudes toward this subject is drill-and-practice exercises, and rote learning procedures. Students dislike memorizing rules and formulas. In addition, lack of understanding skills is associated with maladjustment and delinquent behavior in class. Meaningful methods in teaching mathematics, however, produces positive change toward arithmetic and mathematics, and significant gains in arithmetic computations and reasoning.

In conclusion, the literature review revealed that more research is needed to determine the impact of problem-centered curricula such as the Interactive Mathematics Program (IMP) on mathematical problem-solving performance and attitude toward mathematics in urban high schools with high percentage of remedial students. Research needs to be conducted to find out if IMP is an effective environment for teaching mathematical problem-solving skills and concepts. Currently, there are not enough studies of a successful high-school mathematical curricula for teaching problem-solving skills. Although it seems reasonable that IMP may accomplish this goal, students have never been measured explicitly regarding their problem-solving abilities. Equally important, the attitudes of the students who have been taking IMP classes in urban high schools, and considered to be low achievers have not been measured or compared with students who are taking the traditional mathematics courses. The present study is exploratory because it investigates a previously unexamined topic: the effect of problem-based curricula (IMP) on high-school students' mathematical problem-solving performance and attitude in an urban high school with high percentage of remedial students.

Chapter III

Methodology

This chapter begins with the statement of the study's purpose and the research questions. Then the design and the variables are presented. The students who participated in the study and the instrumentation that was used are described in separate sections. A statement of compliance with protection of human subject's regulations is included. Then follows a section that contains details of the procedure that was employed in the study. Last, a brief section sets forth data analyses procedures that were used.

This study was conducted in September and November of 1998.

Purpose of the Study

The purpose of this study was to compare and correlate the mathematical problem-solving skills and attitudes using two different teaching approaches: (a) problem-centered learning approach utilized in the Interactive Mathematics Program (IMP) and (b) teacher-guided approach utilized in the First-Year Algebra course.

Research Questions

This study attempted to answer the following research questions:

1. To what extent does implementing the problem-centered learning approach enhance high-school students' mathematics performance compared with using the teacher-guided approach?

2. To what extent does implementing the problem-centered learning approach enhance high-school students' attitude toward mathematics compared with using the teacher-guided approach?

Design and Variables

This study is a comparison study with two intact groups: First Year Algebra and First Year Interactive Math Program (IMP) students using a pretest-posttest design. The two groups were compared on their problem-solving performance and attitude toward mathematics.

This study had one independent variable: the type of instruction. This independent variable was measured at two levels: the problem-centered learning approach in the first-year IMP classes and the teacher-centered approach in the first-year Algebra classes.

The study had two dependent variables: mathematical problem-solving performance and attitude toward mathematics. A mathematical problem-solving test (Van Akkeren, 1995) was used to measure the mathematical problem-solving performance. The attitude measures consisted of 14 items to measure the students' attitude toward mathematics (Mitchell & Gilson, 1997).

Subjects

The subjects in this study were selected from 5 IMP and 6 Algebra classes in a public high school in California. The total number of IMP students was 150. Whereas, the total number for Algebra students was 180 students. Only 37 IMP and 24 Algebra students obtained their parental approval and consequently were able to participate in this study (more details will be given in the procedure section).

The researcher was told by the counselors that students were assigned to IMP or Algebra classes based on the recommendations from their middle-school counselors or based on their scores of the mathematics section the Comprehensive Test of Basic Skills (CTBS). Students who scored at the 70 percentile or above on the mathematics section of the CTBS were assigned to first-year Algebra classes and those who scored below the 70 percentile were assigned to first-year IMP classes. To justify their decision regarding the students' placement in either IMP or Algebra classes, the counselors indicated that the IMP curriculum contains a variety of teaching strategies that meet the needs of the advanced students as well as the below average students, whereas the Algebra curriculum is a challenging curriculum that is limited with regard to a variety of teaching strategies, thus it does not meet the needs of the below average students.

In Table 1, the percentage and the frequency of the ethnicity of the students in the school that was selected for this study as well as for the students who participated in the study are provided.

Table 1
Ethnicity of Students in the Study Broken Down By Mathematics Curriculum

	School		IMP		Algebra	
Ethnicity	f	%	f	%	f	%
African-American	342	24.4	10	27.0	6	25.0
Asian	335	23.9	8	21.6	7	29.2
Hispanic	463	33.1	11	29.7	5	20.8
Other White	148	10.6	2	5.4	3	12.5
Other Non White	112	8.0	6	16.3	3	12.5
Total	1400	100.0	37	100.0	24	100.0

The students were 14 to 18 years old and came from middle to lower class families.

Protection of Human Subjects

This study has been approved by the Institutional Review Board for the Protection of Human Subjects at the University of San Francisco. The Approval Number is: 970143.

This study has also been approved by district of the school that was selected for the study. The approval letter is found in Appendix D.

Because the school district required active consent, the researcher sent letters to parents to obtain their parental permission for allowing their children to participate in the study prior to conducting the research (see Appendix C).

To ensure that the confidentiality of the students' responses on the attitude survey and their scores on the problem-solving performance test, the researcher did the following things. First, students were not asked to write their names on the survey or the test. Instead, they were identified by numbers. Those numbers were only used to match their pretest with posttest scores. Second, no one in school was allowed to see the students' responses, or their scores except in the form of summery results (e.g., 45% of the students agreed that they like mathematics).

Instrumentation

Three instruments were used in this study including a mathematical problem-solving test, an attitude survey, and the Stanford Achievement Test (Ninth Edition, Form T).

The Stanford Achievement Test is a test that is given once a year to measure the students' basic skills and concepts in English and mathematics. The scores of the mathematics section of the Stanford Achievement Test from the previous year were used to determine if the students in both IMP and Algebra groups are identical with regard to their mathematical problem-solving skills prior to conducting the instructions.

In this study, a problem-solving performance test developed by Van Akkeren (1995) was used to measure mathematical problem-solving skills. This test consisted of four problems. A copy of the complete test is found in Appendix B.

There were three reasons for using this problem-solving test. First, the test was used before and proven to be reliable. Second, the difficulty of the test problems matched the mathematical problem-solving ability of the subjects who participated in the study. Third, the type of the assignments given in both IMP and Algebra classes were totally different from the questions on this test. An example of such assignments used in both IMP and Algebra classes is given in Appendix F.

The instrument that was employed to measure problem-solving performance in mathematics was piloted with five high-school mathematics teachers during the Spring of 1998. Two of these five teachers served as scorers in the actual study. Using problems worked on by students in the pilot study, these teachers used the instrument to practice scoring problems, some of which were used in the study.

An interrater reliability coefficient of .80 or higher was sought before the instrument was used in the formal study. In this study, interrater reliability referred to the percentage of agreement between the two scorers. Percentage of agreement was calculated by dividing the total number of problems for which agreement occurred by the total number of problems examined. For any given problem, agreement was said to occur if either of two conditions were met: (a) there was a perfect match of total points given by scorers or (b) there was a difference of only one point between the total points given by the scorers.

The researcher worked with the scorers to refine and improve the scoring instrument. Toward the end of the pilot study, an effort was made to establish the content validity for the instruments. In other words, the attempt was to determine if the scores were good indicators that the instruments would measure what it purported to measure.

An example of the type of problems that was used in this test is included below:

> *Hellen is training for an important swim test. She will be swimming against 20 other swimmers from 5 different teams. On the first day of training, she swam 1 lap. She swam 4 laps on the second day. On the third day, she swam 7 laps. If Hellen continues to improve in this way, on what day will she research her goal of swimming at least 30 laps in a single day?*

In each of the four problems of the test, the students were asked to do the following tasks:

-State in writing what you' re trying to figure out.
-Write all the useful facts in a list. Do not list useless facts.
-State in writing your strategy for solving the problem.
-Solve the problem. Show all your work.
-State in writing what your answer is and label it correctly.

Because the researcher defined problem-solving performance as the students' abilities to carry out Polya's four phases of problem solving (including understanding the problem, making a plan, carrying out the plan, and looking back at the completed solution), this study used Polya's system to evaluate performance.

In this study, Polya's four steps in problem solving was used to score each problem. A problem received a score from zero to seven points based on the following rubric developed by Van Akkeren (1995):

1. Understanding the problem: identifying the question and relevant facts.

2. Devising a plan: writing the appropriate solution strategy.

3. Carrying out the plan: implementing the solution strategy.

4. Answering: looking back at the completed solution to check results for accuracy.

Each of the above steps was assigned point values as follows:

1. Understanding the problem:

2 points: -Question correctly stated in writing and all relevant facts are listed (no irrelevant facts included)

1 point: -Question correctly stated in writing or all relevant facts are listed

2. Devising a plan:

1 point: -Appropriate solution strategy

0 points: -Inappropriate solution strategy or no solution strategy

3. Carrying out the plan:

3 points: -Strategy fully carried out with inclusion of appropriate drawings, lists, tables, graphs, and/or computations; strategy adequately explained in writing

2 points: -Strategy fully carried out with inclusion of appropriate drawings, lists, tables, graphs, or and computations; strategy not adequately explained in writing.

1 point: -Strategy partially carried out with the inclusion of appropriate drawings, lists, tables, graphs, and/or computations; strategy not adequately explained in writing.

0 points: -Inappropriate solution strategy or no solution strategy.

4. Answer

1 point: -Correct answer (explicitly stated with appropriate label).

0 points: -Incorrect answer

The total score for a given problem can range from zero to 7 points.

An example of an expected good answer that may earn 7 points on the example problem given on page 101 is as follows:

1. Understanding the problem:

To earn the 2 points in this section of the above problem, students should have an answer similar to the following answer:

Stated question: On what day will she reach her goal of swimming
30 laps in 1 day?

Useful facts: 1st day 1 lap
2nd day 4 laps
3rd day 7 laps

2. Devising a plan:

To earn the one point in this section, students should come up with a solution strategy. An example of an appropriate solution strategy for the above problem is making a "table" and "finding a pattern" based on this table. Students can come with a table similar to the following

DAY	1	2	3	4
LAP	1	4	7	10

3. Carrying out the plan:

To earn the 3 points on this section, students should have come up with:
- a complete table (i.e., up to 30 laps)

- a complete explanation to the strategy used: " I made a table and found a pattern increasing by 3 laps everyday.

4. Answer:

To earn one point given for the answer section, students should clearly state and label the correct answer, for example, students should write something similar to the following statement to show their answer:

On the 11th day, Anne swam 30 laps.

Problem-solving performance on both the pretest and posttest was determined in the following manner for each student. First, the sum of total points received from both scorers was obtained for all four problems. Because there were four problems and two scorers, this meant that there were eight scores to sum. That total was divided by the number of scorers. This calculation yielded a performance average for each student.

To measure the attitude of students toward mathematics, an Interest Survey (IS) was used (Mitchell & Gilson, 1997). The instrument contains three scales. These scales measured individual interest in mathematics (II), mathematics anxiety, and situational interest in mathematics (SI). A complete copy of the survey is found in Appendix A.

The purpose of the survey was to find out to what extent taking a mathematics class utilizing a problem-centered approach such as IMP enhances the students' attitudes toward mathematics in comparison with taking a mathematics class utilizing the guided-practice approach in an Algebra class.

The 14 items in the survey were constructed using a 6-point Likert scale ranging from strongly disagree (1) to strongly agree (6). Each of the scales was composed of 4 to 5 items with approximately half positively worded, half negatively worded. An example of an item from each of these three scales is provided below.

I think mathematics is really boring. (II)

Strongly agree agree slightly agree slightly disagree disagree strongly disagree

Our math class is fun. (SI)

Strongly agree agree slightly agree slightly disagree disagree strongly disagree

When I am in a math class, I usually feel very much at ease and relaxed. (Anxiety).

Strongly agree agree slightly agree slightly disagree disagree strongly disagree

In Table 2, the classification of each item as Anxiety, SI, or II is provided.

Table 2
Classification of the Survey Items

Item #	Item	Classification
1	Mathematics is enjoyable for me	II
2	When the teacher says he/she is going to ask some questions to find out how much I know about math, I worry that I will do poorly.	Anxiety
3	I feel I am most successful when I learn something new.	SI
4	I think mathematics is boring.	II
5	Taking math scares me.	Anxiety
6	When I am in math class, I usually feel very much at ease and relaxed.	Anxiety
7	Compared to other subjects, mathematics is exciting to me.	II
8	When the teacher is showing the class how to a problem, I worry that other students might understand the problem better than me.	Anxiety
9	I am not interested in mathematics.	II
10	I Feel most successful when I do the work better than other students.	SI
11	I actually look forward to going to math class next year.	SI
12	Compared to how much I know in other classes I am taking, I know a lot about math.	SI
13	I feel most successful when all of the work is easy.	SI

Table 2 continued

Item #	Item	Classification
14	When I am taking a math test, I usually feel very much at ease and relaxed.	Anxiety

As can be seen from Table 2, five items are classified as Anxiety (i.e., item number 2, 5, 6, 8, and 14), five items are classified as SI (i.e., item number 3, 10, 11, 12, and 13), and four items are classified as II (i.e., item number 1, 4, 7, and 9).

The survey was short and took the students less than 10 minutes to complete.

An attitude score was obtained in the following manner for each student. First, the student's response to the survey statements was given the following points: 6 points if the response is strongly agree, 5 points if the response is agree, 4 points if the response is slightly agree, 3 points if the response is slightly disagree, 2 points if the response is disagree, and 1 point if the response is strongly disagree.

If the statement indicated a negative attitude toward mathematics such as "taking math scares me," then the above order was reversed. In other words, 1 point was given to strongly agree, and 6 points was given to strongly disagree.

Second, each student received a score for each scale on the survey. Because there are 4 items that measured the individual interest (II), the total points on this scale was 24. Points obtained from the II items are added together and then divided by 4 to yield an II average for each student.

Third, the same procedure was repeated for the other two scales (i.e., situational interest (SI) and Anxiety). This calculation yielded a SI average and an anxiety average for each student.

The internal consistency of all the scales was checked in a previous research as well as in this study. Scales were to be used only if their internal consistency was greater than .70. The internal consistency coefficients (Cronbach's alphas) for the scales range from .77 to .93 (Mitchell, 1993) with fourth-grade students to measure the relationship between anxiety, situational interest, and individual interest and mathematical performance. In this present study as well as Mitchell's study, the attitude scales were to be used only if their internal consistency was greater than .70. The internal consistency coefficients (Cronbach's coefficient alphas) for the scales range from .77 to .93 (Mitchell, 1993).

Construct validity of the Interest Survey also was checked. This was done by comparing the viability of a variety of different models to explain the observed measures (Mitchell, 1993). Mitchell argued that construct validity often proceeds by disconfirming alternative models and hoped that the primary model that contained all the scales would describe the data better than any reasonable countermodel.

Procedure

Students from 5 first-year IMP and 6 first-year Algebra classes were selected in the Fall of 1998 from a public-high school in a school district in California. The researcher made an agreement with the teachers who thought these classes and discussed the study with them. A complete description of both IMP and Algebra programs in the school district is found in Appendix E.

Parents of the students who participated in the study received a letter from the researcher 3 weeks before the beginning of the study (Appendix C). Each student participating in the study was given a letter and instructed to give it to his or her parents. In this letter, the researcher explained to the parents the purpose of the study and how the study would be conducted and asked for their approval regarding having their children participate in the study. In this letter, the researcher asked the parents to sign and return the letter to the mathematics teacher if they wanted their child to participate. Only those students who returned the approval letter signed by their parents were allowed to participate in the study. Other students worked on a mathematics assignment given by their mathematics teacher on the day when the data were collected for the study.

At the end of August 1998, students were assigned by their counselors to either first-year Algebra class or first-year IMP class. Assigning students to either an IMP or Algebra class depended on the recommendations of the students' middle-school counselors or students eighth-grade CTBS scores (70th percentile or higher Algebra; below 70th percentile IMP).

A total of 150 IMP and 180 Algebra students enrolled in these classes. Only 50 IMP and 61 Algebra students returned their parental approval letter and consequently were able to participate in the study. Out of those 111 students, 37 IMP and 24 Algebra students took the mathematical problem-solving pretest. Eighty-four percent of the 37 IMP students (i.e., 34 students) and eighty-seven and 87.5% of the 24 (i.e., 21 students) Algebra students came from one Algebra and two IMP classes that were taught by the researcher. The rest of the students who participated in this study came from one Algebra and three IMP classes. The posttest was administered only to the 61 students who took the pretest. If a student was absent on the day the posttest was given, the researcher arranged with his or her mathematics teacher to have him or her take the posttest at some other time.

All students participated in this study took two identical tests (i.e., pretest and posttest). Each test was consisted of a survey (Mitchell & Gilson, 1997; Appendix A)and a mathematical problem-solving test (Van Akkeren, 1995; Appendix B).

In order to ensure a match between a pretest and posttest with regard to the students' names, each student was given a different number and asked to write it on both the pretest and posttest.

The survey and the problem-solving test were given in one class period during a mathematics class. All students who returned their parents letter were asked to go to a certain class at a specific period to take the test. The test was administered to all of them at once. First, students were given 10 to 15 minutes to fill out the survey. Then the students were given a mathematical problem-solving test. This test consists of four problems. The students were given 40 minutes to work on the test. The researcher administered both the survey and the mathematical problem-solving test. The researcher's role was to read the instructions to the students, pass out the survey and the test, give the students enough time to complete the survey and the test, and then collect them. The survey and the problem-solving performance test were corrected by two mathematics teachers, and the scores obtained from both IMP and Algebra classes were analyzed using the appropriate statistical tests to determine if there is a statistically significant difference between these two groups.

Data Analysis

The researcher used the Statistical Package for the Social Sciences (SPSS) to analyze the data. The level of significance that was employed was at .05.

The researcher calculated and reported the means, the standard deviations, and the effect sizes for all relevant data in order to calculate the statistical significance for the groups participating in the study.

Analyzing, using analysis of variance (ANOVA) the scores from the pretest determined the similarity of the IMP and the Algebra groups before the treatment both in terms of problem-solving skills and mathematics attitudes. Analyzing, using ANOVA, the scores from the posttest determined if statistically significant differences existed between the two groups after the instruction both in terms of problem-solving skills and mathematics attitudes.

Chapter IV

Findings

This study investigated the effect of problem-centered learning model proposed by Wheatley (1991) on high-school students' mathematical problem-solving performance and attitudes toward mathematics. The impact of problem-centered learning model was compared with the effect of the teacher-guided approach. In this study, the problem-centered learning model was utilized in the Interactive Mathematics Program (IMP), whereas the teacher-guided approach was utilized in the first-year algebra class that served as a control group. Two hypotheses were tested in this study including (a) the IMP group will perform statistically significantly better than the comparison group on the mathematical problem-solving test and (b) IMP students will have better attitudes toward mathematics than the Algebra group.

There are five sections in this chapter. First, reliability data concerning the problem-solving test and attitudes toward mathematics are provided. Second, analyses of the data of the Stanford Achievement Test are included. Third, data concerning the pretest and posttest are presented. Fourth, correlational information between attitudes toward mathematics and mathematical problem-solving performance are given. Last, the gender effect in both the Algebra and IMP groups is analyzed and reported.

Reliability Data

To estimate the interrater reliability for the mathematical problem-solving test, 10 students' pretests were selected randomly. Each test consisted of four problems. A total of 40 problems were thus given to two scorers to correct. In this study, interrater reliability was defined as the percentage of agreement between the two scorers. Percentage of agreement was calculated by dividing the total number of problems for which agreement occurred by the total number of problems scored. Agreement between the two scorers was said to occur if there was a perfect match of total points or if there was a difference of only one point between the total points given by the scorers. A student could receive a score between 0 to 7 points for any of the four test problems. The agreement between the two scorers, on the pretest, is reported in Table 3. As can be seen, there was 55% exact agreement (i.e., difference of the two scores on a given problem equals zero) between the two scorers, 32.5% a difference of one point, and 12.5% a difference of more than one point. Thus, percentage of agreement for the mathematical problem-solving pretest was 87.5% - a more than the adequate level of reliability, given the definition of agreement used in this study. The percentage of agreement also was calculated for the mathematical problem-solving posttest. The agreement between the two scorers, on the posttest, is reported in Table 4. The percentage of agreement for the posttest also was more than the adequate level of reliability (i.e., 98.5%).

Table 3

Percentage of Agreement Between the Two Readers on the Mathematical Problem-Solving Pretest.

Difference between scorers	# of problems	Percentage
Exact (zero difference)	22	55.0
One point off	13	32.5
More than one point off	5	12.5
Total	40	100.0

Table 4

Percentage of Agreement Between the Two Readers on the Mathematical Problem-Solving Posttest

Difference between scorers	# of problems	Percentage
Exact (zero difference)	23	57.5
One point off	16	40.0
More than one point	1	2.5
Total	40	100.0

The internal-consistency reliability was assessed for the attitude survey by comparing all ($n = 61$) students' responses to the survey questions. This internal-consistency estimate of reliability was calculated for the three attitudes scales including Anxiety, individual interest (II), and situational interest (SI) for both the pretest and posttest surveys. The consistency estimates are reported in Table 5. In this study, the acceptable value of the internal-consistency reliability was set at .70. The reliability estimates for both Anxiety and Individual Interest scales at the pretest are acceptable (i.e., .78 and .88, respectively). The reliability estimates for both the Anxiety and Individual Interest at the posttest are also acceptable (i.e., .72 and .82, respectively).

The Situational Interest scale had 5 items in the attitude survey that was given as a pretest and posttest. When the reliability estimate was calculated for all of these 5 items, the value obtained was lower than .70. The reliability estimate for the Situational Interest at the pretest was found to be .45 and the posttest was .54. The last item of the situational interest had a very low correlation (i.e.. -.06) at the pretest and (-.10) at the posttest with the other items in the scale. Therefore, the reliability for Situational Interest was calculated based on 4 rather than the 5 items. After disregarding the last item of the

Situational Interest scale, the reliability estimate of the Situational Interest scale at the pretest was still below .70 (i.e., .61). The reliability estimate for the Situational Interest scale at the posttest, however, was acceptable (i.e., .70).

The test-retest correlation coefficients for Anxiety, Individual Interest, and Situational Interest scales were calculated and reported in Table 6.

Table 5

Reliability Estimates for the Three Attitude Scales For Both Pretest and Posttest (n = 61)

Scale	Pretest	Posttest
Anxiety	.78	.72
Individual Interest	.88	.82
Situational Interest	.61*	.70*

* Reliability is calculated based on 4 rather than 5 items.

Table 6

Test-Retest Correlation Coefficients for the Attitude Scales (n = 61)

Scale	Preanxiety	Pre II	PreSI
Post Anxiety	.75	.49	.31
Post II	.34	.70	.50
Post SI	.26	.46	.59

As can be seen, the test-retest correlation coefficients for Anxiety and Individual Interest scales are acceptable (.75 and .70, respectively), whereas the test-retest correlation coefficient for the Individual Interest scale is moderate (.59).

Data Analysis for Stanford Achievement Test

Sixteen out 24 Algebra students and 24 out of 37 IMP students took the Stanford Achievement Test. The mean, standard deviations, and eta square of this test for both the IMP and Algebra groups are reported in Table 7.

Table 7

Stanford Achievement Test Means, Standard Deviations, and Effect Sizes for IMP and Algebra Students (n = 40)

Statistics	IMP	Algebra	eta^2
Mean	37.25	55.75	.15
SD	21.88	22.56	
Range	0 - 100	0 - 100	
n	24	16	

The Algebra group mean is 55.75, whereas the IMP group mean is 37.25. Analysis of variance (ANOVA) results show that the Algebra mean score is statistically significantly greater than the IMP mean score. These are shown in Table 8. The Stanford Achievement Test cannot be used, however, as a covariate in this study, because the correlation coefficients between this test and the mathematical problem-solving pretest and posttest are very low (i.e., .05 and .00, respectively). Instead of using the Stanford Achievement Test, the mathematical problem-solving test was used in determining if the IMP and Algebra groups were identical to each other with regard to their problem-solving skills prior to conducting the instruction.

Table 8

Analysis of Variance Summary for the Stanford Achievement Test (n = 40)

Source of Variation	df	SS	MS	F
Between	1	3285.60	3285.60	6.67*
Within	38	18649.50	490.78	
Total	60	21935.10		

* Statistically significant at .05 level.

<u>Data Analysis for the Pretest and Posttest</u>

In this section, the results of the pretest and posttest of both the IMP and Algebra groups are reported. First, the results of the mathematical problem-solving test are reported. Then, the results of the attitude scales are analyzed.

A pretest was used in this study to determine if the IMP group was identical to the control group (Algebra) with regard to (a) the problem-solving skills and (b) attitudes toward mathematics prior to the instruction.

A pretest consisting of four mathematical problems was administered to both IMP and Algebra groups. Each problem has a 0- to 7-point scale with a total of 28 points. The average score per problem was used when reporting results. Pretest means, standard deviations, and effect sizes for both IMP and Algebra groups are shown in Table 9. The pretest mean for the IMP group was slightly higher than the pretest mean score for the Algebra group (3.82 vs. 3.77, respectively) with a small value of eta square (.00).

Table 9

Mathematical Problem-Solving Test Means, Standard Deviations, Range of Test Scores and Effect Size for IMP and Algebra Students for The Pretest (n = 61)

Statistics	IMP	Algebra	eta^2
Mean	3.82	3.77	.00
SD	1.96	1.58	
Range	0 - 7	0 - 7	
n	37	24	

The magnitude of the measure of association between IMP and Algebra group is shown in Table 9 as eta square. The value shown in Table 9 indicates a small value in eta square (i.e., .00).

An analysis of variance for pretest problem-solving test scores for IMP and Algebra groups is displayed in Table 10. The analysis determined that the IMP group was not statistically significantly different from the Algebra group with regard to the problem-solving skills before the instruction was conducted.

A posttest was given 6 weeks after administrating the pretest. Both IMP and Algebra groups were given a posttest identical to the pretest. Means, standard deviations, and effect sizes of the posttest for both IMP and Algebra groups are presented in Table 11. As can be seen, both IMP and Algebra means dropped: the IMP means dropped from 3.82 to 3.62, whereas the Algebra mean dropped from 3.77 to 3.33. An analysis of variance for the mathematical problem-solving posttest is displayed in

Table 12. The analysis determined that the IMP mean was not statistically different from the Algebra mean. The range of the scores for each problem on the posttest was from 0 to 7.

Table 10

Analysis of Variance Summary for The Mathematical Problem-Solving Pretest.

Source of Variation	df	SS	MS	F
Between	1	.04	0.04	.01
Within	59	195.10	3.31	
Total	60	195.14		

Table 11

Mathematical Problem-Solving Test Means, Standard Deviations, and Effect sizes for IMP and Algebra Students for Posttest (n = 61)

Statistics	IMP	Algebra	eta^2
M	3.62	3.30	.00
SD	2.38	2.32	
n	37	24	

Maximum score is 7

An analysis of variance for the mathematical problem-solving posttest is displayed in Table 12. The analysis determined that the IMP mean was not statistically different from the Algebra mean.

Problem-by-problem analyses for both the pretest and posttest also were conducted. Means and standard deviations for each problem on the pretest and posttest are reported in Table 13. The range of the scores for each problem on the posttest is 0 to 7.

Table 12

Analysis of Variance Summary for the Mathematical Problem-Solving Posttest (n = 61)

Source of Variation	df	SS	MS	F
Between	1	1.49	1.49	.27
Within	59	327.33	5.55	
Total	60	329.82		

Table 13

Means and Standard Deviations for Pretest and Posttest (n = 61)

		P1		P2		P3		P4	
Group		Pre	Post	Pre	Post	Pre	Post	Pre	Post
Algebra	M	5.50	4.53	2.50	2.96	2.75	2.71	4.33	2.96
	SD	2.04	2.65	2.28	2.56	2.49	2.91	2.63	3.16
IMP	M	5.81	4.78	3.62	3.30	3.03	2.70	2.84	3.70
	SD	2.20	2.86	2.64	2.68	2.69	2.61	3.09	3.36

Comparing the mean scores of the pretest with the posttest for the Algebra group shows that on the first, third, and fourth problem the mean drops (e.g., 5.50 to 4.43, 2.75 to 2.71, and 4.33 to 2.96, respectively). On the second problem, the mean increases from 2.50 to 2.96.

Comparing the mean scores of the pretest with the posttest for the IMP group shows that on the first, second, and third problem the mean drops (i.e., from 5.81 to 4.78, from 3.62 to 3.30, and from 2.69 to 2.61, respectively). On the fourth problem the mean increases from 3.09 to 3.36.

Comparing the IMP with Algebra groups on each of the problems of the pretest shows that on the first three problems the IMP means are greater than the Algebra means, whereas on the last problem the Algebra mean is greater than the IMP mean (4.33 and 2.84, respectively). To determine if these changes are statistically significant, an analysis of variance was conducted on each of the pretest problems. The results of the analyses are reported for problem 1, problem 2, problem 3, and problem 4, in Table 14, 15, 16, and 17, respectively. Analyses of variance show that there no statistically significant differences between the IMP and Algebra groups with regard to their performance on each of the problems of the mathematical problem-solving pretest.

Comparing the IMP with the Algebra groups on each of the problems of the mathematical problem-solving posttest shows that the IMP group means on the first and second problem appear to be greater than the Algebra means, whereas the Algebra means on the third and fourth are greater than the IMP means. The analysis of variance was conducted to determine if these differences are statistically significant. Tables 18, 19, 20, and 21 contain the variance analysis for problems 1, 2, 3, and 4, respectively. The results of these analyses show that there are no statistical significant differences on these posttest for the four problems.

Table 14

Analysis of Variance for the Mathematical Problem-Solving
Pretest Problem 1 (n = 61)

Source of Variation	df	SS	MS	F
Between	1	1.41	1.41	.31
Within	59	269.68	4.57	
Total	60	271.09		

Table 15

Analysis of Variance for the Mathematical Problem-Solving
Pretest Problem 2 (n =61)

Source of Variation	df	SS	MS	F
Between	1	18.31	18.31	2.91
Within	59	370.70	6.28	
Total	60	389.01		

Table 16

Analysis of Variance for the Mathematical Problem-Solving Pretest Problem 3 (n = 61)

Source of Variation	df	SS	MS	F
Between	1	1.12	1.12	.16
Within	59	403.47	6.84	
Total	60	404.59		

Table 17

Analysis of Variance for the Mathematical Problem-Solving Pretest Problem 4 (n = 61)

Source of Variation	df	SS	MS	F
Between	1	32.56	32.56	3.82
Within	59	502.36	8.51	
Total	60	534.92		

Table 18

Analysis of Variance for the Mathematical Problem-Solving Posttest Problem 1 (n = 61)

Source of Variation	df	SS	MS	F
Between	1	0.58	0.58	.07
Within	59	456.10	7.73	
Total	60	456.68		

Table 19

Analysis of Variance for the Mathematical Problem-Solving Posttest Problem 2 (n = 61)

Source of Variation	df	SS	MS	F
Between	1	1.67	1.67	.24
Within	59	408.69	6.92	
Total	60	410.36		

Table 20

Analysis of Variance for the Mathematical Problem-Solving Posttest Problem 3 (n = 61)

Source of Variation	df	SS	MS	F
Between	1	0.00	0.00	.00
Within	59	440.69	7.47	
Total	60	440.69		

Table 21

Analysis of Variance for the Mathematical Problem-Solving Posttest Problem 4 (n = 61)

Source of Variation	df	SS	MS	F
Between	1	8.07	8.07	.75
Within	59	634.69	10.76	
Total	60	642.76		

A similar analysis was conducted to determine if the IMP group was identical to the Algebra group with regard to their attitudes toward mathematics prior to conducting the instruction. An attitude survey was administered as a pretest to both the IMP and Algebra groups. In this study, the attitude was measured on three scales including Anxiety, Individual Interest (II), and Situational Interest (SI). Means, standard deviations, and effect sizes of the pretest for the IMP and Algebra groups are presented in Table 22. The IMP group means are less than the Algebra group means. The Anxiety

The Anxiety scale means are 3.71 and 4.06 with a small effect size of .03 in favor of the Algebra group. The Individual Interest scale means are 3.46 and 3.82 with an effect size of .02 in favor of the Algebra group. Finally, the Situational Interest scale means are 4.11 and 4.41 with an effect size of .05 in favor of the Algebra group.

Table 22

Attitudes Toward Mathematics Means, Standard Deviations, and the Effect Sizes for IMP and Algebra Groups for the Pretest (n = 61)

Scale		IMP (n = 37)	Algebra (n = 24)	eta^2
Anxiety	M	3.71	4.06	.03
	SD	1.10	0.95	
Ind. Interest	M	3.46	3.82	.02
SD	1.19		1.81	
Sit. Interest	M	4.11	4.41	.05
	SD	1.03	0.68	

Maximum score is 6

The analysis of variance results for pretest attitude scores for IMP and Algebra groups is displayed in Tables 23, 24, and 25. Analysis of variance results for pretest Anxiety scores is shown in Table 23. The analysis determined that IMP group mean score was not statistically significant from the Algebra group with regard to anxiety prior to conducting the instruction. Analysis of variance results for pretest Individual Interest and Situational Interest scales are shown in Tables 24 and 25, respectively. The results of the analyses show that IMP group is not statistically significantly different from the Algebra group with regard to both Individual and Situational Interest prior to conducting the instruction.

Table 23

Analysis of Variance Summary for Pretest Anxiety Scale Scores (n = 61)

Source of Variation	df	SS	MS	F
Between	1	1.67	1.67	1.53
Within	59	60.05	1.09	
Total	60	61.72		

Table 24

Analysis of Variance Summary for Pretest Individual Interest Scale Scores (n = 61)

Source of Variation	df	SS	MS	F
Between	1	1.85	1.85	1.32
Within	59	81.59	1.41	
Total	60	83.44		

Table 25

Analysis of Variance Summary for Pretest Situational Interest Scale Scores (n = 61)

Source of Variation	df	SS	MS	F
Between	1	1.30	1.30	1.55
Within	59	48.51	0.61	
Total	60	49.81		

The same attitude survey was given to both IMP and Algebra groups after 6 weeks of the beginning of the instruction as a posttest. Posttest means, standard deviations, and effect sizes for both groups are given in Table 26.

The results of the posttest indicated that the Anxiety scale IMP group mean score has increased from 3.71 to 4.00. Whereas the Anxiety scale Algebra group mean score has dropped from 4.06 to 3.98. As can be seen, the posttest IMP Anxiety scale mean score is greater than the Algebra anxiety (4.00 vs. 3.98, respectively) with a small value of eta square. The IMP Individual Interest (II) scale mean score has dropped slightly from 3.46 to 3.45 as well as the Algebra mean score (from 3.82 to 3.53). As can be seen, the posttest IMP Individual Interest scale mean score is less than the Algebra Individual Interest mean score (3.42 vs. 3.53, respectively). Finally, the SI scale IMP mean score has dropped from 4.11 to 3.93 as well as the Situational Interest scale Algebra mean score (from 4.41 to 4.20). The posttest situational interest scale IMP mean score is less than the Situational Interest scale Algebra mean score (3.93 vs. 4.20, respectively).

Table 26

Attitudes Toward Mathematics Means, Standard Deviations, and Effect Sizes for IMP and Algebra Groups for the Posttest (n = 61)

Scale		IMP	Algebra	eta^2
Anxiety	M	4.00	3.98	.00
	SD	0.97	0.89	
Ind. Interest	M	3.45	3.53	.00
	SD	1.17	0.97	
Sit. Interest	M	3.93	4.20	.02
	SD	1.16	0.87	

Maximum score is 6

An analysis of variance was conducted on the posttest attitudes. The results of the analysis of variance for Anxiety, Individual Interest, and Situational Interest scales are shown in Tables 27, 28, and 29, respectively. The results of the analyses show that the IMP group is not statistically significantly different from the Algebra group with regard to Anxiety, Individual Interest and Situational Interest scales on the attitude measures.

Table 27

Analysis of Variance for the Anxiety Scale Posttest Scores (n = 61)

Source of Variation	df	SS	MS	F
Between	1	0.01	0.01	.01
Within	59	51.01	0.88	
Total	60	51.02		

Table 28

Analysis of Variance Summary for the Individual Interest Scale Posttest Scores (n =61)

Source of Variation	df	SS	MS	F
Between	1	0.09	0.09	.08
Within	59	69.77	1.20	
Total	60	72.67		

Table 29

Analysis of Variance Summary for the Situational Interest Scale Posttest Scores (n = 61)

Source of Variation	df	SS	MS	F
Between	1	1.03	1.03	.92
Within	59	64.57	1.11	
Total	60	65.60		

Correlations Between Attitude and Performance

This study measured three attitudes using Anxiety, Individual Interest, and Situational Interest scales. In this section, the correlation coefficients between these three scales and mathematical performance are reported. In addition, the correlations between the pre- and postmeasures of all the variables are given. Intercorrelations for the pretest, posttest, and pre- and posttest are displayed in Tables 30, 31, and 32, respectively.

At the pretest level, the correlation coefficients between performance and anxiety and performance and Situational Interest scales are low and negative (-.02 and -.08, respectively). Positive and low correlation coefficient is found between performance and Individual Interest scale (.14).

At the posttest level, the correlation coefficients between performance and Anxiety scale and performance and Situational Interest scale increased to -.02 and -.08 and to .17 and .43, respectively. The correlations between performance and Individual Interest scale increased from .14 at the pretest to .44.

At the pre- and posttest level, the correlation coefficient between pretest performance and posttest performance is low (.35). The correlation coefficient between pretest performance and posttest anxiety scale is close to zero (-.03). The correlation coefficients between pretest performance and both the Situational Interest and Individual Interest scales are low (.06 and .16, respectively). The correlation coefficients between posttest performance and pretest Anxiety, pretest Individual Interest, and pretest Situational Interest scales are low (.21, .23, and .21, respectively).

Table 30

Correlation Coefficients Between Attitudes and Performance Pretest Scores (n = 61)

Scale	Perf.	Anxiety	SI	II
Perf.	1.00			
Anxiety	-0.02	1.00		
SI	-0.08	0.31	1.00	
II	0.14	0.51	0.58	1.00

Table 31

Correlation Coefficients Between Attitudes and Performance Posttest (n = 61)

Scale	Perf.	Anxiety	SI	II
Performance	1.00			
Anxiety	0.17	1.00		
SI	0.43	0.22	1.00	
II	0.44	0.43	0.63	1.00

Table 32

Correlation Coefficients Between Attitudes and Performance Pre- and Posttest (n= 61)

Posttest Scales	Performance	Anxiety	II	SI
Performance	.35	.21	.23	.21
Anxiety	-.03	.75	.49	.31
II	.16	.34	.70	.50
SI	.06	.26	.46	.59

<u>Gender Effect for Both the Algebra and IMP Students</u>

Out of the 24 students in the Algebra group, there are 15 males and 9 females. The IMP group has 25 male and 12 female students. Means and standard deviations for the Algebra males, Algebra females, IMP males, IMP females are reported in Table 33. The Algebra male mean is close to the Algebra female mean (3.78 vs. 3.75, respectively), whereas the IMP male mean is lower than the IMP female mean (3.61 vs. 4.27, respectively). Analysis of variance using results indicated that there are no statistically significant difference between the IMP males and IMP females. Analysis for variance results for the IMP male and IMP female students are reported in Table 34.

Table 33

Means and Standard Deviations for Algebra Males, Algebra Females, IMP Males, and IMP Females Pretest (n = 61)

Group	Statistics	Males	Females
Algebra	Mean	3.78	3.75
	SD	1.83	1.15
	n	15	9
IMP	Mean	3.61	4.27
	SD	1.96	1.95
	n	25	12

Table 34

Results of Analysis of Variance of IMP Females and IMP Males for Pretest (n = 37)

Source of Variation	df	SS	MS	F
Between	1	3.54	3.54	.92
Within	35	134.19	3.83	
Total	36	137.73		

Means and standard deviations for Algebra male mean, Algebra female mean, IMP male mean, and IMP female mean for posttest are reported in Table 35. As can be seen, both the Algebra males mean and the Algebra Females mean dropped (the Male mean dropped from 3.78 to 3.28 and Female mean dropped from 3.75 to 3.33). The IMP male mean dropped from 3.82 to 3.62, whereas the IMP female mean increased from 4.27 to 4.88. Analysis of Variance for the IMP male and IMP female means is reported in Table 36.

Table 35

Means and Standard Deviations for Algebra Males, Algebra Females, IMP Males, and IMP Females Posttest (n = 61)

Group	Statistics	Males	Females
Algebra	Mean	3.28	3.33
	SD	2.09	2.80
	n	15	9
IMP	Mean	3.02	4.88
	SD	2.39	1.88
	n	25	12

As can be seen, analysis of variance indicates that on the posttest the IMP females did statistically significantly better than the IMP males students. The measure of practical importance, eta square, is .14.

Table 36

Analysis of Variance of IMP Females and IMP Males for Postest (n = 37)

Source of Variation	df	SS	MS	F
Between	1	27.90	27.90	5.55*
Within	35	5.02		
Total	36	32.92		

* Statistically significant at .05 level.

Summary

The results of the pretest and posttest given to both IMP and Algebra groups are reported in this chapter. The results indicated that there are no statistically significant differences between IMP and Algebra groups with regard to their mathematical problem-solving performance both prior and instruction. This means that the first hypothesis (i.e., IMP group will perform statistically significantly better than the Algebra group on the mathematical problem-solving test) was not supported in this study.

The results of the attitude survey indicated that the second hypothesis (i.e., IMP students will have better attitudes toward mathematics than the Algebra students) were statistically not supported in this study.

Chapter V

Discussion, Limitations, Implications, and Recommendations

This study investigated the impact of the problem-centered learning model proposed by Wheatley (1991) on high-school students' mathematical problem-solving performance and attitudes toward mathematics. The problem-centered learning model was utilized in the Integrated Mathematics Program (IMP) and was compared with a more traditional program (First-Year Algebra). Two hypotheses were proposed in this study.

This chapter is divided into five sections. First, the limitations of the study are discussed. Second, a discussion of the two hypotheses is presented. Third, a reflection on the scoring instruments is reported. Fourth, the implications of this study are discussed. Finally, recommendations for further research are presented.

Limitations of the Study

This study was restricted to the high-school students attending a large school district in California. This study also was restricted to those students who were taking the first year of the Interactive Mathematics Program (IMP) or a high-school freshman Algebra course. Variables being studied were limited to high-school students' mathematical problem-solving skills and attitudes toward mathematics. Attitudes toward mathematics were limited to three scales including anxiety, individual interest, and situational interest. The effect of the different instructions was measured only twice- prior to the instruction and 6 weeks after the instruction. The problem-centered learning model was utilized in IMP. IMP contains all features that the problem-centered learning model includes except for one feature. This feature is selecting problems that are at the students' level. The problems included in the IMP curriculum, however, are very challenging for remedial students.

Mathematical Problem-Solving Performance

To determine if the IMP and Algebra groups were similar to each other with regard to their mathematical problem-solving skills prior to the delivery of the instructions, a four-problem pretest was given to both groups. The scores on the mathematics section of the Stanford Achievement Test (SAT) from the previous year (Spring 1997) were used as well to determine if both groups were similar to each other. On one hand, the results of the pretest analysis of variance showed that that there was no statistical significant difference between the IMP and the Algebra group prior to instruction. On the other hand, analyzing the scores of the Stanford Achievement Test indicated that the Algebra group mean is statistically significantly greater than the IMP group mean. These two tests, however, are different from each other in two aspects. First, the Stanford Achievement test measures students' mathematical knowledge, concepts, and basic skills, whereas the pretest measures specifically assesses the students' mathematical problem-solving skills as defined by Polya (1954), such as guess and check, constructing a table, and making a chart. Second, the Stanford Achievement test is a multiple-choice test, and thus it focuses only on the product (i.e., the answer), whereas the pretest is an constructed-response test in which students were asked to write a plan that will help them answer the problem and then to show the way they carry out their plans. Thus, the focus of the pretest is not only on the product but also on the process. In addition, when the pretest was correlated with the Stanford Test, the correlation coefficients were very low. Therefore, in this study, the Stanford Achievement Test was not used a covariate. Moreover, the IMP and Algebra groups were identical to each other only with regard to their mathematical problem-solving skills as defined by Polya (1954) prior to conducting the instructions.

After 6 weeks of instructions, both groups were given a posttest identical to the pretest. Results of analyzing the data of the posttest showed that the mean of both tests had dropped (i.e., the IMP mean dropped from 3.82 to 362 and the Algebra group from 3.82 to 3.2). Analyzing the posttest data using ANOVA indicated that the two groups were not statistically significantly different from each other after 6 weeks of the instructions.

Many factors might have contributed to these results including the teacher's effect, Polya's model, problem-centered learning model, and the IMP curriculum.

The teacher effect may not be a contributing factor to the results' of this study, because the majority of the students in this study (i.e., 84%) had one teacher. Polya's four-step model (1954), however, may be a major contributing factor to the results of this study. In this study, mathematical problem-solving skills were defined by Polya's model. Therefore, the students were tested prior to and after 6 weeks of the instructions on their abilities to apply Polya's model to solve mathematical problems. The IMP and Algebra groups, however, have not learned explicitly how to apply Polya's model to solve mathematical problems during the 6 weeks of instructions. In addition, there are other strategies than Polya's that the students could have learned to solve mathematical problems and were not able to apply them during the posttest. Thus, students in both groups may have improved their mathematical problem-solving skills, but the way the posttest was written and scored did not give the students the opportunity to demonstrate this improvement.

The problem-centered learning model and the IMP curriculum might be other factors that contributed to the results of this study. In this study, the problem-centered learning model was utilized in IMP. Wheatley (1991) argued that the core of problem-centered learning is a set of problematic tasks that focus attention on the key concepts of the discipline that will guide students to construct effective ways of thinking about mathematics. It utilizes the use of small groups of students working together as mathematicians to solve mathematical problems. Review of the literature reveals the importance of the usage of small groups in mathematics classes (Bishop, 1985; Doise & Mugny, 1984; Haste, 1987; Sigel, 1981; Yackel, Wood, Wheatley, & Markel, 1990). The problem-centered learning model allows students to present their solutions, inventions, and insights to the whole class. Wheatley (1991) described the importance of sharing mathematical ideas in mathematics classes and indicated that in order to do mathematics students must learn how to carry on a scientific discussion. The review of literature shows the success of problem-centered model in enhancing students' understanding of mathematics (Cobb et al., 1991; Trowell, & Wheatley, 1995; Yackel, Cobb, & Wood, 1991). These studies were conducted using students who are different from those in the present study. For example, Cobb et al. and Yackel et al. used second-grade students, whereas Trowell and Wheatley used undergraduate college students majoring in mathematics. In the present study, students were high-school students with a poor mathematics background. That is, the IMP mean on the Stanford Achievement Test was less than 40^{th} percentile.

In this study, the problem-centered learning model was utilized in the Interactive Mathematics Program (IMP). IMP includes all the features that the problem-centered learning model includes such as working in small groups and presenting solutions to the class except for one feature. This feature is selecting problems that are at the students' levels. The problems included in the IMP curriculum, however, are very challenging for the students in this study.

Research studies have shown that IMP students were doing statistically significantly better than the Algebra students not only on national tests such as SAT, but also on achievement tests (Alper et al., 1995). These studies were conducted in magnet schools at which students are considered to be high achievers.

In this study, however, IMP students did not do statistically significantly better than the Algebra group in the mathematical problem-solving test. There are two issues to be considered in explaining the fact that the IMP mean on the mathematical problem-solving test was not statistically significantly different from the Algebra mean scores. On one hand, the IMP students were tested 6 weeks after the treatment began. If the posttest, however, was given after a longer period of time (e.g., one semester), then the IMP students might have had enough time to develop their mathematical problem-solving skills and consequently show an improved performance on the posttest compared with the Algebra group. On the other hand, as indicated above, the previous studies were conducted in magnet schools using high-achieving students. The present study, however, was conducted in an urban high school whose students are considered to be low achievers. One possible conclusion is that IMP may be an effective curricula for high-achieving students and less effective for low-achieving students. The IMP curriculum utilizes a discovery approach without providing examples or providing opportunities of practicing basic-mathematical skills. Students, working in small groups, are expected to discover mathematical rules and formulas or come up with solutions to certain mathematical problems without having examples that show them how to do so. Remedial and low-achieving students, such as those in the present study, need step-by-step explanations, to see examples, and to be given the opportunity to practice their basic mathematical skills.

IMP may be a very challenging curriculum for low-achieving students such as those in this study. Therefore, one possible conclusion, based on the results of this study, is that in order to serve the remedial students better and meet their needs, educators need to employ a less challenging curriculum than IMP. A curriculum that provides students with concrete explanations to the major concepts in the lesson or the chapter, show examples of how to approach and do the problems, provide review sections when appropriate, and finally, give students some meaningful problems to practice in small groups and individually and to present their solutions to their classmates. Therefore, the problem-centered learning model may be still be viable when used with remedial students, but should be applied with a less challenging and more concrete curriculum.

Finally, an analysis of the gender's effect was conducted. The results of this analysis reveal the following things. First of all, there was no statistically significant difference between the Algebra males and Algebra females on both pretest and posttest. When the analysis was conducted on the IMP group, however, results of the analysis of variance indicated that there is a statistically significant difference between the IMP utilizes a discovery approach without providing examples or providing opportunities of practicing basic-mathematical skills. Students, working in small groups, are expected to discover mathematical rules and formulas or come up with solutions to certain mathematical problems without having examples that show them how to do so. Remedial and low-achieving students, such as those in the present study, need step-by-step explanations, to see examples, and to be given the opportunity to practice their basic mathematical skills.

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Finally, an analysis of the gender's effect was conducted. The results of this analysis reveal the following things. First of all, there was no statistically significant difference between the Algebra males and Algebra females on both the pretest and posttest. When the analysis was conducted on the IMP group, however, results of analysis of variance indicated that there is a statistically significant difference between the IMP males and IMP females on the posttest in favor of the female studnets. There was no statistically significant difference on both groups on the pretest. More important, the IMP female increased whereas the IMP male mean dropped. Thus, the IMP statistically significantly benefits the IMP females with regard to their mathematical problem-solving skills.

Attitudes Toward Mathematics

The second hypothesis predicted that students in the IMP program would have statistically significantly better attitudes toward mathematics than those in the Algebra program. In this study, attitude toward mathematics was measured on three scales including anxiety, individual interest, and situational interest.

The results of the pretest indicated that the IMP group had a lower anxiety mean score than the Algebra group (3.71 vs. 4.06, respectively). Analysis of variance of the pretest indicated that this difference between the IMP and Algebra mean scores was not statistically significant. The results of the posttest showed that the IMP group anxiety mean score increased to 4.00. In other words, IMP students responded that they were less anxious, whereas the anxiety mean score for Algebra students dropped down to 3.98 (i.e., Algebra students responded that they were more anxious). Analysis of variance of the posttest indicated that there is no statistically significant different between the IMP and the Algebra group with regard to Anxiety.

The second attitude scale measured in this study was Situational Interest. The Situational Interest scale pretest mean for IMP students was lower than the Algebra Situational Interest scale pretest mean (4.11 vs. 4.41, respectively) with no statistical significant difference reported. The results of the posttest indicated that the IMP Situational Interest scale mean dropped (from 4.11 to 3.93) as well as the Algebra mean (from 4.41 to 4.20) with no statistically significant difference reported between the two groups.

The third attitude scale measured in this study was Individual Interest. The Individual Interest pretest scale mean for IMP students was lower than the Individual Interest scale mean for the Algebra group (3.46 vs. 3.82, respectively) with no statistically significant difference reported. More important, comparing the results of the pretest and posttest indicated the Individual Interest scale mean dropped for both the IMP group (3.46 to 3.45) and the Algebra group (3.82 to 3.53) with a still higher mean for Algebra than IMP mean. The results of the analysis of variance of the posttest and pretest score means indicated that there was no statistically significant difference between the IMP and the Algebra groups with regard to the Individual Interest scale. Thus, the IMP instruction did not improve the students' individual interest.

In summary, the analysis of the results of this study have shown that the there were no statistically significant differences between the IMP and Algebra group with regard to Anxiety, Situational Interest, and Individual Interest scales for both the pretest and posttest.

Previous research studies indicated that there is a strong relationship between the curriculum, instructional quality, and student attitudes. For instance, a study by Reynolds and Walberg (1992) reported a strong correlation between instructional practices and mathematics attitudes. A study by Hembree (1990) indicated that there is a strong inverse relationship observed for an enjoyment of mathematics and attitudes (self-confidence) toward the subject.

A study by Mitchell (1994) showed that environments (classes) with high situational interest were associated with substantial increases in the mean Individual Interest scale scores of students and had a beneficial impact on decreasing mathematics anxiety. More important, Mitchell argued that teachers and learning environments that are effective at increasing students' motivation to learn mathematics are likely helping students in many ways by increasing students' interest in mathematics inquiry.

The instructional techniques used in IMP were new to most students. It takes longer time than 6 weeks to get adjusted to IMP. Students were not used to the type of assignments in IMP. The assignments were difficult and challenging word problems. In elementary and middle schools, students were exposed mainly to drill-and-practice mathematical problems. The IMP curriculum did not provide students with sufficient examples to enable them to solve the problems. Instead, it utilized the discovery approach as an instructional tool for learning. All these factors combined together were contributing factors for not obtaining statistically significant differences between IMP and the Algebra groups on the attitude scales of Anxiety, Situational Interest, and Individual Interest.

Problem-Solving Performance Instrument

In this study, the mathematical problem-solving test was used for the first time with high-school students. In order to determine if this instrument can be utilized again with future research the following three issues will need to be discussed. First, a discussion of what the instrument measured will be presented. Second, the scoring system utilized with this instrument is discussed. Finally, a discussion of the reliability of this instrument is presented.

To measure problem-solving performance, a four-problem test was used. This test was given as a pretest and posttest. This test was used to measure mathematical process as well as product. In the literature, process was defined as activities that direct the solution research, whereas the product was defined the actual solution (Kantowski, 1977). In this study, process was defined as pointing out what was the unknown, providing a list of all relevant information, coming up with a plan, and finally carrying on the plan. The product was defined as providing and labeling the answer to the given problem. A total of 7 points was assigned to each problem, and only one point was given to the correct answer. Thus, 6 points were for the process and only one point was given for the product. The reason for emphasizing process as well as product is that the study operated on the premise that mathematical problem solving ought to be scored in a way that recognizes supportive thinking as well as answer correctness.

An easy-to-use scoring system was adapted for this study (Van Akkeren, 1995), which when employed by two scorers consistently yielded similar results. To estimate the interrater reliability for the mathematical problem-solving test, 10 students' pretests were selected randomly and were given to two scorers to correct. Interrater reliability was defined as the percentage of agreement between the two scorers. Percentage of agreement was calculated by dividing the total number of problems for which agreement occurred by the total number of scored. Agreement was said to occur if there was a perfect match of total points or there was a difference of only one point between the total points given by the scorers. Because the interrater agreement of .88 (87.5%) was found for the mathematical problem-solving pretest and .98 (98%) was found for the posttest, the performance for process problem solving can be reliably measured in such a manner that process as well as correctness is taken into account.

Four earlier studies employed instruments that attempted to measure problem-solving process as well as answer correctness (Charles & Lester, 1984; Guernon, 1989; Putt, 1978; Van Akkeren, 1995).

Putt (1978) investigated the effect of heuristics instruction on fifth-grade students' performance for solving process problems. A 6-point instrument was used to measure performance in three sections including understanding the problem (0 to 2 points), planning (0 to 2 points), and results (0 to 2 points). To test reliability, 18 posttests were selected randomly and scored by two scorers. A correlation of .86 was found between the two scorers, which may not be a stable estimate of the reliability because of the small size of the sample.

Charles and Lester (1984) studied the impact of heuristics instructions on performance with fifth- and seventh-grade students. Problem-solving performance was measured by an analysis of students' written work. Using a scoring system similar to Putt (1978), Charles and Lester obtained similar high interrater correlations that ranged from a low of .88 to a high of .94.

Guernon (1989) examined the effects of teaching heuristics on the problem-solving performance of eighth-grade students. Performance was obtained using a scoring system of either 0,1,3, or 5 points for understanding the problem and either 0,1, 3, or 5 points for problem execution. For the pilot study, four scorers scored the problems, and reliability was .79 for exact match and .92 when scorers were within two points of one another.

Van Akkeren (1995) studied the effect of cognitive modeling on fourth-grade students' performance. Van Akkeren used the same scoring system and instrument utilized in this study. Van Akkeren measured students ' performance at the end of the treatment and after 2 weeks. A correlation of .98 was found for both the posttest and the follow-up test.

Implications

The importance of improving high-school students' mathematical problem-solving skills and their attitudes toward mathematics has become a major issue among educators in 1980s and 1990s (California Department of Education, 1992; Lester, 1994; Mitchell, 1993; National Council of Teachers of Mathematics, 1989). The educational significance of this present study is centered on the need to provide teachers with an effective instructional method that will enhance students' mathematical problem-solving performance skills as well as their attitudes toward mathematics.

The present study yielded negative results for IMP curricula with regard to both mathematical problem-solving performance and attitudes toward mathematics when utilized in mathematics classes with remedial students with high-mathematics anxiety. More research is needed to confirm these results. If other studies corroborate the findings of this study, then there will be some implications for those institutions involved with mathematical problem-solving instruction. First, high-school districts in California similar to the district used in this study have to offer more than one curriculum in order to meet the needs of students in mathematics classes. This implication is an important issue, because there is no single curriculum that meets the needs of all students. For instance, IMP has proven to be a successful curriculum when applied in magnet schools with high-achieving students. In this study, when subjects were remedial and low-achiever students, the IMP instruction after 6 weeks failed to produce positive results regarding mathematical performance and attitudes. Second, in order to ensure success in those programs, schools have to provide appropriate training and staff development for their high-school mathematics teachers. Third, educators should expose both elementary- and middle-school students to mathematical-problem curricula similar to IMP so as they enter high school they are already familiar with such curricula. Fourth, credential programs have to demonstrate to students in their programs the different mathematics curriculum and instructional strategies that meet the needs of all students at both ends of the spectrum (i.e., low and high achievers).

Recommendations for Future Research

Results and limitations of this present study promoted the following recommendations for future research in mathematical problem-solving.

First, because the 6-week period between pretest and posttest did not produce positive results for IMP treatment, this study should be replicated with a longer treatment span.

Second, because this study examined mathematical problem-solving performance in first-year mathematics and it takes long time to develop mathematical problem-solving skills, this study should be replicated at other grade levels such as fourth-year high-school mathematics.

Third, because the present study found a low reliability coefficient for the Situational Interest scale for both pretest and posttest (.45 and .54, respectively), and another study conducted by Mitchell (1993) using the same instrument at fourth-grade level found a higher reliability, future research should be conducted to explore the issue of situational interest reliability among high-school students. Such research should examine the relationship that may exist between reliability of the Situational Interest scale and the age of students.

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Appendix A

Survey

Mathematics Survey

There are no right or wrong answers to the questions on this survey: it asks you only for your perceptions and opinions. Your responses to this survey will not be seen by your teacher or anyone else at your school, except in the form of summery results (e.g. 45% of the students agreed that they like mathematics).

Your responses on this survey are part of a research project in which the researcher is trying to better understand how students experience mathematics.

All of the items in this survey are in a multiple-choice format. For each of these items, please CIRCLE the one choice that best describes your opinion or feelings about the statement.

For example, the circle around "disagree" in the following item:

I like homework.

strongly agree   agree   slightly agree   slightly disagree   disagree   strongly disagree

means you Do not agree with the statement

Number:________________

Gender: _______ Male _________ Female

1. Mathematics is enjoyable for me.
strongly agree agree slightly agree slightly disagree disagree strongly disagree

2. When the teacher says he/she is going to ask me some questions to find out much I know about math, I worry that I will do poorly.
strongly agree agree slightly agree slightly disagree disagree strongly disagree

3. I feel I most successful when I learn something new.
strongly agree agree slightly agree slightly disagree disagree strongly disagree

4. I think mathematics is boring.
strongly agree agree slightly agree slightly disagree disagree strongly disagree

5. Taking math scares me.
strongly agree agree slightly agree slightly disagree disagree strongly disagree

6. When I am in math class, I usually feel very much at ease and relaxed.
strongly agree agree slightly agree slightly disagree disagree strongly disagree

7. Compared to other subjects, mathematics is exciting to me.
strongly agree agree slightly agree slightly disagree disagree strongly disagree

8. When the teacher is showing the class how to do a problem, I worry that other students might understand the problem better than me.
strongly agree agree slightly agree slightly disagree disagree strongly disagree

9. I am not interested in mathematics.
strongly agree agree slightly agree slightly disagree disagree strongly disagree

10. I feel most successful when I do the work better than other students.
strongly agree agree slightly agree slightly disagree disagree strongly disagree

11. I actually look forward to going to math class this year.
strongly agree agree slightly agree slightly disagree disagree strongly disagree

12. Compared to how much I know in other classes I am taking, I know a lot about math.
strongly agree agree slightly agree slightly disagree disagree strongly disagree

13. I feel most successful when all of the work is easy.

strongly agree agree slightly agree slightly disagree disagree strongly disagree

14. When I am taking a math test, I usually feel very much at ease and relaxed.

strongly agree agree slightly agree slightly disagree disagree strongly disagree

Thanks for your help!

When your are done, raise your hand and the person conducting this survey will collect your completed form.

Appendix B

Problem-Solving Test

Problem-Solving Test

Number:______________________

Instructions: Please answer each of the following 4 problems. You have the entire period to complete this test.

Problem #1

Anne is training for an important swim event. She will be swimming in the 14-year-old category against 20 other swimmers from 5 different swim clubs. On the first day of her training, Anne swam 1 lap. She swam 4 laps on the second day. On the third day, she swam 7 laps. If Anne continues to improve this way, on what day she will reach her goal swimming at least 30 laps in a single day?

Please do the following on this paper (use the back if needed):

- State in writing what you're trying to figure out.
- Write all the useful facts in a list. Do not include useless facts.
- State in writing your strategy for solving the problem.
- Solve the problem. Show all your work. Do not erase anything.
- State in writing what your answer is and label it correctly.

Problem #2

In August, the annual city softball tournament will be held in San Francisco. Each participating team must have won at least 20 games during the regular season and not lost more than 5 games. There will be 7 teams playing in the San Francisco softball tournament this year. Each team is scheduled to play every other team once. How many games are scheduled for the all the teams?

Please do the following on this paper (use the back if needed):

- -State in writing what you're trying to figure out.
- -Write all the useful facts in the list. Do not list useless facts.
- -State in writing your strategy for solving the problem.
- -Solve the problem. Show all your work. Do not erase anything.
- -State in writing what your answer is and label it correctly.

Problem #3

A local park has 25 squirrels living in it. The squirrels like to eat acorns. One particular squirrel found a total of 50 acorns over a period of 5 days. During this five-day period, each day the squirrel found 3 more acorns than the day before. How many acorns did the squirrel find on each of the 5 days?

Please do the following on this paper (use the back if needed)

-State in writing what you're trying to figure out.
-Write all the useful facts in a list. Do not list useless facts.
-State in writing your strategy for solving the problem.
-Solve the problem. Show all your work. Do not erase anything.
-State in writing what your answer is and label it correctly.

Problem #4

Sam wants to rent a car to use for a three-day weekend holiday. He wants to save the most amount of money on this car rental. After careful research, Sam found two competitive car-rental companies that offer the cheapest rates in town. Company A rents their cars for $18 a day plus $0.10 for each mile. Company B rents their cars for $30 a day and does not charge for mileage. Which of those two companies would you recommend Sam to rent from?

Please do the following on this paper (use the back if needed)

- -State in writing what you're trying to figure out.
- -Write all the useful facts in a list. Do not include useless facts.
- -State in writing your strategy for solving the problem.
- -Solve the problem. Show all your work. Do not erase anything.
- -State in writing what your answer is and label it correctly.

Appendix C

Parent-Information Letter

March 2nd, 1998

Dear Parent:

My name is Samer Malouf, and this is my third year teaching at McAteer High School. Besides teaching mathematics and physics, I am currently working toward the completion of my doctorate degree in Learning and Instruction at the University of San Francisco. My research focus is on problem solving in mathematics.

With the support and approval of your teenager's principal and mathematics teacher, I have planned a study in his/her mathematics class. I am looking at students' mathematics performance and attitudes toward mathematics in the first few years of high school. Your child is attending one of these classes. As a participant in this study, your teenager will complete an attitude survey and a mathematical problem-solving test. This test will be administered to your teenager during his or her mathematics class sometime in March or April.

The attitude survey will take less than 10 minutes to complete and consists of simple nonthreatening statements such as 'I like mathematics'. The problem-solving test will take one class period and will include a few interesting problems for students to work on.

I need your permission to allow your teenager to participate in the study, and to obtain his or her score from the mathematics section of the Comprehensive Test of Basic Skills (CTBS).

The results of this study may greatly help us better understand how students develop problem-solving skills.

Your son or daughter's scores will be kept strictly confidential and will not be seen by anyone at their school. Tests are administered without names.

If you choose not to have your teenager participate in this study, he or she will be working individually on a mathematics assignment.

If you have any questions regarding this study, please feel free to call me at (415)-695-5700.

I appreciate your cooperation and support.

Sincerely,

Samer Malouf

Samer Malouf

Consent to Participate in Malouf's Problem-Solving
Research Study

Please complete this portion by March 10th, and return it to your student's math teacher, as soon as possible.

Student's Name: ________________________________
Parent's Name: ________________________________
Parent's Signature: ____________________________
Date: __________

- ☐ Yes, my child can participate in this study, I understand that all results are confidential.
- ☐ No, I do not want my child to participate in this study.

Appendix D

SFUSD Approval Letter

SFUSD

Francisco Unified School District 135 Van Ness Avenue San Francisco California 94102-5299

December 15, 1997

Samer G. Malouf
1275 El Camino Real #301
Millbrae, CA 94030

Dear Mr. Malouf,

Thank you for submitting your request to conduct research to assess student learning in the IMP program. We have reviewed your request and are approving it. Our approval is at a central, District office level. It does not obligate any school site or teacher to participate in your study. You must still obtain their cooperation.

Please feel free to contact me if you have any questions. Please note that we must receive a copy of the results of your study when it is completed. In addition, in keeping with the District's commitment to professional development, it is critical that you share your work with the school community that assisted you in the course of your study. Good luck with your work.

Sincerely,

Bill Penuel

Bill Penuel, Ph.D.
Education Integration Specialist

Appendix E

Course Description

INTERACTIVE MATHEMATICS PROGRAM
First-Year IMP

Program Description:

The Interactive Mathematics Program is a four year college-prep program developed by the Interactive Mathematics Project. The program is in its sixth year of implementation. This program is partially funded by a National Science Foundation Grant. First-year IMP is the first year of a set of four-year long courses.

IMP's primary innovation is a curriculum that is problem-based rather than subject based. As a result, Algebra, geometry, trigonometry, probability and other areas of mathematics are interconnected with each other, with their applications and with other subjects.

The units vary in length from four to eight weeks, and each unit is organized around a central problem or theme. The learning of concepts and skills is motivated by this central focus.

Each unit includes a variety of smaller problem, both routine and non-routine, that develop the underlying skills and concepts needed to solve the central problem in that unit. These smaller problems can be either class activities or part of daily assignments.

For assessment of IMP students, teachers use open-ended questions, group and whole class discussion, student portfolios, oral presentations, and self-assessments.

Text and other books:

The text materials are a series of booklets that include:

* Patterns
* The Overland Trail
* The Game of Pig
* The Pit and Pendulum
* Shadows

Graduation requirements:

Each year long course fills ten of the twenty credits of mathematics required toward graduation from San Francisco Unified School District.

Algebra 1 & 2

Course Description

This is a first year course in elementary algebra. This course will emphasize the study of functions as the general and unifying concept which ties many algebraic topics together.

Learning Outcomes

Students will learn to use symbols and language of algebra, real number properties and their operation, functions with emphasis on linear and quadratic solution and graphing, polynomial expressions and their operation, factoring, exponents and radicals, slope formulas, and systems of linear equations.

Textbook

Algebra One--Merrill

Graduation Requirements

This on year course fulfills 10 credits of the 20 credit SFUSD math graduation requirement.

University of California a-f Requirements

This course satisfies the first year of the three year "c" requirement for math.

Related Courses

Algebra may be preceded by preparatory courses Math A and/or Math B, and is followed by Geometry and Advanced Algebra.

Appendix F

Sample Assignments

IMP Sample

Homework 4: Running with Tennis Shoes on the Overland Trail

Do the following steps for each of the problems below:

- Define an appropriate variable.
- Write an equation that represents the problem.
- Solve both the equation and the problem.

1. If Phillipe had $7 more, he could buy a $30 pair of tennis shoes. How much money does he have?

2. Yolanda jogged 2 miles to a lake, ran twice around the lake, and then jogged 2 more miles home. Altogether she traveled 10 miles. How far is it around the lake?

3. An Overland Trail family is carrying 5 gallons of water per person in its wagon. Then, unexpectedly, two stragglers ask to join the group. The family decides to include them, and figures out that this now means that there are only 4 gallons per person.

   How many people were in this Overland Trail family? (*Hint:* Start with a guess, and see if it works. Use the arithmetic steps you follow in testing your guess as a model for developing an equation.)

Oral Exercises

Tell whether the two expressions have the same value or different values.

1. $4 \times 4,\ 4 + 4$
2. $2 + 2,\ 2 \times 2$
3. $(1 + 1)1,\ 1 + 1 \cdot 1$
4. $0 \cdot 1 + 1,\ 0(1 + 1)$
5. $\frac{8 + 6}{2},\ 8 + \frac{6}{2}$
6. $\frac{12 - 9}{3},\ 4 - 3$

Tell which operation to perform first. Then simplify each expression.

7. $9 + 1 \cdot 5$
8. $(9 + 1)5$
9. $18 - 3 \div 3$
10. $(18 + 3) \div 3$
11. $3 + 2^3 - 1$
12. $(2 \cdot 3)^2 - 4^2$
13. $\frac{9 + 6}{9 \div 3}$
14. $5 \cdot 3^2 - 9 \cdot 2$
15. $[5(3^2 - 9)]2$

Written Exercises

Copy each partial statement. In Exercises 1-4, find the number that makes the statement true. In Exercises 5-14, use one of the symbols = or ≠.

A

1. $7 \times 8 = \underline{\ ?\ } \times 7$
2. $6 + 3 = 3 + \underline{\ ?\ }$
3. $5 \div \underline{\ ?\ } = 5$
4. $\underline{\ ?\ } \times 10 = 10$
5. $8 \times 1 \underline{\ ?\ } 8 \div 1$
6. $4 \div 2 \underline{\ ?\ } 2 \div 4$
7. $(5 + 4) + 9 \underline{\ ?\ } 5 + (4 + 9)$
8. $6(2 + 7) \underline{\ ?\ } 6 \cdot 2 + 7$
9. $5(4 - 1) \underline{\ ?\ } 5 \cdot 4 - 5 \cdot 1$
10. $(3 + 8) \div 1 \underline{\ ?\ } 3 + 8 \div 1$
11. $\frac{10 + 5}{10 - 5} \underline{\ ?\ } \frac{2 + 1}{2 - 1}$
12. $\frac{9 + 3}{3 + 3} \underline{\ ?\ } \frac{9}{3}$
13. $\frac{4^2 + 4}{4} \underline{\ ?\ } 16 + 1$
14. $10 - 2^2 \div 2 \underline{\ ?\ } (10 - 2)^2 \div 2$

Simplify.

15. $18 - (6 - 2)$
16. $18 - 6 - 2$
17. $126 \div 6 \div 3$
18. $126 \div (6 \div 3)$
19. $2 - [(7 - 3) - 2]$
20. $[4 - (5 - 2)] - 1$
21. $\frac{15 + 7}{15 - 4}$
22. $\frac{8 + 20 - 4}{8 - 4}$
23. $3(6 - 2^2)$
24. $3^2(6 - 2)$
25. $5 \cdot 3 - 2 \cdot 7 + 6 \cdot 0$
26. $2[(5^2 + 1) - (4^2 - 1)]$
27. $5[(2^2 - 1) - (2^2 - 2)]$
28. $3(10^2 - 8^2)$
29. $24 \div 12 + 12 \div 4$
30. $4 \cdot 7 - 2^3 \div 2$
31. $[2(7 - 2) - 3^2]3$
32. $8[2(2 + 3) - (3^2 + 1)]$
33. $2 \cdot 4^3 - 3 \cdot 4^2 + 5 \cdot 4 - 7$
34. $5 \cdot 2^3 + 1 \cdot 2^2 - 9 \cdot 2 + 1$
35. $\frac{5 + 2 \cdot 5}{2^2 + 1} + 5^2 - 5$
36. $\frac{3^2 \cdot 6 \div 3}{2 \cdot 1 + (3 + 1)^2}$