2019

EFFECTS OF THINK-ALOUD PROTOCOL ON THE MATHEMATICAL PROBLEM-SOLVING SKILLS OF SEVENTH- AND EIGHTH- GRADE STUDENTS WITH LEARNING DISABILITIES

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EFFECTS OF THINK-ALOUD PROTOCOL ON THE MATHEMATICAL PROBLEM-SOLVING SKILLS OF SEVENTH- AND EIGHTH-GRADE STUDENTS WITH LEARNING DISABILITIES

A Dissertation Presented to

The Faculty of the School of Education

Learning and Instruction Department

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Education

by

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San Francisco, California

May 2019

THE UNIVERSITY OF SAN FRANCISCO

Dissertation Abstract

Effects of Think-Aloud Protocol on the Mathematical Problem-Solving Skills of Seventh- and Eighth-Grade Students with Learning Disabilities

The purpose of this study was to examine the effect of a cognitive- and metacognitive-strategy instruction on the mathematical problem-solving performance and metacognitive experience of 22 seventh- and eighth-grade students with learning disabilities. When solving mathematical word problems, students with learning disabilities typically lack self-regulation processes (Larson & Gerber, 2002) tend to respond impulsively, to use trial and error, and fail to evaluate or verify their solutions (Bryant, Bryant, & Hammermill, 2000).

This study used the Metacognitive Experience Survey (MES), two sets of three mathematical-word-problem probes of varying complexity levels, and think-aloud protocols to measure intervention effect. The first research question probed the effect of the intervention on the mathematical-problem-solving performance of the participants as measured by their metacognitive verbalizations collected through think-aloud protocols. Qualitative analysis of the transcripts revealed four emerging themes: students with high metacognition were more successful in performing tasks correctly even when their nonproductive metacognitive verbalizations were above 25.0%; students in the high- and average-metacognition categories successfully solved the 3-step probe, whereas students in the low-metacognition category were not successful in solving the 3-step probe; students in the low-metacognition category used less productive metacognitive verbalizations as the complexity level of the probe increased; and students from all the

metacognition categories extensively used cognitive- metacognitive strategies compared with preintervention observations.

The second research question probed the intervention effect on the mathematical-problem-solving performance of the participants as measured by the change from pre- to postintervention scores on two sets of three mathematical probes. A dependent-samples *t* test revealed no strong statistically significant relationships. One weak but statistically significant relationship was found for students' performance on the 1-step probe. There was an increase in the means for the 1-step and 3-step probes from pre- to postintervention. For the 2- and 3-stepstep probes, however, the change from pre- to postintervention was not statistically significant.

The third research question probed the effect of the strategy instruction on the metacognitive experience of the participants as measured by the MES. A dependent-samples *t* test results indicated an increase in the participants' metacognitive experience means from pre- to postintervention but the postintervention mean was not statistically significantly different than the preintervention mean.

Notwithstanding that statistically significant changes were not realized across the MES and the mathematical-word-problem probes, important insights were obtained from the think-aloud protocols.

Signatures

This dissertation, written under the direction of the candidate's dissertation committee and approved by the members of the committee, has been presented to and accepted by the Faculty of the School of Education in partial fulfillment of the requirements for the degree of Doctor of Education. The content and research methodologies presented in this work represent the work of the candidate alone.

Nasa L. Cole May 16, 2019

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CHAPTER I

INTRODUCTION TO THE STUDY

Progressively, being adept at mathematical problem solving is vital to success in mathematics curriculum (Krawec, Huang, Montague, Kressler, & de Alba, 2012) as well as to a student's achievement in school and career (Hudson & Miller, 2006). Solving mathematical word problems entails that students possess declarative and procedural knowledge (Montague & Applegate, 2001). The declarative or conceptual knowledge in mathematics relates to a student's ability to recognize and apply mathematical operations and algorithms in various situations, and the procedural knowledge entails the ability to apply declarative knowledge effectively as well as to coordinate multiple cognitive and metacognitive processes associated with proficient problem solving (Mayer, 1985; Zawaiza & Gerber, 1993). Montague (2001) maintained that students need conditional knowledge (conceptual and procedural) to enable them to select and implement appropriate strategies and to adjust their behaviors to changing mathematical problem-solving demands. Both knowledge bases, however, are impaired in students' with learning disabilities who demonstrate a lack in strategy knowledge and use (Bornert & Wilbert, 2015; Kraai, 2011; Krawec et al., 2012), who apply strategies inconsistently and ineffectively (Kraai, 2011; Rosenzweig, Krawec, & Montague, 2011), who manifest a lack in self-efficacy processes (Larson & Gerber, 2002) as they tend to respond impulsively, who use trial and error, who fail to evaluate their answers or verify their solutions (Bryant, Bryant, Hammil, 2000; Kraai, 2011), and who display an inability to transfer known strategies to different tasks in comparison with students without learning disabilities (Bornert & Wilbert, 2015; Fuchs et al., 2003; Hessels, Hessels-Schlatter, Bosson, & Balli, 2009).

Students with learning disabilities are poor problem solvers who also exhibit deficits in working memory, processing speed, and executive function (Geary, 2004; Johnson, Humphrey, Mellard,Woods, & Swanson, 2010). The situation is exacerbated when students' instruction in mathematical problem solving is restricted to textbook models and composed of sequenced list of activities for solving problems (e.g., *read, decide what to do, solve, and check the problem*) as is evident in most mathematics classrooms (Jitendra & Star, 2011). Consistently, these cognitive activities are inadequate for students with LD who possess limited knowledge of appropriate problem-solving strategies (Montague & Applegate, 2001), immature ways to identify the type of problem (Garcia, Jimenez, & Hess, 2006), and constrained ability to represent the problem visually (Booth & Thomas, 2000).

Mathematical problem solving, which correlates highly with success in mathematics, presents a challenge for students with LD (Geary, 2003). Metacognition, defined as reflective abstraction (Paiget, 1985), as well as one's thinking about own thinking (Flavell, 1976), was identified as crucial to mathematics success, to problem solving, and to overall academic achievement (Trainin & Swanson, 2005). Metacognition, further, was identified as a better index of learning performance than intelligence (Vennman & Spans, 2005). Luit and Kroesbergen (2006) contended that, although intelligence contributes about 25% of the explained variance in performance, metacognition contributes approximately 75% of the explained variance in performance. The performance variance between intelligence and metacognition occurs because metacognition entails higher-order-thinking skills that enable students to monitor and control cognitive processes employed during metacognitive activities (Langeli & Cabrele, 2006). One form of metacognitive processing is the think-aloud protocol (TAP). TAP is used in a variety of disciplines to facilitate meaningful learning. TAP entails a conscious unveiling of the thought processes used by students during problem-solving activities. TAP provides rich verbal data useful for instructional and assessment purposes in the scientific and quantitative reasoning of typically-developing students (Thelk & Hoole, 2006), in the nursing arena (Offredy & Meerabeau, 2005), in the cognitive processes of graduate engineering students (Litzinger, Van Meter, Firetto, Passmore, & Masters, 2010), as well as in the cognitive processes of students with learning disabilities (Desoete, Roeyers, & Buysse, 2001; Rosenzweig et al., 2011).

A review of literature suggested that previous research on using think alouds focused primarily on a cognitive or a metacognitive strategy when investigating the achievement of students with learning disabilities in mathematical problem solving. Further, other studies that used the think-aloud protocol indicated positive effect in the interaction of cognitive, metacognitive, or affective variables (Rosenzweig et al., 2011; Shuell, 1990) on the mathematical problem-solving skills of students with learning disabilities. To date, scant investigation focused on the effect of instruction in cognitive skills and metacognitive strategies on the mathematical problem-solving skills of seventh- and eighth-grade students with learning disabilities in an intact resource class. This study filled a gap in the thinking-aloud self-efficacy learning-strategies research and offered an instructional approach that can remediate the mathematical problem-solving skills of students with learning disabilities.

Purpose of the Study

The purpose of this study was to investigate the effect of implementing a cognitive- and metacognitive-strategy instruction on the word-problem-solving performance and self-efficacy perception of seventh- and eighth-grade students with learning disabilities in relation to their mathematical word-problem-solving skills (perception and performance). This study was conducted over a 7-week period (2 weeks of assessment, 5 weeks of intervention) in the students' resource classroom. During the study, students learned how to apply the think-aloud protocol as they attempted mathematical word problems. Students used Montague's (1992) model comprised of the seven cognitive skills and the three metacognitive processes to solve word problems.

To examine the efficacy of the intervention, this study used a pre- and postintervention design. Learning was measured using quantitative and qualitative methods. The qualitative data served to confirm and augment data gathered through quantitative measures. The quantitative instruments included the Metacognitive Experience Survey (MES) and two sets of three word-problem probes. The Metacognitive Experience Survey assessed students' self-efficacy beliefs about their ability to solve mathematical word problems. The first set of three word-problem probes and the MES were used as pre- and postintervention items. The verbalizations of six students were audio-recorded as they solved the second set of three word problems. Transcription and coding of the verbalizations were analyzed to assess students' perception of mathematical performance, attitude toward mathematics, and attitude toward mathematical word problems. The descriptive information generated by analyzing the audio recordings of students' verbalizations as they solved mathematical word problems served as the qualitative component of this study. The analyses of the data obtained in this study provided empirical information on the effect of cognitive- and metacognitive-strategy instruction on the mathematical-word-problem-solving performance of students with learning disabilities. The analyses of the think-aloud-protocol data enabled exploration of the role of students' verbalization during mathematical-problem-solving event.

Background and Need

Swanson and Sachs-Lee (2001) argued that problem solving encompassed a complex behavior in relation to the cognitive development of adolescents. Problem solving was defined as the ability to employ cognitive processes to tackle and resolve intricate cross-disciplinary tasks (U.S. Department of Education National mathematics Advisory Panel, 2008). Expert problem solvers, further, executed problem-solving tasks by employing and integrating cognitive, metacognitive, and motivational elements (Schoenfeld, 1983). The three components termed *skill, metaskill,* and *will* (Mayer, 1998) referred to possessing domain-specific knowledge, possessing strategy on how to apply and monitor the knowledge, and possessing the intrinsic motivation and task-related interest respectively. Embedded in the definition of an expert problem solver was the presumption that the individual obtained extensive training in a specific domain and, therefore, was considered highly knowledgeable in that specific area. Given that students with learning disabilities receive instruction in the general-education setting where instruction in mathematical problem solving is restricted predominantly to textbook models and composed of sequenced list of activities for solving word problems (e.g., read, decide what to do, solve, and check the problem), it is imperative that students with learning disabilities receive extensive cognitive and metacognitive strategy instruction in the domain of mathematical problem solving.

An international measure used to assess the performance of participating students worldwide is the Trends in International Mathematics and Science Study (TIMSS). Established in 1985, TIMSS provides a comparative perspective of student achievement in mathematics and science curricula for fourth and eighth graders every 4 years. TIMSS was designed to align broadly with mathematics and science curricula in the participating countries and education systems (TIMSS, 1995, 1999, 2003, 2007, 2011) thereby producing results that indicated the degree to which students acquired the mathematics and science concepts and skills taught in school. Over the 16-year period of implementation, TIMSS has provided an unexcelled data resource for trends in mathematics and science achievement (Foy, Arora, & Stanco, 2013). At both fourth- and eighth-grade levels, the mathematics framework was organized around a content dimension that indicated the mathematics subject matter to be assessed and a cognitive dimension that specified the thinking processes to be assessed (Mullis, Martin, Ruddock, O'Sullivan, & Preuschoff, 2009). TIMSS' three cognitive domain processes include *knowing facts, procedures, and concepts; applying knowledge and understanding; and reasoning.* Mullis and associates (2012) suggested that *knowing* pertained to the student's knowledge base of mathematics facts, concepts, tools, and procedures. *Applying* emphasized the student's ability to apply knowledge and conceptual understanding in a problem-solving situation, and *reasoning* went beyond finding the solution of typical problems to unfamiliar situations, complex contexts, and multistep problems (p. 140). For the TIMSS, the cognitive domains assessed were the same for fourth and eighth grades and comprised the scope of cognitive processes used in solving mathematical problems in the primary- and middle-school years.

In the 2011 study, participants in TIMSS comprised internationally representative samples of students in 63 countries. The TIMSS 2011 International Results in Mathematics illustrated data on the trend of student performance in mathematics over the five assessments since 1995, on student performance in the mathematics content domains (algebra, geometry, etc.), and on student proficiency in handling the problem-solving tasks in these mathematical contexts (Gonzales et al., 2011; TIMSS). A comparative synopsis of the eighth-grade students' performance in 2011 indicated that the U.S. average mathematics score (509) was higher than the international TIMSS scale average (500) for that same year and 17 score points higher than the U.S. average mathematics score in 1995 (509 vs. 492). In regard to the fourth-grade students, the US average score in mathematics (541) was higher than the international TIMSS scale average (500), and when compared with the 1995 and 2007 scores, the U.S. 2011 fourth-grade average mathematics score (541) was 23- and 12-score points higher, respectively. Across the fourth and eighth grades, however, the U.S. students demonstrated higher proficiency in knowing mathematics (i.e., recalling, recognizing, and computing) than in applying mathematical knowledge, and reasoning (problem solving). In fact, a comparison between the US and select countries (Singapore, Korea, Hong Kong, Chinese Taipei, Japan, Northern Ireland, and Belgium) across the cognitive domains indicated that, although the US has made improvements, fourth- and eighth-grade students in the US lagged behind in mathematics cognitive and metacognitive domains (Kastberg, Ferraro, Lemanski, Roey, & Jenkins, 2013).

Over a period of 2 decades (1980-2000), members of the National Council of Teachers of Mathematics (NCTM), the largest organization concerned with mathematics education, initiated mathematics reforms that were structural, curricular, or instructional in nature. The Council maintained that *problem solving* was at the core of any substantive mathematics curriculum (NCTM, 1980). Council members stressed the need to replace the emphasis on computational fluency with a proficiency in higher-order conceptual skills because the latter constituted a better measure of mathematical competence (NCTM, 1980). Another area the Council advocated for was *reasoning.* Council members argued that a student who was imparted with the reasoning (logic) associated with a mathematical procedure was more likely to learn and apply the procedure appropriately than a student who attempted to apply rules without regard to their reasonableness (NCTM, 2003). Similarly, Council members highlighted *communication* as an important assessment tool that students use to explain, write, draw, or otherwise demonstrate what they have learned. Accordingly, they recommended that teachers formulated alternate means of assessing the communication of students with learning disabilities or students with limited-English abilities who encountered academic challenges as a result of their identified language and processing impairments. As stated previously, students with learning disabilities struggled in the identified areas (problem-solving, reasoning, communication) and, therefore, required intervention that addressed the specific domains.

Predictably, students with learning disabilities were more affected, comparatively, by the US lag in achievement in the mathematics cognitive and metacognitive domains. Ample research confirmed that cognitive and metacognitive domains were difficult for students with learning disabilities (Fuchs & Fuchs, 2002; Geary, 2003; Hanich, Jordan, Kaplan, & Dick, 2001; Montague & Applegate, 1993). Researchers agreed that when solving mathematical word problems, students with learning disabilities responded impulsively, used trial and error, and failed to verify solution path more than their typically-achieving peers (Bryant, Bryant, & Hammermill, 2000; Fuchs & Fuchs, 2002; Gonzalez & Espinel, 2002; Geary, 2004; Johnson et al., 2010). Yet, students with LD characteristically overestimated their ability compared with their peers (Garette, Mazzocco, & Baker, 2006; Montague, 1997). Mayer (1985) noted that to solve mathematical word problems, students needed to be able to represent the problem, develop a solution path, and execute the solution. Mayer (1985) further stated that cognitive processing and metacognitive strategies (e.g., visualizations, estimations, self-questioning) were integral to representing the problem. In other words, mathematical problem solving entailed a proper synthesis and execution of metacognitive strategies and cognitive processes. These processes, however, were challenging for students with learning disabilities who typically experienced processing deficits as a manifestation of their disability (Montague, 2004, 2008). As a result, students with learning disabilities were less likely to use task-appropriate metacognitive strategies when solving mathematical word problems (Stone & May, 2002), although metacognition enhanced the implementation of the cognitive strategy as well as student learning (Azevedo & Cromley, 2004).

In 2010, California joined 45 other states across the nation to adopt the Common Core State Standards (CCSS) in Mathematics and English Language Arts (ELA). The CCSS provided consistent, clear, and challenging standards for what students were expected to learn and be able to do in mathematics from kindergarten (K) through Grade 12. The California Assessment of Student Performance and Progress (CAASPP) system, the assessment arm of the CCSS, replaced California's Standardized Testing and Reporting (STAR) program in July 2013. The STAR program had measured the achievement of California Content Standards for grades 2 through 11 using five benchmarks to indicate a student's proficiency in English Language Arts and Mathematics. The students took part annually in statewide testing, and schools were assigned an Academic Performance Index based on results from STAR testing. Under the No Child Left Behind (NCLB, 2001) mandate, the Academic Performance Index also was used to evaluate schools for Adequate Yearly Progress toward the growth in curricula instruction and assessment of all students, including students with learning disabilities. The No Child Left Behind Act (NCLB, 2001) and the Individuals with Disability Education Act (IDEA, 2004) were standards-based reforms that mandated the accountability of and access to the general curriculum for all students, including students with learning disabilities. Nolet and McLaughin (2000) contended that the purpose of standards-based reform was to align special-education programs and policies with the larger national school-improvement efforts. Nolet et al. (2000) argued that the reforms linked academic achievement and accountability for all students and increased the prospects in educational planning for students with learning disabilities.

A school district in Northern California administered the Smarter Balanced Assessments Consortium (SBAC) to students in April 2015. The SBAC is part of the California Assessment of Student Performance and Progress (CAASPP). This was the first year that all California students in grades 3 through 8 and 11 were administered the test. As a result, students' scores were considered baseline performance against future test scores. Students with learning disabilities at this middle school historically lagged in the domain of mathematics problem solving. For instance, the results from the STAR 2012-13 indicated that of the 94 students with disabilities assessed on the mathematics domain, 15% scored within the *Proficient or Above* range compared with 52% of the 566 students with no identified disabilities. In the 2015 CAASPP, 181 seventh- and 172 eighth-grade students with no identified disabilities were tested on the *Problem Solving & Modeling/Data Analysis* domain. Thirty-six percent of the seventh graders and 49% of the eighth graders scored in the *Standard Exceeded* and *At or Near Standard* range. Twenty-three seventh- and 21 eighth-grade students with identified learning disabilities were tested on *Problem Solving & Modeling/Data Analysis* domain. Only 13% of the seventh graders and 15% of the eighth graders scored in the *Standard Exceeded* and *At or Near* Standard range.

The CCSS was designed as a blueprint for mathematics instruction in the general-education classroom (National Governors Association Center for Best Practices, 2010), and considering that many students with learning disabilities receive their mathematics instruction in the general- education classroom, concerns arose relating to effective strategies to use for instruction on mathematics problem-solving tasks for students with learning disabilities while simultaneously adhering to the Common Core principles

(Fuchs & Fuchs, 2013). Teachers nationwide claimed that, although the benefits of the rigorous CCSS reached their districts, the positive effect was not evidenced in their classrooms (Gates Foundation, 2012). Foundation informed that even in states that had begun to provide professional development and support, teachers still struggled with the progression of the complex mathematics skills across grade level and across disabilities (Gates Foundation, 2012). Teachers stressed that only a few teaching strategies were available to teach students with disabilities content that linked to the CCSS or other state standards in mathematics (Browder, Jimenez, et al., 2012). As a result, the achievement of students with learning disabilities continued to lag behind the achievement of their peers without learning disabilities. There was a need, therefore, to develop more empirical interventions for students with learning disabilities as scant studies addressed the effect of cognitive and metacognitive skills on the achievement of students with learning disabilities especially bearing the rigorous CCSS framework in mind.

The framework of the CCSS in mathematics delineated eight (see Table 1) instructional practices (i.e., practice standards) for teachers to implement during instruction. Teachers were encouraged to provide opportunities to apply the practice standards throughout as they taught the mathematical content standards (Russell, 2012). Research on the use of cognitive strategies in solving mathematical word problems indicated that students with learning disabilities typically experience processing deficits as a manifestation of their disability (Montague, 2004, 2008; Sweeney, 2010). Students with learning disabilities were consequently less likely to use task-appropriate metacognitive strategies when solving the CCSS mathematical word problems.

Azevedo et al. (2004) noted that metacognitive processing equally enhanced the implementation of a cognitive strategy and student learning. It was vital, therefore, that teachers employed instructional practices that incorporated metacognitive processing coupled with cognitive strategy to deliver instructions on the new CCSS mathematics standards. In this way,

Table 1
Common Core Standards for Mathematical Practice

Number	Standard
1	Make sense of problems and persevere in solving them
2	Reason abstractly and quantitatively
3	Construct viable arguments and critique the reasoning of others
4	Model with mathematics
5	Use appropriate tools strategically
6	Attend to precision
7	Look for and make use of structure
8	Look for and express regularity in repeated reasoning

students with learning disabilities would learn how to employ metacognitive strategies (i.e., self-questioning) to monitor cognitive processing during mathematical problem solving. For this reason, this study examined the effect of a metacognitive strategy (TAP) with eighth-grade students with learning disabilities on their mathematical problem-solving skills.

Theoretical Rationale

This study used think-aloud protocols to investigate the effect of the cognitive and metacognitive processes on the mathematical problem-solving skills of seventh- and eighth-grade students with learning disabilities. To examine the mathematical-problem-solving performance and perception of students with learning disabilities, the current study synthesized Montague's 1992 theory of mathematical problem solving with

Flavell's 1979 theory of metacognition. This section presented the two theories related to the foci of this study: metacognition and model of mathematical problem solving.

In a meta-analysis, Lester (1994) identified two implications that correlated metacognition and mathematical problem solving. The first was that effective metacognitive activity during mathematical problem solving entailed knowing *what* and *when* to monitor as well as *how* to monitor, and the second specified that teaching students to become aware of their cognition, including the ability to monitor their mathematical-problem-solving actions, should take place in the context of learning specific mathematics concepts and strategies. Lester (1994) further posited that delivering a general metacognitive instruction, in isolation, was likely to be less effective. As a result, this investigation was conducted within the theoretical framework of Flavell's (1976) theory of metacognition (Figure 1) and Montague's (1992) model of effective mathematics problem solving (Figure 2). Students with learning disabilities were chosen as the

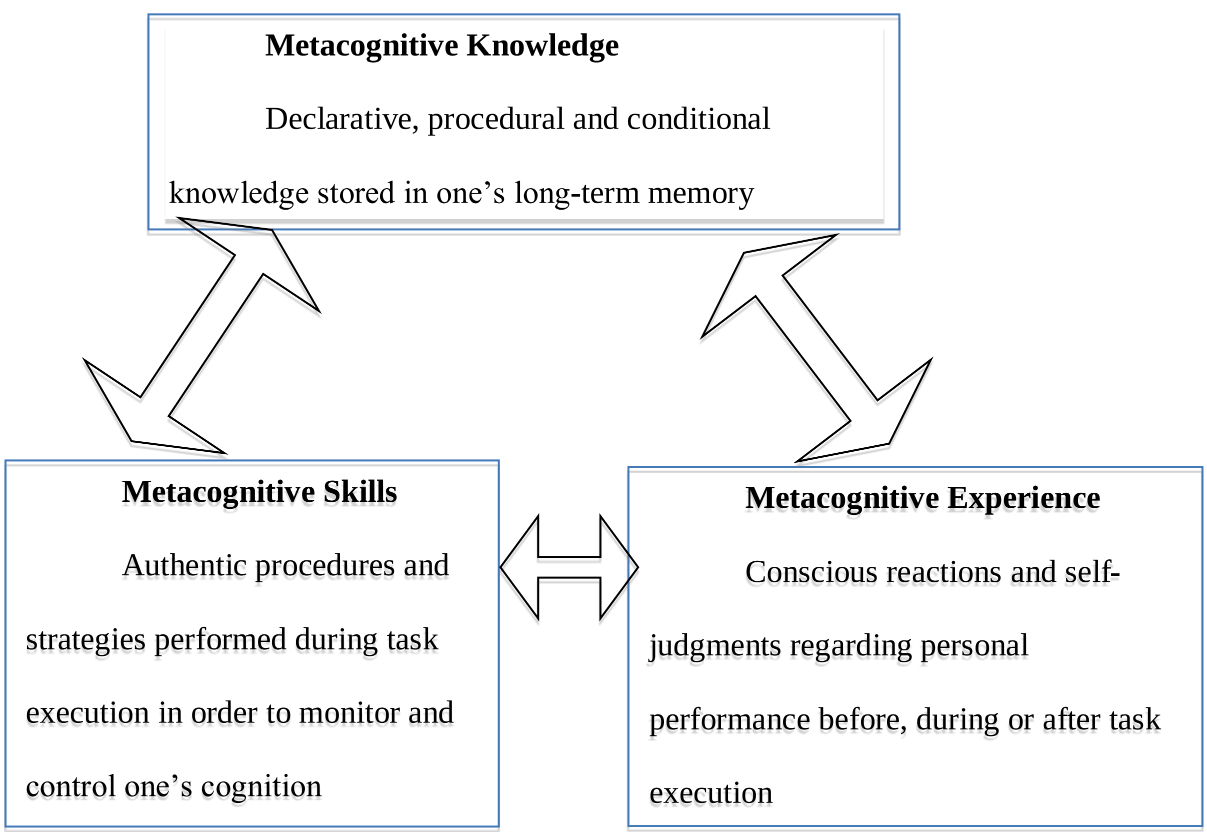

Figure 1. Flavell's (1979) Theory of Metacognition illustrating how the three elements of metacognition relate

population for this investigation because of the unique deficits in processing, organization, and persistence that characterized their disabilities in the area of mathematical problem solving.

Flavell's Theory of Metacognition (1979)

John Flavell (1971), a cognitive developmental psychologist, developed the concept of metamemory (now known as metacognition). Flavell (1971) described metamemory as an individual's ability to manage and monitor the endeavor of inputting, storing, searching, and retrieving information from one's own memory. Flavell (1971) theorized that the process of metamemory was deliberate, conscious, and strategic and was aimed at achieving a goal or outcome. In a later article, Flavell (1976) extrapolated that monitoring and regulation were two aspects of metacognition, which he defined as follows:

In any kind of cognitive transaction with the human or nonhuman environment, a variety of information processing activities may go on. Metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects or data on which they bear, usually in service of some concrete goal or objective (Flavell, 1976).

Flavell's (1979) model of metacognition highlighted the interaction between four components of metacognition namely (a) metacognitive knowledge, (b) metacognitive experiences, (c) tasks or goals, and (d) strategies or skills. The last two components have been combined into *metacognitive skills* (Bannert & Mengelkamp, 2008; Desoete &

Roeyers, 2002; Lucangeli & Cabrele, 2006) as will be used henceforth in this proposal (Figure 1).

Metacognitive knowledge relates to one's knowledge or beliefs about the factors that effect cognitive activities. Flavell (1979) argued that metacognitive activity typically preceded and tracked cognitive activity; both were closely and mutually dependent.

An example of metacognitive knowledge would be when a student determined through prior knowledge that using the order Parenthesis, Exponent, Multiplication, Division, Addition, and Subtraction to solve a computation problem entailed multiplying prior to dividing terms. Metacognitive knowledge also apply to a person's awareness or perception that he or she was a visual rather than an auditory learner. Flavell (1979) concluded that one's beliefs about himself or herself as a learner helped or hindered his or her performance in learning situations.

Metacognitive experience pertains to an individual's internal response to or monitoring of his or her own knowledge or strategies. Metacognitive experience serves as an internal feedback mechanism to the individual about his or her current progress, future expectations of progress or completion, degree of comprehension, connecting new information to old, and many other events (Flavell, 2009). In this way, a metacognitive experience would cover the affective response that an individual demonstrates when encountering a task. For instance, a person's willingness or interest to undertake a future tasks may be determined by his or her perceived success or failure, task difficulty, frustration or satisfaction, and confidence with prior similar tasks.

Metacognitive skills refer to the strategies and procedures (e.g., self-observation, self-monitoring, self-questioning, and so on) used during task execution to facilitate monitoring and controlling one's cognition (Efklides, Kiorpelidou, & Kiosseoglou, 2006). Some of these strategies are valuable in assessing how individuals demonstrate their knowledge when tackling novel as well as complex tasks (Lucangeli et al., 2006).

In summary, using the three components of metacognition (i.e., knowledge, experience, and skill) as a framework for this study enabled persuasive and robust assessment of the metacognitive knowledge, experience, and skills that students with LD employed as they performed mathematics word problems. Metacognitive knowledge, experience, and skill highlighted *what* students knew about their own knowledge, *who* they were in terms of self-regulating their task performance, *why* they persevered through or relinquished from a task, and the basics of *how* they performed the task at hand.

Montague's Model of Effective Problem Solving in Mathematics

Montague's (1992) model of effective problem solving was developed from robust research conducted in the area of self-regulation, general and mathematical problem solving, as well as in other affective variables that facilitate successful problem-solving endeavor (Montague, 1992, 1997; Montague & Applegate, 1993, 2000; Montague & Bos, 1986). This model (Figure 2) holds that to employ self-regulation techniques in solving mathematical problems, expert problem solvers purposefully and actively monitor their performance (metacognitive) as they select from a collection of applicable strategies (cognitive). Montague's (1992) seven cognitive strategies and three metacognitive processes illustrate best practices for successful mathematical problem solving.

The seven cognitive processes enhance solving mathematics word problems and comprise of: ***R****ead* (for understanding), ***P****araphrase* (in your own words), ***V****isualize* (draw a picture or diagram), ***H****ypothesize* (a plan to solve the problem), ***E****stimate* (make a prediction), ***C****ompute* (do the arithmetic), and ***C****heck* (make sure everything is right). The metacognitive

processes guide and monitor the application of the cognitive strategies. They include *self-instruction*, *self-questioning*, and *self-monitoring*.

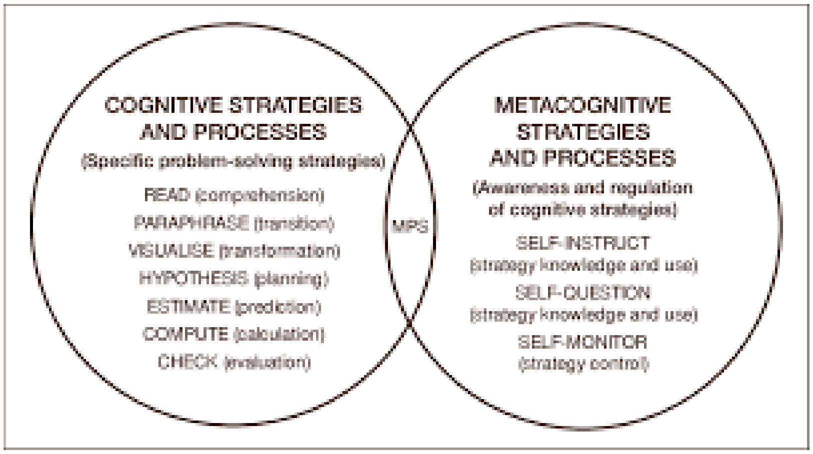

Figure 2. Montague's (1992) Model of Mathematical Problem Solving

To facilitate acquisition of the cognitive skill, students were instructed explicitly through teacher modeling during the intervention phase of this study. Montague's (1992) framework for the protocol analysis of problem solving in mathematics differentiates explicitly between cognitive and metacognitive problem-solving behaviors observed within the different events of problem solving. Montague's (1992) framework indicates a synthesis of the cognitive and metacognitive levels of problem-solving behaviors studied within cognitive psychology by Flavell (1981). Flavell's (1979) theory of metacognition and Montague's (1992) model of effective problem solving provided the grounding for this study. Synthesizing what students knew (metacognitive knowledge), how they related and reflected the knowledge (metacognitive skills), and to what they attributed the outcome (metacognitive experience), with a practical model of effective problem solving in mathematics yielded a theoretical basis used to evaluate the effect of a cognitive- and metacognitive-strategy instruction on the problem-solving performance and perception of students with learning disabilities.

Educational Significance

This investigation addressed two main gaps in the research. A review of literature suggested that previous research on think alouds were focused primarily on using a cognitive (Krawec, Montague, Kressler, & de Alba, 2012) or a metacognitive strategy (Rosenzweig et al., 2011) when investigating the achievement of students with learning disabilities on mathematics word problems. Further, other studies that used think-aloud protocol indicated positive effect in the interaction of cognitive, metacognitive, and affective variables (Rosenzweig et al., 2011; Shuell, 1990) on the mathematical problem-solving skills of students with learning disabilities. Specifically, the current study investigated how using both cognitive and metacognitive strategies on the mathematical problem-solving skills effected the performance and perception of seventh- and eighth-grade students with learning disabilities.

Students with learning disabilities often over- or underestimate their mathematics abilities (Garrett et al., 2006), and this deficiency in self-regulation hinders how educators can assist them to acquire critical self-help skills. When practitioners understand the connections between what a student says he or she knows (metacognitive knowledge), how he or she applies that knowledge (metacognitive skill), and what motivates him or her toward their performance (metacognitive experience), they are equipped to provide students with the remediation that is germane to the specific area(s) of deficit in mathematical problem solving. Think-aloud protocols provide educators access to students' thought processes through the students' verbalizations when they encounter difficult tasks, novel tasks, or both. Using the think-aloud approach to solve mathematical problems, therefore, enables students to become aware of their own thought processes and enables teachers concurrently to gain access into their students thought processes and patterns. Furthermore, problem solving is critical to navigating higher education and employment. Research has shown that problem solving is difficult for students who are low achieving or have learning disabilities (Fuchs & Fuchs, 2002; Geary, 2003; Hanich et al., 2001; Montague & Applegate, 1993). Swanson and Saez (2003) argued that even with available federal funding and educator efforts, students with learning disabilities tend to experience more problems in transitioning to higher education or to workforce placement. The low proficiency of students with learning disabilities in regulating cognition, metacognition, and motivations in learning activities may be a critical factor in explaining their unsatisfactory school performance and challenges in transitioning to higher education or to workforce placement (Wagner, 2005). Therefore, facilitating the cognitive and metacognitive processes needed to understand and solve mathematical word problems would help students with learning disabilities to compete in the global economy in pursuit of educational opportunities and worthwhile careers.

Similarly, analyzing the metacognitive strategies that student use during problem-solving activities may enable practitioners to teach students to acquire, internalize, and apply metacognitive skills effectively. This study attempted to further the research on how students with learning disabilities utilized cognitive processes and metacognitive skills to enhance their mathematical problem-solving performance. Positive effects from this study would support the need for educators to foster both cognitive and metacognitive strategies when teaching students with learning disabilities how to solve mathematics word problems.

Research Questions

This pre- and postintervention experimental study examined the effect of cognitive- and metacognitive-strategy instruction, using think-aloud protocols as the instrument, on the problem-solving performance of seventh- and eighth-grade students with learning disabilities in an intact resource room. The following research questions guided the study:

1. To what extent are the seventh- and eighth-grade students with learning disabilities using the cognitive and metacognitive strategies solving mathematical word problems?
2. To what extent does using cognitive and metacognitive strategies improve the mathematical problem-solving performance of seventh- and eighth-grade students with learning disabilities as measured by the change from pre- to postintervention scores on two sets of word-problem-solving probes?
3. To what extent does cognitive and metacognitive strategies improve the metacognitive experience of seventh- and eighth-grade students with learning disabilities as measured by the Metacognitive Experience Survey (MES)?

Definition of Terms

This section contains the definition of main terms and concepts that were used in this investigation. The definitions provided are specific to this study as there may be other definitions that are used for the terms.

Cognition entails the mental processes and abilities that learners engage on a daily basis (Montague, 2008). Examples of the cognitive processes engaged are memory, learning, problem-solving, evaluation, reasoning, and decision making (Montague et al.,1991, 2009, 2011, 2014). For a learner, cognition helps to generate new knowledge through mental processes and also helps in the utility of knowledge in daily activities.

Cognitive strategies refer to a learner's interaction with the material to be learned; how the learner manipulates the information mentally (as in making mental images or relating new information to previously acquired concepts or skills) or physically (as in grouping items to be learned in meaningful categories or making summaries of important information to be remembered; O'Malley & Chamot, 1987).

Conceptual knowledge is a grasp of the mathematical concept and ideas that are not problem-specific and consequently can be applied to any problem-solving situation (Jayanthi, Gersten, & Baker, 2008).

Conditional knowledge is the ability to discern under what circumstance it is appropriate to use a specific strategy (Schraw et al., 2006).

Declarative knowledge is symbolic knowledge (Broadbent, 1989) that enables individuals to retrieve stored information using associations (Squire & Knowlton, 1995). The creation of new memories can alter declarative knowledge although declarative knowledge is not substantive until it is retrieved by cues and prompts (e.g., questioning). Accessing or retrieving declarative knowledge is not intentionally as the individuals can only perceive the products of this process.

Higher-order thinking skills pertain to learning experiences that are focused around analysis, evaluation, and synthesis. Aspects of higher-order thinking skills, as used in this study, entail developing problem-solving skills such as inferring, estimating, predicting, generalizing, and reflecting (Dillon, 2002; Zohar & Dori, 2003; Zoller, Dori, & Lubezky, 2002).

Learning Strategies are thoughts, behaviors, beliefs, or emotions that facilitate the acquisition, understanding, or later transfer of new knowledge and skills (Weinstein et al., 2000).

Mathematical problem solving is a complex cognitive activity involving a number of processes and strategies. Montague (1996) informed that problem solving comprises *problem representation* and *problem execution*. Obtaining a solution to the mathematical problem is grounded on appropriately representing the problem. Montague (1996) hypothesized that students who struggle with correctly representing the problem will have challenges with solving the problem because problem representation underlies understanding the problem and devising a plan to solve the problem.

Mathematical problem-solving performance is the dependent variable. Mathematical problem-solving performance entails: (a) organizing the mathematical operations, (b) choosing the most effective method, (c) monitoring and controlling operations carried out, and (d) evaluating the reasonableness of the solution obtained (Montague, Warger, & Morgan, 2000; Victor, 2004). In this study, mathematical problem-solving performance incorporated interpretation and analysis (Passolunghi, Mazocchi, & Fiorillo, 2005), problems that required single and multiple steps to solve (Fuchs, Fuchs, & Prentice, 2004; Montague & Applegate, 1993), and word problems set up in contextually simple as well as complex formats that included irrelevant information (Fuchs & Fuchs, 2002; Passolunghi et al., 2005). Participants were administered two identical sets of three mathematical-word-problem probes pre- and postintervention to assess the effect of the cognitive- and metacognitive-strategy instruction on their performance. The three word-problem probes were of varying difficulty levels (1-step, 2-step, and 3-step). The resulting preintervention and postintervention scores ranged from 0 to 6 points, respectively with the latter indicating the effect of the treatment.

Mathematical-problem-solving performance was analyzed using descriptive narratives. The second set of three mathematical word problems were administered individually postintervention to six students based on their Metacognitive Experience Survey scores. Selection criterion was two students each who score high (50 to 60 points), average (40 to 49 points), and low (Below 40 points) on the Metacognitive Experience Survey. Descriptive analyses of student verbalization and performance yielded cognitive profiles that informed on students' knowledge of mathematical problem-solving, and knowledge, use, and control of the seven problem-solving processes (Daniel, 2003; Montague, Bos, & Doucette, 1991).

Metacognition refers to higher-order thinking that involves active control over the cognitive processes engaged in learning (Brown, 1978; Flavell, 1979). Activities such as planning how to approach a given learning task, monitoring comprehension, and evaluating progress towards the completion of a task are metacognitive in nature (Flavell, 1979).

Metacognitive strategies pertain to the executive processes in planning for learning, monitoring one's comprehension and production, and evaluating how well one has achieved a learning objective (O'Malley & Chamot, 1987).

Procedural knowledge involves knowing how to use a particular learning strategy. Procedural knowledge pertains to awareness and management of cognition, including knowledge about strategies (Kuhn & Dean, 2004; Schraw et al., 2006). Procedural knowledge enables students to execute the necessary action sequences to solve problems (Rittle-Johnson & Star, 2007).

Self-efficacy deals with an individual's beliefs that he or she is capable of successfully performing a given task (Zimmerman et al., 1996). In this study, self-efficacy was assessed using the Metacognitive Experience Survey. The Metacognitive Experience Survey comprised a 5-item Likert-style questionnaire with four response scores ranging from *Not at all True* (1 point); *Hardly True* (2 points); *Mostly True* (3 points); and *Absolutely True* (4 points), and three mathematical problem-solving probes differentiated by difficulty types (one step, two step, and three step). Questions on the Metacognitive Experience Survey included, *I have seen this type of question before; I understand what the problem asks me to do; The problem is going to be difficult to solve; I will need to use a lot of effort to solve the problem;* and *I am confident that I will solve this problem correctly*. The Metacognitive Experience Survey provided a measure of students' self-efficacy beliefs and the effectiveness of cognitive- and metacognitive-strategy instruction on students' perception in their ability to solve the three word problems. The Metacognitive Experience Survey was administered individually to all students pre- and postintervention by the researcher and assistant during the student's resource session. Each student received one question-type on a sheet of paper. After perusing each question type, the student responded to the 5-item survey. Each student, working solely with the researcher or assistant, received three mathematics question types on three separate sheets delivered one after the other. Each student, therefore, read and responded to the survey questions three times each for the pretest and the posttest data-collection processes. Metacognitive Experience Survey scores, consequently, ranged from 15 to 60 possible points for the survey. This study, therefore, examined the effect of a metacognitive strategy instruction on students' rating of self-efficacy using the Metacognitive Experience Survey.

Think-Aloud Protocol (TAP) is a method to garner insight into metacognition by asking students to verbalize their thoughts while working on a mathematics word-problem. The verbalizations subsequently are recorded, transcribed, and systematically assessed (Veenman et al., 2005). In this study, the researcher trained the participants by modeling thinking out loud. Six students comprised of two students each with high, average, and low scores on the MES were audiotaped solving three mathematics word problems while thinking aloud. Montague's (2003) model of seven cognitive and three metacognitive processes were the instruments used to measure students' verbalization as they solved three word problems with varying difficulty level. The qualitative data gathered was analyzed in narrative format in this study.

Summary

This chapter outlined the purpose of the study, the research problem and its significance, and the two conceptual theories that framed this study. Metacognitive theory and the Model of Effective Mathematical Problem Solving were described and presented as channels to capture the challenges that students with disabilities face as they solve mathematical word problems. In addition, the research questions and the definition of terms were summarized in this chapter. In chapter II, the review of literature examined the recent relevant research findings in the area of metacognition and think-aloud protocols. In chapter III, the methodology for this study was explained and included a description of the research design, the treatment, procedures for data collection, and the data analysis. Chapter IV reported the results of the data analysis for the research questions that guided this mixed-method study. Chapter V presented a discussion of the study findings, limitations, and recommendations for future research.

CHAPTER II

LITERATURE REVIEW

According to Flavell (1979) and colleagues (Lucangeli & Cabrele, 2006), metacognition comprises metacognitive knowledge, metacognitive experience, and metacognitive skills. The purpose of the current study was to examine the effect of cognitive- and metacognitive-strategy instruction on the mathematical problem-solving performance and perception of middle-school students with learning disabilities. The three components of Flavell's (1979) model of metacognition and Montague's (1992) seven cognitive and three metacognitive processes (see Figure 2) provided the framework for this study. This review of literature presented research that examined metacognition, and the cognitive- and metacognitive-functioning of students with learning disabilities during mathematical problem-solving task performance. Metacognition ranks high among the cognitive processes extolled and recommended in mathematics education. Anderson, Corbett, Koedinger, and Pelletier (1995) stressed that understanding and controlling cognitive processes, a function of metacognition, are fundamental skills that classroom teachers can help learners develop. Metacognition is described as the ability to develop one's self-knowledge as well as the ability to learn how to learn (Desoete, 2007; 2008; Desoete & Roeyers, 2006; Desoete & Veenman, 2006).

Mathematical problem solving is an increasingly critical skill in the 21st century mathematics curriculum because success in mathematical problem solving is correlated with overall mathematics achievement (Bryant, Bryant, & Hammermill, 2000). Similarly, the need to develop proficiency in the mathematics domain is relevant to students' success in school and beyond. Problem-solving skills span the five curricular content standards and are a means and a goal of learning mathematics (National Council of Teachers of Mathematics, 2000); furthermore, mathematical problem solving comprises a skill set that has become central to success in 21st century workplaces (Hudson & Miller, 2006). A prevalent goal in mathematics education is for students to become adept in mathematical thinking (Greeno, 1997). Academic curriculum in the

Kindergarten through 12th (K through 12) grades require grounding in mathematics ability as a symbol of progression in learning, and the workforce equally requires problem-solving skills as a symbol of creativity and success (Hudson & Miller, 2006).

Due to the influence of cognitive psychology, mathematical proficiency has become paramount in policy-level recommendations such as Common Core State Standards (CCSS, 2012) and the National Council of Teachers of Mathematics (NTCM) standards (NCTM, 1989, 2000). Both the CCSS and the NCTM stressed a focus on conceptual understanding and a problem-solving approach to teaching mathematics. NTCM advised mathematics teachers to engage students in meaningful discussions about mathematics in order to develop students' ability to understand and make connections across mathematics concepts. Developing students' ability to conceptualize and problem solve during a mathematical episode encapsulates the microcosm of mathematical culture (Schoenfeld, 1987): critical-thinking skills that enable students to relate classroom mathematics to everyday life. Schoenfeld (1980) further argued that metacognition is the process that students employ to achieve a linkage between mathematical education and everyday mathematical implications. Other researchers agree that mathematical problem solving is one of the domains for which metacognition consistently predicts the learning performance (Desoete, 2009; Desoete & Veenman, 2006; Fuchs et al., 2010; Harksamp & Suhre, 2007). Schoenfeld (1992) also informed that metacognition monitors the solution processes and regulates the problem-solving events. Schoenfeld (1992) elucidated further that the problem-solving events include analyzing and exploring a task, making a solution plan, implementing the plan, and verifying the answer.

Competent problem solvers integrate and implement cognitive and metacognitive components (Brown, 1978; Mayer, 1985, 1998; Montague, 2001; Montague & Applegate, 1993) because proficiency in either component or in isolation is insufficient to successful problem solving. The researchers maintained that students need declarative knowledge of mathematical concepts, procedural knowledge to apply declarative knowledge and to coordinate cognitive and metacognitive processes, and conditional knowledge to discern and adapt their attitudes to the changing demands of the tasks. As a result of the intricate interaction among cognitive, metacognitive, and attitudinal factors, average-achieving students and students with learning disabilities, in particular, continue to struggle with mathematical problem solving (Gonzalez & Espinel, 2002; Montague & Applegate, 1993; Morris & Mather, 2008). A critical aspect of learning that benefits students with learning disabilities is the adoption of self-regulatory practices (Butler, 2003). Because students with learning disabilities typically use strategies inefficiently (Butler, 2003), instruction that incorporates learning strategies, therefore, enhances the efficiency of strategy awareness and implementation for students with learning disabilities (Montague, 2003).

The purpose of this study was to investigate the effect of the cognitive and metacognitive strategies on the mathematical problem-solving skills of seventh- and eighth-grade students with learning disabilities. This literature review, therefore, examined the literature and research that dealt with the effect of cognitive, metacognitive, and affective processes on the mathematical word-problem-solving of students with LD. The instructional implications of the three concepts (cognition, metacognition, and self-efficacy), in relation to remediating the problem-solving skills of students with learning disabilities, were presented.

Metacognition

Notwithstanding its enduring stance in educational psychology, the term metacognition is interpreted in multiple ways in the literature (Livingston, 1997). Flavell (1976) described metacognition as thinking about thinking, and later (Brown, 1978; Flavell, 1979) as knowledge about and regulation of an individual's cognitive activities in the learning processes. Flavell (1971) introduced the concept within the framework of developmental psychology and research on metamemory (Simons, 1996), defining metacognition as "one's own cognitive processes and products or anything related to them" (Flavell, 1976, p. 232). Piaget (1985) referred to the act of thinking about thinking as "reflective abstraction" that develops in children through an awareness of different viewpoints and an experience of self-conflict when challenged conceptually (Fisher, 1998). Lesh Livingston (1997) further described metacognition as higher-order thinking that entails active control over the cognitive processes engaged in learning. More recently, Ormrod (2006) described metacognition as an individual's knowledge and beliefs about his or her cognitive processes and the resulting attempts to regulate those cognitive processes in order to maximize learning and memory.

Other researchers, additionally, have stressed the importance of metacognition on the mathematics-learning process and performance (Desoete & Veenman, 2006; Ozsoy & Ataman, 2009; Stel, Veenman, Deelen, & Haenen, 2010), on enabling learners to be flexible and intentional in accordance to the problem-solving tasks, demands, and contexts (Paulus, Tsalas, Proust, & Sodian, 2014); and on influencing cognitive behavior at all phases of mathematical problem solving (Fuchs & Fuchs, 2005; Krawec, Huang, Montague, Kressler, & de Alba, 2012). Roberts and Erdos (1993) defined metacognition as an individual's knowledge and awareness of his or her own cognitive process. In all, although diverse definitions of metacognition exist in the literature, the recurring theme on metacognition is that metacognition pertains to individuals having information about their cognitive structure and processes and being able to organize this structure (Aktürk & Şahin, 2011; Dunlosky & Hertzog, 2000; Georghiades, 2004; Steinbach, 2008; Veenman, Van Hout-Wolters, & Afflerbach, 2006). In theory, metacognition entails planning of the information on cognitive processes before fulfilling a task, comprehending the reasoning and learning that facilitates task implementation, regulating actions and decisions that pertain to the task, and evaluating task completion (Scott, 2008). In a meta-analysis, Lester (1994) contended that during mathematical problem solving, effective metacognitive activity comprises knowing what to monitor, when to monitor, and how to monitor task execution. In addition, Lester (1994) recommended that instruction in cognitive and metacognitive strategies occur within the context of learning specific mathematics concepts and techniques.

Metacognition and Cognition

This subsection differentiates between cognition and metacognition to highlight the levels of cognitive behaviors students demonstrate during mathematical-problem-solving events. John Flavell (1976), credited with founding the concept of metacognition through research, initially theorized that metacognition is the individual's knowledge about his or her cognitive processes and products, and later Flavell (1979) conceptualized metacognition as the learner's perception of his or her own cognition. Schraw (2001) posited that students require cognition to carry out a task, and metacognition to understand how a task will be performed. Cognition, therefore, involves an awareness and understanding of a situation, whereas metacognition involves being aware and knowledgeable about how one learns as well as being aware and understanding of a situation (Senemoglu, 2005). Expatiating on cognition and metacognition, Gourgey (1998) informed that cognition is necessary to form and apply the learning process and information whereas metacognition enables the individual to develop, apply, check, and evaluate current processes, knowledge, and experience about a task. Metacognition, therefore, is fundamental for cognitive effectiveness, occurring before cognitive activities (planning), during activities (monitoring) or after activities (evaluating and checking; Akturk & Sahin, 2011). Flavell (1979) acknowledged that cognitive knowledge and metacognitive knowledge are similar and only differ in the way that the information is used. He argued that cognitive strategies are procedures implemented to help attain a particular goal whereas metacognitive strategies are used to plan, monitor, control, and evaluate the cognitive processes to ensure that the desired goal is attained.

Two studies have examined the isolated and combined effects of cognitive and metacognitive processes in the mathematical-problem-solving performance of freshman-college students (Bayata & Tamizi, 2010) and in the problem-solving of elementary- and middle-school students (Forster, 2014). Bayata and colleague (2010) used a descriptive correlational design to investigate the cognitive and metacognitive processes used by 86 randomly selected college-students with a Mathematics major in a Malaysian university while they solved Algebra problems. Algebra problem-solving performance was measured using a test based on problems discussed in their tutorial class. This test was comprised of seven algebra questions: four questions were routine problems and three questions were nonroutine problems. The researchers operationally-defined *nonroutine* algebra problems as problems that require critical thinking because the problems are unfamiliar to the students, whereas *routine* problems were operationally-defined as algebra problems used in the class on a regular basis. Additionally, mathematical achievement in algebra was measured and based on the cumulative final score of the MTH 3200 course taken by the students during the semester. Cognitive strategy and metacognitive strategy were assessed using self-report instruments. The cognitive strategy instrument, consisting of 18 items, assessed two types of cognitive strategies: *shallow cognitive strategy* (e.g., highlighting, underlining, copying, repeating items in a list) and *deep cognitive strategy* (e.g., paraphrasing, summarizing, creating analogies, and note-taking). The study used a 5-point Likert scale ranging from "1-*never*" to "5 = *very often*" to elicit students response to each statement in relation to how they learned algebra and how they solved algebraic problems. The 52-item Metacognitive Awareness Inventory (MAI) was used to measure students' opinions about their metacognitive processes (e.g., self-instruction, self-questioning, and self-monitoring) as they solved algebraic problems in the MTH 3200 course. The students were required to give a "true" or "false" response to each item.

The results indicated no statistically significant correlation between Algebra problem-solving performance and *shallow cognition strategy* ($r = -.13$). Likewise, there was no statistically significant relationship between the students' performance and *deep cognitive strategy* ($r = .12$). Results showed, however, that there was statistically significant correlation between overall metacognitive strategies and performance on

Algebra problem solving (r = .39). In addition, there was a statistically significant and positive effect between the metacognitive strategies (self-instruction, self-questioning, and self-monitoring) and metacognition subscales (knowledge, planning, and evaluation) and students' performance in the MTH 3200 course (r = .39). Cognitive strategies, however, indicated minimal effect on mathematical problem-solving performance of university students in the MTH 3200 course. This finding revealed that metacognitive strategies had an effect on algebra problem solving, and positive effects on the metacognition subscales of knowledge, planning, and evaluation.

Forster (2014) examined the existence and relationships between students' cognitive skills (verbal, spatial, and problem-solving) and mathematical problem-solving performance. The sample comprised of 98 students from the fifth through eighth grades. Fifty students attended the public charter school: seventh grade (n=25) and eighth grade (n=25); 48 students attended the private Montessori-based school: fifth grade (n=11), sixth grade (n=10), seventh grade (n=15), and eighth grade (n=12). The instruments used were the Problem-Solving Test (PST) and the Cognitive Test. Participants were administered the Cognitive Test instruments that measured verbal skills, spatial skills, and logical skills and the problem-solving test (PST) instrument that consisted of a verbal (PST-Verbal) subtest and a spatial (PST-Spatial) subtest. The researcher used multiple-regression analysis to analyze the students' scores on the problem-solving instrument and each of the assessments.

The results indicated statistically significant relationships between students' cognitive skills and problem-solving performance on the verbal subtest, the spatial subtest, and overall problem-solving performance. In the Problem Solving Test (PST), stronger relationships were found between spatial skills and verbal performance than between verbal skills and verbal performance. Similarly, stronger relationships were found between verbal skills and spatial performance than between spatial skills and spatial performance. The pairwise analyses indicated statistically significant relationships among the cognitive skills, with the strongest pairwise relationship existing between verbal and analytical skills. Results of Foster's (2014) study suggest that verbal skills align with analytical or logical reasoning skills. The present study combined verbal skills, a component measure of cognition, with analytical skills, component measure of metacognition (i.e., students' ability to determine whether or not a conclusion is logically correct). Because mathematical problem solving entails an ability to draw conclusions from the information provided, the present study used Think Aloud Protocols and Metacognitive Experience Survey to investigate the effect of cognitive- and metacognitive-stratey instruction on the mathematical problem-solving performance and perception of students seventh- and eighth-grade students with learning disabilities.

Desoete (2008) conducted a multimethod study to assess metacognition in third-grade elementary-school students. The students solved tests on mathematical reasoning and numerical facility. Desoete's (2008) study assessed metacognitive skills through think-aloud protocols, prospective and retrospective student ratings, teacher questionnaires, and calibration measures. The result indicated that, whereas metacognition correlates with intelligence, planning measured by teacher ratings was a better predictor of task correctness than Intelligence Quotient (IQ). The researchers expressed that, although intelligence and metacognition are related, it is more appropriate to assess them separately. In addition, the results showed the value of an experienced teacher as actual measure of metacognitive planning skills. There was convergent validity for prospective and retrospective child ratings, but no statistically significant relationship with the other metacognitive measures. Metacognitive skillfulness combined with intelligence accounted for between 52.9% and 76.5% of the mathematics performances. The current study combined cognitive and metacognitive measures to assess the problem-solving performance of middle-school students with learning disabilities. Based on the findings of Desoete's (2008) study, the present study assumed that combining metacognitive skillfulness and cognition processing would yield positive effects on the performance and perception of the students with learning disabilities.

Metacognition and Learning

This literature review presents research that investigated the effect of Flavell's components of metacognition (knowledge, skill, and experience) on the metacognitive functioning of students within a mathematical-learning context. Metacognition refers to higher- order thinking that involves active control over the cognitive processes engaged in learning (Livingston, 2003). Many researchers have investigated the relationship between metacognition and the learning process through the lens of metacognitive knowledge, metacognitive skills, and metacognitive experiences (Flavell, 1979, 1987; Lucangeli & Cabrele, 2006; Metallidou, 2009).

Metacognitive knowledge pertains to acquired knowledge that affects cognitive processes. Metacognitive knowledge, that is, one's thought processes about learning, provides a platform from which the learner can select strategies for the regulation of learning (Efklides, 2009). Flavell (1979) split knowledge further into *knowledge of person variables*, *knowledge of task variables*, and *knowledge of strategy variables*.

Knowledge of person variables addresses general knowledge of how human beings learn and process information and also includes personal knowledge of one's own learning processes. An instance is when a student discerns that studying in the library would yield better results than studying at home with infinite distractions. *Knowledge of task variables* refers to knowledge about the nature of the task and includes knowledge about the type of processing demands required for the individual to execute the task. An example would be the awareness that it will take longer to read and understand a chemistry textbook than a novel. *Knowledge about strategy variables* refers to an individual's knowledge about cognitive and metacognitive strategies as well as knowledge of when and where it is appropriate to use such strategies. *Knowledge about strategy characteristics* comprises knowing what needs to be done, how one will go about doing the task, and applying the right strategy. In relation to mathematical-problem-solving tasks, research findings indicate that metacognitive knowledge is not delineated as finely into the three categories (person, task, and strategy) but involves interactions among the three components (Teong Su Kwang, 2000). For example, person-by-strategy interactions are demonstrated by a student's confidence and preference to use a specific strategy, and task-by-strategy interactions include awareness that mathematics problems involving order of operation can be solved using the Parentheses, Exponents, Multiplication, Division, Addition, and Subtraction (PEMDAS; Schrock & Morrow, 1993) heuristic.

Research on metacognitive knowledge investigated what students know about learning and what strategies they employ to help them learn. Swanson (1990) conducted a study to investigate the relationship between general academic aptitude and metacognition. Swanson's research sought to identify the role of metacognition in improving cognition of 56 participants in fourth and fifth grades by analyzing their think-aloud protocols. Participants were stratified into high- and low-cognitive-ability groups based on their scores on the Cognitive Abilities Test (Klondike & Hagen, 1978). Students subsequently were grouped based on performance on a 17-item questionnaire that assessed metacognition in the domain of mathematical-word-problem solving. The resulting groups comprised high- and low-metacognitive groups. In all, there were four ability groups: high aptitude-high metacognition (HA-HM), high aptitude-low metacognition (HA-LM), low aptitude-high metacognition (LA-HM), and low aptitude-low metacognition (LA-LM). Students were audio-recorded solving a pendulum task and a combinatorial task while thinking out loud. The think-aloud protocols were transcribed and coded based on 24 mental components. Results indicated that, regardless of aptitude, high- metacognitive students outperformed low-metacognitive students on solving a pendulum task and a combinatorial task. Moreover, students in the LA-HM group performed statistically significantly better than students in the HA-LM group. In relation to heuristics and strategy use, students in the HA-HM group consistently employed hypothetico-deductive reasoning to problem solve. Swanson's (1990) finding that metacognition may be more predictive of future success with mathematical problem-solving than aptitude and general intelligence is supported by more current research (Dignath & Buttner 2008; Van der Stel & Veenman 2010; Veenman & Spaans, 2005).

The current study used think-aloud protocols to examine the effect of cognitive and metacognitive training on the problem-solving skills (perception and performance) of students with learning disabilities who, predominantly, possessed lower academic aptitude than their average-performing peers. Based on Swanson's (1990) findings, using think-aloud protocols should reveal the mathematical problem-solving skills (perception and performance) of students with learning disabilities.

In summary, metacognitive knowledge can effect the selection, evaluation, and revision of cognitive tasks, goals, and strategies. Metacognitive knowledge equally can guide individual's interpretation of the meaning and implications of metacognitive experiences along the same lines as metacognitive experiences can add, delete, or reverse one's metacognitive knowledge store (Nelson, 1992) as suggested in Piaget's (1952) model of assimilation and accommodation. Metacognitive knowledge, metacognitive experience, and metacognitive skills are, moreover, complementary and interdependent. Papaleontiou-Louca (2003) inferred that metacognitive knowledge lends credence to proper interpretation of and action on metacognitive experience, whereas and conversely, the latter lends information about persons, tasks, and strategies to the metacognitive-knowledge database. For example, the skill or knowledge of playing a card game might be acquired simply by experiencing (forming some ideas and feelings about the game while watching) the game. One can surmise, therefore, that the three components of metacognition (knowledge, skill, and experience) inform and stimulate one another during the execution of problem-solving (cognitive) tasks.

The third component of metacognition, metacognitive skills, refers to a person's procedural knowledge for regulating problem-solving and learning activities (Veenman, 2005). Lester (1994) suggested that metacognitive skill relates to how well one monitors the process of doing a task and how well the observation guides the problem-solving task. Metacognitive skills are the conscious controls that involve planning, process progress monitoring, effort allocation, strategy use, and regulation of cognition (Efklides, 2002; Papaleontiou-Louca, 2003). To measure students' strategy use, concurrent verbal reports (i.e., think-alouds) are recommended as researchers use the think-aloud protocols (TAPs) to gain access to students' mental processing during authentic task performance. TAPs incorporate verbal thought and, therefore, metacognitive skills, because TAPs verbally manifest students' abilities to control, monitor, and self-regulate behaviors during problem-solving activities. Students are required to verbalize thought, feelings, and actions during think-aloud procedures to enable researcher access to, and assessment of, the cognitive and metacognitive processes that underlie task performance (Sweeney, 2010). Montague and Applegate (1993) conducted a random study that used TAPs to evaluate the self-regulation and strategy use of 81 eighth-grade students comprised of varying ability groups: students with learning disabilities (LD), students with average abilities (AA), and students with gifted abilities. The students (learning disabiled, n = 28; average achievers, n = 25; gifted, n = 28) received 10 minutes of think-aloud instruction using two verbal-reasoning problems. Students subsequently were asked to solve three mathematical word problems consisting of one-step, two-step, and three-step difficulty levels. The results indicated no differences in the cognitive and metacognitive verbalizations of the students for the one-step problem. Gifted students, however, made more cognitive but not metacognitive verbalizations than students with learning disabilities in two-step problem, and more cognitive and metacognitive verbalizations than students with learning disabilities and average-achieving students in the three-step problem. These findings support the hypothesis that metacognition is triggered when individuals are confronted with more challenging tasks. Students' personal perception of task complexity, however, may determine the self-regulatory checks and the metacognitive strategies employed. The present study contributed to the limited research on the use of think-aloud protocols to study the mathematical problem-solving performance of seventh- and eighth-grade students with learning disabilities within the resource setting.

Metacognitive experience relates to students' attribution and perception of personal academic achievements and failures, in the past and the present, with the performance of a task. Efklides (2009) defined metacognitive experiences as an awareness or knowing that enables the learner to feel, estimate, or assess the related features of the learning task, of the cognitive processing as it takes place, or of its outcome. A key attribute of metacognitive experience is its access to the cognitive and the affective regulatory loop of learning behavior. In relation to the affective loop, metacognitive experience is related to motivation and self-efficacy processes. As part of the cognitive loop, metacognitive experience is connected to metacognitive knowledge and metacognitive skills.

With the connection of metacognitive experience to metacognitive knowledge (one's thinking about learning), metacognitive skills (self-monitoring and self-regulating), and the affective loop (motivation and self-efficacy), metacognitive experience inspires intrinsic awareness that links the present to past learning experiences. The connection of the present to past learning experiences, consequently, either facilitates or inhibits self-regulation of learning in the present as well as in the future. By implication, because students with learning disabilities overestimate academic skills (Stone & May, 2002), the intrinsic trigger to seek out helpful metacognitive strategies is impaired. Garrett, Mazzocco, and Baker (2006) and Desoete and Roeyers (2002) analyzed the prediction and evaluation skills of students with learning disabilities (*n*=17) and students without learning disabilities (*n*=179) in mathematical problem solving. The researchers focused on the metacognitive skills that either preceded or followed task engagement, as opposed to focusing on the processes that occurred during a task. Participants were required to predict which of several mathematics problems they could solve correctly and subsequently were required to solve the problems. Finally, participants were asked to evaluate their solution to each of another set of problems for correctness. Results showed that students with learning disabilities were less accurate than students without learning disabilities in predicting and evaluating skills in mathematical problem solving. Students with learning disabilities also were less accurate in predicting and evaluating the correctness and incorrectness of solutions. Although students with learning disabilities were confident in their ability to solve problems correctly, they were less accurate at predicting which problems they could solve correctly (Garret et al., 2006). Finally, students with learning disabilities were as accurate as their peers in predicting that they could not solve certain types of mathematical problems. These findings lend credence to the claim that relative to their peers, the accuracy of students with learning disabilities at predicting the difficulty of mathematics problems may not be a valid measure that the student can determine accuracy regardless of whether completed mathematics problems were solved correctly.

The studies on the metacognitive experience of evaluation and prediction highlight the importance of motivation and self-efficacy in students with learning disabilities. The ability to assess and predict accurately whether a problem is difficult or easy enables students to determine which problems require more skill or strategy to complete (Garrett et al., 2006). Students with good prediction skills are able to distinguish between real and seeming challenges with mathematical problem-solving task when predicting future performance (Desoete & Roeyers, 2002).

Metacognition and Learning Disabilities

Ample research indicates that students with learning disabilities demonstrate deficits when completing academic tasks that require the use of cognitive and metacognitive processes across academic domains (Chalk, Hagan-Burke, & Burke, 2005; Kraai, 2011; Montague & Applegate, 1993a; Roberts, Torgesen, Boardman, & Scammacca, 2008). Kraai (2011) used interview data and found that elementary-school students with learning disabilities had difficulty identifying and selecting effective strategies during a spelling test. The students in the study manifested limited ability to monitor, regulate, or correct their performance even when applying familiar strategies. Montague and Applegate's (1993a) study indicated deficits in the ability of middle-school students with learning disabilities to solve word problems accurately due to the students' inability to identify effective strategies to apply to the tasks. In the reading domain, Roberts et al. (2008) noted deficiencies in the ability of students with learning disabilities to monitor their comprehension on reading passages, and Chalk et al. (2005) found similar patterns of weaknesses in students with learning disabilities during the writing process.

Despite the evidence that conceptual understanding of mathematics facilitates the development of mathematical-problem-solving skills, students with learning disabilities continue to receive mathematics instruction in the general-education setting where rote learning of mathematics facts and procedures is the paradigm (Rosenzweig, Krawec, & Montague, 2011). To solve mathematics word problems successfully, Mayer (1985) contended that students must be able to represent the problem, develop a path to the solution, and then execute the solution. Mayer (1985) reasoned that effectively solving mathematical word problems entails several cognitive processes as well as metacognitive strategies (e.g., visualization, estimation, self-questioning). Evidence of the importance of metacognition to academic success as well as to success with problem solving is substantiated in the literature (De Corte, Greer, & Verschaffel, 1996; Flavell, 1979; Graham & Butler, 2006; Lucangeli & Cornoldi, 1997; Montague, 2008; Montague & Applegate, 1993a; Trainin & Swanson, 2005; Veenman et al., 2006; Zimmerman, 2002). Research informs that metacognition may be a better predictor of learning performance than general aptitude (Veenman & Spaans, 2005). In the reading domain, students with learning disabilities demonstrated inadequate metacognitive strategy awareness, application, and control (Mason, Meadan, Hedin, & Corso, 2006; Wong et al., 2006). In the mathematics domain, however, there is scant research focusing on the metacognitive functioning of students with learning disabilities during a problem-solving event (Carr, Alexander, & Folds-Bennet, 1994; Montague & Applegate, 1993b). Furthermore, in the domain of mathematical problem solving, students with learning disabilities relied only on strategies (e.g., rereading a problem or switching computations) to solve word problems (Montague & Applegate, 1993b; Montague, Bos, & Doucette, 1991).

Additionally, students with learning disabilities demonstrated difficulty with applying metacognitive strategies in their approach to word problems as compared with average-achieving, low- achieving, and gifted students (Lucangeli, Coi, & Bosco, 1997;

Montague & Applegate, 1993b; Rosenzweig et al., 2011). Lucangeli et al. (1997) discovered that Italian fifth-grade students with learning disabilities displayed lower metacognitive awareness when compared with proficient mathematical problem solvers. Montague and Applegate's (1993b) results indicated that students with learning disabilities verbalized fewer metacognitive strategies as the problem difficulty increased in contrast to gifted students who verbalized more as the word problems increased in difficulty. The researchers' findings further revealed that academic performance may be dependent on cognitive, metacognitive, and noncognitive factors (e.g., self-efficacy).

Bandura (1986) informed that self-efficacy relates to an individual's beliefs and attitudes about ability and capability to learn and perform a task at a designated level. The performance of students with learning disabilities, in terms of the effort expended and the level of persistence sustained on a given task, were influenced by self-efficacy attributes (Montague, 2000). Students with learning disabilities are characterized by low self-esteem, a flawed evaluation of the difficulty of a mathematical word problem, and an attribution of failure to diminished ability (Borkowski, Weyhing, & Carr, 1988). Students with learning disabilities, consequently, persisted less than average-achieving peers on perceived difficult word problems. Montague (2000) further noted that persistence correlates highly with success in mathematical problem solving. Graham and Harris (1989) suggested that students with learning disabilities possessed diminished self-efficacy for cognitive competence compared with average-achieving peers due to past failures at achievement tasks. The current study, therefore, examined the effect of a cognitive- and metacognitive-strategy intervention on students' rating of self-efficacy, as measured by the Metacognitive Experience Survey (MES), in performing mathematical-word-problem tasks. The relationship of the self-ratings to measures of performance after intervention strategy instruction was examined.

Metacognition and Pedagogy

Swan (2008) recommended that teachers involve problem-oriented strategies in their classroom instruction that require conscious attention and that are not employed automatically with all learners without teaching (p. 265). To tackle the need for problem-solving proficiency, policies have been implemented to reform the mathematics curriculum from an emphasis on rote skills and procedural knowledge to problem analysis, interpretation, and conceptual understanding (National Council of Teachers of Mathematics, 2000). Pedagogical changes stress student engagement through discussions, explorations, and multiple representations, primarily through problem-solving activities (Goldsmith & Mark, 1999). Yet, even with the increased interest channeled toward mathematical problem solving by educators and researchers, students with learning disabilities continue to struggle. Difficulties in working memory and processing speed (Fuchs & Fuchs, 2002), identifying the correct operation and performing the computation (Huinker, 1989; Montague & Applegate, 1993a), higher-order reasoning (Maccini & Ruhl, 2001), and the comprehension demands integral in word problems, blend to make mathematics word-problem solving one of the most challenging parts of the curriculum for students with learning disabilities (Lerner, 2000). Metacognition, clearly, plays a pivotal role in successful learning; consequently, researchers study metacognitive activity and development to investigate how students can be instructed to apply their cognitive resources through metacognitive control and regulation.

Theoretical and empirical research has focused on the metacognitive strategy model as a teaching approach for use with cognitive procedures to boost mathematical problem solving for students with LD (Bayat & Tarmizi, 2010; Desoete et al., 2006; Maccini & Hughes, 2000; Montague, Enders, & Dietz, 2011; Özsoy & Ataman, 2009). In the learning and instruction domain, the three components of metacognition (knowledge, skill, and experience) are important in the acquisition of problem-solving skills (Martinez, 2006). Cognitive- and metacognitive-strategy instruction helps learners to control and monitor problem-solving behaviors (Lin et al., 2005) by activating and extending independent and intentional thinking processes, especially among students with learning disabilities (Anderson et al., 2002; Lambert, 2000). Knowing how to learn and apply appropriate strategies are valuable skills that proficient problem solvers possess (Cano, 2009). In the present study, the researcher modeled thinking aloud using logical-reasoning problem set that demonstrated metacognitive processes such as self-questioning, progress monitoring, and the use of affective statements that relate to the problem set. The students subsequently thought aloud while solving an identical set of mathematical word problems. Student verbalizations were audio-recorded, transcribed, coded, and analyzed to delineate the effect of think-aloud protocols on the mathematical problem-solving performance of students with disabilities.

One instructional approach, cognitive-strategy instruction (CSI), has been proven to improve the knowledge and application of effective processes and strategies to increase problem-solving performance (Case, Harris, & Graham, 1992; Montague, 2008; Montague et al., 2011). CSI emphasizes the development of thinking skills and processes as foundation to enhance learning, and focuses on enabling students to be strategic, independent, flexible, and productive learners (Scheid, 1993). CSI boosts students' memorization, application, and internalization of a cognitive routine by combining elements of explicit instruction (i.e., modeling, verbal practice, and scaffolded instruction) with metacognitive strategies (i.e., self-instruction, self-questioning, and self-monitoring) thereby improving task performance (Harris & Graham, 2009; Krawec & Montague, 2012; Montague & Dietz, 2009). As evidenced in the research, students with learning disabilities experience difficulty with retrieving and applying cognitive and metacognitive processes effectively (Montague & Applegate, 1993b; Roberts et al., 2008; Rosenzweig et al., 2011). CSI is grounded on the assumption that cognitive strategies, associated with successful learning (Borkowski, Carr, & Pressley, 1987; Garner, 1990) and utilized by expert problem solvers, can be taught to students with learning disabilities (Halpern, 1996). Adopting the CSI model entails teaching students how to identify and select cognitive processes and metacognitive skills appropriate for the context of the task while self-monitoring the task execution (Montague, 2008).

Montague (2003) developed a cognitive-strategy intervention tool, Solve it!, to assist middle-school students with learning disabilities with strategy knowledge in mathematical problem solving. Solve it! uses an instructional routine that supports explicitly teaching the cognitive processes and metacognitive strategies adopted by expert problem solvers to solve mathematical word problems (Montague et al., 2000). A key requirement of the CSI model is that students reach 100% mastery of Montague's seven cognitive processes and what each process entails; for example, read entails understanding, visualize entails pictorial representation, and so on. Attaining mastery of the seven cognitive processes is followed by teacher modeling through thinking out loud to demonstrate how to apply metacognitive strategies (self-instruction, self-questioning, and self-monitoring) to the cognitive processes. Students ultimately become independent, flexible, and proficient in applying the CSI routine over time. The CSI model, fundamentally, stresses teaching cognitive processes and metacognitive skills within the context of the task thereby enabling students to select, apply, monitor, and reflect on the execution of the appropriate strategies (Montague, 2008). In relation to this, the present study used Montague's (1992) cognitive and metacognitive model of mathematical problem solving and Flavell's (1976) theory of metacognition to investigate the effect of think-aloud protocols on the mathematical problem-solving performance of seventh- and eighth-grade students with learning disabilities. The present study further assumed that using cognitive and metacognitive strategies simultaneously underlied proficient mathematical problem solving and can be taught to students with disabilities who manifest deficits in cognitive and metacognitive strategies compared with their peers with average- and gifted-achievements.

CSI results in single-subject studies (Montague, 1992; Montague & Bos, 1986) and randomized control studies with teachers delivering direct instruction in the inclusive classroom (Montague et al., 2012; Montague et al., 2011) indicated that students with learning disabilities increased problem-solving accuracy, as a result of the CSI intervention, to an extent that was superior to average-achieving peers (Montague et al., 2011). Montague et al. (2012) conducted an efficacy study with 20 pairs of middle schools matched on the Florida Comprehensive Achievement Test (FCAT) performance grade and socioeconomic status. The researchers randomly assigned one school from each pair to the intervention, Solve it! The intervention group comprised of 644 students and the comparison group comprised 415 students. The variables assessed include: ability level (students with learning disabilities (SLD), low-achieving students (LAS), and average-achieving students (AAS), Gender (males, females), Ethnicity (European American, Hispanic American, and African American), and Free or Reduced Lunch (Yes, No). The intervention was implemented for 7 months with periodic progress monitoring.

The research on CSI's effect on students' strategy use indicated statistically significant main effects for the condition. Students in the intervention group reported using more strategies than students in the comparison group. As measured by the Mathematics Problem Solving Assessment (MPSA; Montague, 2003), students in the intervention group showed statistically significant improvements from pretest to posttest on strategy use, whereas the comparison group indicated no statistically significant changes from pretest to posttest. Considering whether ability level moderated the effects of solve it, the results indicated a uniform intervention effect across the three ability groups. Similarly, results from Krawec, Huang, Montague, Kressler, and De Alba's study (2012) indicated that Solve It! is effective for students irrespective of ability levels. The researchers hypothesized that students in the intervention increased problem-solving accuracy due to an increased repertoire of cognitive and metacognitive strategies. Furthermore, the researchers informed that a comparison of posttest means showed that students with LD, in the treatment group ($M=14.95$, $SD=3.14$), demonstrated increased strategy knowledge more than the average-achieving (AA) students in the comparison group ($M=14.16$, $SD=4.47$). Krawec et al. (2012) surmised that, although the study was focused on strategy use and not problem-solving accuracy, both concepts are interdependent. The present study, however, examined the interdependency of strategy use and problem-solving accuracy.

Ozsoy and Ataman (2009) investigated the effect of implementing metacognitive-strategy training on mathematical-problem-solving achievement. The study used a quasi-experimental design, random assignment to treatment and comparison groups, and pre- and posttest measurements. The dependent variable was “problem-solving achievement” as measured by the Mathematical Problem Solving Achievement Test (MPSAT) and the independent variable was metacognition as measured by Metacognitive Knowledge and Skills Assessment-Turkish version. The study was conducted over a 9-week period with 47 fifth-grade students. Students in the intervention group (n = 24) received strategy instruction to improve their metacognitive skills, whereas students in the comparison group (n = 23) received only their normal lessons. Students were administered pre- and posttests using the MPSA test and Turkish version of Metacognitive Skills and Knowledge Assessment (MSA-TR). The results showed that students in the treatment group statistically significantly improved in both mathematical-problem-solving achievement and metacognitive skills. The metacognitive strategy instruction in the treatment group demonstrated a statistically significant difference [$F(1,45) = 23.39$] between the treatment and comparison group on the level of metacognitive knowledge and skills, and with a large effect size ($\eta 2$=.34). Comparing the students’ performance on the MPSAT, the pretest mean obtained by the treatment group was 25.00 and posttest mean was 46.46, whereas the pretest mean obtained by the comparison group was 29.13 and a posttest mean of 27.83. The mean differential between the treatment and comparison groups suggested a substantial increase in problem-solving achievement by the students in the treatment group than by the students in the comparison group.

Jacobse and Harskamp's (2009) study examined how to improve students' metacognitive and problem-solving skills with a computer program consisting of word problems and metacognitive hints. A total of 49 students comprised the sample with 23 students in the experimental group and 26 students in the comparison group. Students in the experimental group practiced with the computer program, which also incorporated a choice of metacognitive hints during problem solving. The comparison group did not work with the computer program. All the participants had comparable socioeconomic status, had similar average mathematical performance scores on a norm-referenced test, and did not differ statistically significantly on the word-problem-solving pretest. During the course of the study, the comparison and treatment groups used the same mathematics textbook and received instruction on the same content of the textbook at the same pace. Think-aloud protocols of 10 randomly selected students were used to measure the metacognitive skills of the participants. The results indicated that the groups differed statistically significantly on the posttest; the treatment group that used the computer program with metacognitive hints outperformed the comparison group in metacognitive skills and problem-solving skills. Additionally, there were statistically significant effects indicated between mathematical-problem-solving performance and metacognitive-hint use. The results, therefore, support other studies (Bayat & Tarmizi, 2010; Desoete et al., 2006; Maccini & Hughes, 2000; Montague, 2003, 2007, 2013; Montague, Bos, & Doucette, 1991; Montage, Enders, & Dietz, 2011, 2014; Sweeney, Krawec, & Montague, 2011) that the use of metacognitive hints or strategy instruction increases students' performance in mathematical word problem solving. Livingston (2003) asserted that the most effective approaches to metacognitive instruction entails training learners with knowledge of cognitive processes, strategies, and experience; imparting learners with the knowledge or practice in using both cognitive and metacognitive strategies. The present study used think-aloud protocols to assess the effect of cognitive- and metacognitive-strategy instruction on the mathematical problem-solving performance of students with learning disabilities. In the present study, the researcher used modeling to teach seventh- and eighth-grade students with disabilities how to think aloud while solving mathematical word problems of varying levels of difficulty.

Metacognition and Measurement

This section presents current methods of assessing or measuring metacognition, examines the challenges in assessing metacognition, and identifies specific recommendations, from the literature for measuring metacognition.

Research studies in metacognition use quantitative measures to assess metacognitive components (Teong, 2010). Hart (1965), Underwood (1966), and Arbuckle and Cuddy (1969) pioneered the study of the concept of metacognition. Hart (1965) investigated students' perception of their solution to general information questions. The findings indicated that the participants' perceptions about the solution to the problem were a reliable predictor of which answer is correct. Underwood (1966) further examined the participants' perception of the difficulty of each item on the test. The result demonstrated that individual responses could predict personal learning. Arbuckle and Cuddy (1969) investigated individual judgments about learning. Result demonstrated that individuals' accurately judged their own learning.

Metacognition is difficult to measure because it is not an explicit behavior (Akturk & Sahin, 2011). Veenman (2005) delineated the method of metacognitive measurement into three categories: *probable* if implemented prior to task execution, *simultaneous* or synchronic if implemented during the task, and *retrospective* if implemented after the task. The researcher additionally informed that the tools used to measure metacognition can be examined through *reports* as relayed by the participants (questionnaires and interviews) and through objective behavior measurements (i.e., systematic and rigorous observations and think-aloud protocols). The think-aloud procedures enable researchers to access students' covert cognitive and metacognitive processes in instances when such processes cannot be observed (Ericsson & Simon, 1993). Think-aloud procedure, borrowed from cognitive psychology (Ericsson & Simon, 1984, 1993), requires participants or individuals to perform a task and use verbalizations to describe the task-performance process. Think-aloud protocols (TAPs) are the written transcripts generated from the participants' verbalizations. TAPs therefore facilitate researcher access and assessment of students "online" metacognitive ideation.

Because think-aloud methods draw on thoughts in the short-term memory, which is the pathway for all cognitive processes, the conscious thoughts of the individuals can be reported at the time they are processed (Ericsson & Simon, 1993). The researchers further noted that cognitive processes that generate verbalizations ("think alouds") are part of the cognitive processes that generate behavior or action. Therefore, think-aloud protocols are appropriate and valid method for the collection and measurement of metacognitive data (Ericsson & Simon, 1993; Fonteyn, Kuipers, & Grobe, 1993). Reactivity and completeness, however, have been identified as weaknesses that pervade the use of TAPs (Bannert & Mengelkamp, 2008; Branch, 2000). Reactivity deals with whether the cognitive demands of students' thinking out loud concurrent to problem solving interfere with the thinking process. The researchers asserted, however, that the problem can be eased by using retrospective data or postprocess questions. Branch (2000) and Fonteyn et al. (1993) found that asking postprocess questions (students recall what they were thinking immediately following a task) to participants provided information that made data collected through think aloud easier to understand and interpret. Completeness, however, deals with whether students, consciously or unconsciously, are able to convey all the cognitive processes that they think, experience, and feel during problem-solving tasks, through thinking- out loud.

To address the question of reactivity, Ericsson and Simon (1990) contended that thinking-aloud does not interfere with the systems of cognition, rather, thinking-aloud while problem solving slows down the process of cognition. The second issue, completeness, was addressed through the concept of strategy use and automaticity. Crowley, Shrager, and Siegler (1997) suggested that when students experience success with strategy use, the process eventually evolves from being explicit to implicit (automatic) and that is the goal of metacognitive strategies. Similarly, Logan (1988) proposed that when students attended to a stimulus, a new processing episode is created in the storage system. This system consists of a specific combination of the stimulus, the interpretation given to the stimulus, the response, and the task goal. Repeating the stimulus results in the retrieval of the previously-stored processing episodes that, in turn, facilitates task performance if the mapping is consistent or results in impaired performance if the mapping is inconsistent.

Proficient students automatically retrieve information without the need to activate metacognitive strategies, whereas students with learning disabilities need to activate metacognitive strategies to enable control and self-regulation of processing episodes for task completion. In the absence of verbalized thought, it is difficult to determine if the strategies and processes are deficient, delayed, or internalized to automaticity (Sweeney, 2010). Another hurdle implicated by the use of TAPs pertains to the practicality of using TAPs in the laboratory conditions versus in the classroom setting. Due to the nature of TAPs, Scott (2008) reasoned that although TAPs allow researchers to access students' use of metacognitive thinking in a laboratory setting, employing systematic observations and thinking-out loud in the classroom setting are not functional due to the related issues of managing and controlling the metacognitive behaviors of large number of students simultaneously.

For the *probable* and *retrospective* methods of measuring metacognition, the prevalent tools employed are questionnaires (students record their thinking subsequent to completing a task) and interviews (student responses to open-ended or fixed questions about thinking). Questionnaires, the most commonly-used tool for measuring metacognition, pose aspects of positives and negatives in practice. For example, a student responding to a question may be hesitant to express unfavorable ideas or experiences or may not interpret the question correctly (Scott, 2008). The positive attributes of this method are that questionnaires allow researchers to survey large groups of students simultaneously without interfering with classroom experiences and are seamless and objective to evaluate (Tobias & Everson, 1996). Furthermore, questionnaires can be used reliably and efficiently to observe cognitive engagement and motivation in situations where observations of similar events are otherwise hampered (Pintrich & DeGroot, 1990). Interviews enable thorough exploration of students' cognitive and metacognitive processes thus serving as a practical and information-rich tool for measuring metacognition (Paulhus & Vazire, 2007). The shortcomings of interviews, however, include loss of time as a result of the back-and-forth interaction between students and interviewer, dependency on the students' ability to recall information, potential for biased or incomplete "after-the-fact" descriptions of thinking (Hacker, Dunlosky, & Graesser, 1998; Scott, 2008).

Notwithstanding the shortcomings of the methods of measuring metacognition, the methods enable researchers to access cognitive processes that would otherwise be inaccessible. Sigler and Tallent-Runnels (2006) posited that more research is needed to investigate the validity of the methods used to assess the construct. Because gathering data in real time has been identified as problematic, researchers suggested using retrospective data (e.g., interviews) after the think-aloud protocol to mitigate and provide corroborating and clarifying information to TAPs (Branch, 2000; Ericsson & Simon, 1993; Fonteyn, Kuipers, & Grobe, 1993). This two-step process is a practical approach to conducting think-aloud research with students with learning disabilities, a population who demonstrate cognitive difficulty especially in producing the language required to explain mathematical problem-solving processes (Desoete, 2008; Johnstone, Bottsford-Miller, & Thompson, 2006). Because students with learning disabilities were the population sampled in the current study, retrospective data consisting of interviews and questionnaires were conducted and collected after the TAPs to corroborate and facilitate clarification of the think-aloud protocols.

Think-Aloud Protocols

To obtain and analyze students' metacognitive processes during mathematical-problem- solving tasks, Schoenfeld (1985) and Goos and Galbraith (1996) used think-aloud protocols (TAPs) as a tool to measure metacognition. TAPs are proven effective method employed to obtain insight into students' metacognition as students are asked to verbalize their thoughts while working on a task. The verbalized thoughts were recorded and transcribed verbatim, or otherwise, judged by means of systematical observation (Veenman et al., 2005). Think-aloud protocols provide substantive information on the metacognitive processes used during a learning task and are powerful predictors of test performance (Schraw 2010; Veenman et al., 2005). Because the information about metacognitive behavior is collected directly when it is executed, Veenman (2011b) reasoned that the information is less vulnerable to students' memory distortions. In addition, students do not have to judge the appropriateness of their learning processes themselves. Although TAPs may slow down the learning process, thinking out loud does not impair students' learning performance (Bannert & Mengelkamp 2008; Fox et al., 2011). A major drawback of TAPs, however, relates to the gathering and scoring of the data of individual students' think-aloud protocols. The processes that underlie TAPs are complex and time consuming and that, invariably, limits the recommendation for TAPs as a measurement tool by seasoned practitioners and researchers in studies with large samples (Azevedo et al., 2010; Schellings, 2011, 2013).

Maccini and Hughes (2000) examined the effects of a graduated instructional strategy on the word-problem-solving performance of six secondary students with learning disabilities through a multiple- baseline-across-participants study. The treatment consisted of applying the mnemonic STAR (The "S" stands for Search the word problem, "T" for Translate the words into an equation in picture form, "A" for Answer the problem, and "R" for Review the solution) and a graduated instructional phase of concrete, semiconcrete, and abstract (C-S-A) instructional model to algebra problem solving. The mnemonic *STAR* was taught to the participants as a cue for remembering the steps in solving the problems. The cognitive strategy *STAR* was taught through six scripted elements: (a) advance organizer, (b) model, (c) guided practice, (d) independent practice, (e) posttest, and (f) feedback and rewards. The researchers implemented the strategy treatment in four phases: (a) pretest, (b) concrete application, (c) semiconcrete application, and (d) abstract application. In the first phase of concrete applications, students were taught how to represent mathematics word problems using a Workmat (mathematics graphic organizers that enable students to organize visually mathematical concepts and vocabulary) with positive and negative integers. After achieving the study criterion of 80% mastery, students proceeded to the semiconcrete and abstract phases. In the semiconcrete phase, students were taught to use drawings to represent problems and to use numerical symbols in the abstract application phase. The researcher presented five problems each for the guided practice and the independent practice. The researcher created word problems for addition, subtraction, multiplication, and division of integers that were adapted from introductory algebra materials, and think–aloud protocols as the dependent measures. Students completed near-transfer and far-transfer problems after attaining criterion of completing two consecutive probes at the abstract level with 80% accuracy. Near-transfer problems consisted of five problems that were similar to the problems on the instructional measures, and far-transfer items consisted of more complex items than were used in the instructional set. The think-aloud protocols were coded for verbalizations. Students were videotaped and did not receive prompting during verbalizations.

Maccini et al. (2000) evaluated the percentage correct on problem representation, percentage correct on problem solution, and the percentage of strategy use. The results for the multiple baseline across subjects were analyzed based on the stability of baseline conditions, changes in instructional variables between conditions, and changes in mean performance between conditions. Results indicated that students' problem representation accuracy increased from the pretreatment range of 10% to 33% correct to the posttreatment range of 93% to 97% correct for the mathematical functions (addition, subtraction, and division). During the semiconcrete and abstract instructional phases, students maintained a range of 90% to 100% mean accuracy. On measures of problem-solution accuracy, the results indicated a percentage of growth from a pretreatment range of 40% to 60% to a posttreatment range of 91% to 98%. Percent correct on the near-transfer generalization tasks were higher than percentages on the generalization far-transfer tasks. These findings corroborate Hutchinson's (1993) findings. Finally, the students' scores on a measure of maintenance were 75% for problem representation and 91% for problem solution. Results, in addition, showed that five participants learned to solve subtraction, multiplication, and division word problems involving integers using the instructional strategies. The sixth participant, who was absent frequently, was not able to complete all instructional objectives. The results offered initial evidence that students with learning disabilities can be taught to solve word problems through the adoption of strategic processes that can be applied to both near- and far-transfer problems as well as to maintenance problems. The present study, similarly, assumed that students with learning disabilities can be taught to solve mathematics word problems through the adoption of cognitive and metacognitive strategies that, in turn, effected students with learning disabilities' strategy use, solution accuracy, and problem-solving experience.

Ostad and Sorenson (2007) examined the interaction between patterns of private speech and strategy use in students with learning disabilities in second grade through seventh grade (*n*=134). Students thought out loud as they solved mathematics computation problems. Participants were observed individually in two sessions. Results indicated that task-relevant speech positively correlated with metacognition and successful task completion. The students with learning disabilities used more ancillary strategies (e.g., counting on fingers), and students without identified disabilities used advanced retrieval strategies (retrieving information from memory). The researchers concluded that the students with learning disabilities are deficient in mathematical problem solving due to adoption of immature metacognitive skills. Through teacher modeling, this study aimed to help students with learning disabilities develop efficient and effective metacognitive strategies for mathematical problem solving.

Swanson's (1990) study investigated the relationship between metacognition and academic aptitude. The researcher measured metacognitive ability using tape-recorded responses to a metacognitive questionnaire. The study was comprised of 56 students in fourth and fifth grades and assessed for differences in problem-solving processes and strategy use among ability groups. Students were stratified into high- and low-ability groups based on performance on a cognitive-ability test. Participants subsequently were administered a 17-item survey to assess metacognition in the problem-solving domain.

The stratification generated four ability groups: high aptitude-high metacognition (HA-HM), high aptitude-low metacognition (HA-LM), low aptitude-high metacognition (LA-HM), and low aptitude-low metacognition (LA-LM). Participants' verbalizations were audio-recorded during a problem-solving task and a combinatorial task. The think-aloud protocols were transcribed and coded based on 24 mental components. Results showed that, irrespective of aptitude level, students with high metacognition outperformed students with low metacognition. The LA-HM group, additionally, performed better than the HA-LM group. The HA-HM group, however, were the only group who used more heuristics, strategy-subroutines and hypothetico-deductive reasoning to solve problems. Swanson's (1990) study was pivotal in linking metacognition to successful problem solving as results indicated that metacognition was more important for problem-solving success than aptitude. Furthermore, students with low aptitudes-high metacognition (LA-HM) performed as well as students with high aptitude. The implications of Swanson's finding are relevant because educators place a strong emphasis on aptitude throughout the history of psychoeducational assessment. The present study provided instruction focused on increasing the metacognitive skills of students with learning disabilities who, typically, possess lower academic aptitude compared with their average-achieving peers.

Montague and Applegate (1993b) used think-aloud protocols to examine strategy use and self-regulation in students with learning disabilities (*n*=28), with average achievement (*n*=25), and with gifted abilities (*n*=28). The students were trained in thinking out loud using two verbal reasoning problems. Subsequently, students were administered a test consisting of three mathematical word problems (a one-step, a two-step, and a three-step-word problems). Results indicated that there were no statistically significant mean differences in the cognitive or metacognitive verbalizations among the ability groups on the one-step problem. On the two-step problem, all the ability groups made few metacognitive verbalizations, and students with learning disabilities made less cognitive verbalizations than gifted students. On the three-step problem, gifted students made more cognitive and metacognitive verbalizations than the students with LD and than the average-achieving students. The researchers surmised that metacognition is triggered when students perceive mathematical problems as challenging. The students' perception of the difficulty of the problem activates persistence as well as the need to retrieve metacognitive strategies. Students with learning disabilities, however, lack the metacognitive repertoire compared with average-achieving and gifted students and, therefore, may abandon the task altogether. The present study used a teacher-modeling instructional approach to impart strategy knowledge and use to students with learning disabilities and subsequently measured the effect of the strategy awareness and usage on the problem-solving performance of the students.

Rosenzweig, Krawec, and Montague (2011) investigated the processing differences between three ability groups: students with learning disabilities, low-achieving (LA) abilities, and average-achieving (AA) abilities. The 73 participants thought out loud as they solved three mathematical word problems with increasing difficulty. The think-aloud protocols were coded and analyzed to obtain the frequency of participant's cognitive processing and metacognitive verbalizations. The latter was further analyzed for quality of verbalizations (productive or nonproductive). Results showed that the ability groups presented different patterns of verbalizations in accordance with the type of metacognitive activity and problem difficulty. Rosenzweig et al. (2011)

study was conducted with eighth-grade middle-school students who were stratified into three ability groups based on performance on the mathematics section of Florida's Comprehensive Assessment Test (FCAT). The FCAT consists of criterion-referenced tests that measure selected benchmarks in reading, science, mathematics, and writing (Rosenzweig et al., 2011). FCAT-scaled-score ranges include Levels 1 and 2 indicating below-level performance, Level 3 indicating grade-level performance, and Levels 4 and 5 indicating above grade-level performance. To be eligible to participate in the study, students with learning disabilities (n=14) were in the mathematics FCAT Level 1 or Level 2 range, LA students (n=34) equally were in the Level 1 or Level 2 range, and AA students (n=25) were in the Level 3 or 4 range. Students also had to be English proficient as measured by district standards.

A think-aloud protocol was the dependent measure for Rosenzweig et al.'s (2011) study. Students were audiotaped thinking out loud while solving one-step, two-step, and three-step mathematical word problems that required knowledge of whole numbers, decimals, and the four basic arithmetic operations (addition, subtraction, multiplication, and division). Participants were assessed individually as they thought out loud while solving three mathematics problems with varying levels of difficulty. The researchers transcribed audio-recording verbatim using Montague's (2003) seven cognitive processes (i.e., read, paraphrase, visualize, hypothesize, estimate, compute, and check) and three metacognitive strategies (i.e., self-instruction, self-questioning, and self-monitoring) as basis for coding. For data analyses, factorial analysis of variance (ANOVA) was conducted to investigate differences in metacognitive verbalizations between the ability groups. When the type of metacognitive verbalizations and problem difficulty were examined, results indicated that students across ability levels differed in the patterns of metacognitive verbalizations. For example, all participants did not differ statistically significantly on the quantity of metacognitive verbalizations regardless of the problem difficulty. AA students used more productive metacognitive verbalizations than nonproductive verbalizations. Students with learning disabilities, in comparison, used more metacognitive verbalizations than AA; however, the verbalizations were nonproductive predominantly. Additionally, students with learning disabilities and LA had increased nonproductive verbalizations as the problem difficulty increased indicating a lack in appropriate strategies for solving problems. Students with LD used more nonproductive verbalizations than AA and LA on the three-step problem indicating increased frustration with the problem (Montague et al., 2011, p. 515).

The present study aimed to extend the findings of Montague et al.'s 2011 study. This study examined the effect of cognitive, metacognitive and affective processes on the mathematical-problem-solving performance of seventh- and eighth-grade students with learning disabilities and explored the role of metacognition during mathematical-word problem solving. This study is important because it contributes to the understanding of students with learning disabilities in relation to cognitive and metacognitive processes during mathematical-problem-solving episodes. Findings from this study provided information on the phases in the teaching and task-performance processes when information deviates from being internalized or useable. For example, deficits in metacognitive skills may suggest a deficiency in strategy knowledge and usage, whereas deficits in metacognitive experience may suggest deficiency in self-efficacy.

Summary

The literature presented in this review supported the appropriateness of providing cognitive- and metacognitive-strategy instruction for students with learning disabilities for use as they solve mathematical word problems. The literature showed that students with learning disabilities are challenged by the rigors and complexities inherent in mathematical problem solving (Garrett et al., 2006; Rosenzweig et al., 2011). Deficit in cognitive and metacognitive processes may contribute to the challenges students with learning disabilities face (Bayat & Tamizi, 2010; Bornet & Wilbert, 2015; Krawec et al., 2012). This review of the literature indicated a strong correlation between mathematical problem-solving achievement and cognitive-metacognitive strategy knowledge, skill, and experience (Bayat & Tamizi, 2010; Krawec et al., 2012; Montague, 2008).

The first section of the review examined the relationship between metacognition and other constructs (cognition, learning, learning disabilities, and pedagogy). A discussion of the current methods of assessing or measuring metacognition was provided. The discussion addressed the challenges in assessing metacognition and identified specific recommendations, from the literature for measuring metacognition (Arturk et al., 2011; Branch, 2000; Fonteyn et al., 1993). This section summarized how the construct under study, metacognition, related to other constructs that are critical and germane to mathematical-word problem solving. Metacognition was shown to correlate positively with cognition (Bayata & Tamizi, 2010; Forster, 2014), learning (Montague & Applegate, 1993a; Swanson, 1990; Teong, 2003), and pedagogy (Jacobse et al., 2009; Montague, 2008; Montague et al., 2011, 2012; Ozsoy & Atamanet, 2009), and negatively with learning disabilities (Chalk et al., 2005; Kraai, 2011; Montague et al., 1993; Roberts et al., 2008).

The literature review presented studies that confirmed that when students learned how to implement cognitive and metacognitive strategies on problem-solving tasks, they improved their academic achievement (Montague et al., 2011; Trainin & Swanson, 2005), increased their self-efficacy (Montague et al., 2011), and improved their motivation for learning mathematics (Krawec et al., 2012). Learning cognitive and metacognitive strategies helped students with learning disabilities to meet the complex curricular requirements of mathematical word-problem solving in a way that is comparable with their average-achieving peers (Krawec et al., 2012; Montague et al., 2011).

When students solve mathematical word problems, they need to plan, execute, monitor and reflect on the task and the resulting solution (Montague, Enders, & Dietz, 2011; Scott, 2008). The literature reviewed indicated that when students did not know the appropriate strategy to use, applied the strategy inefficiently, or did not adequately reflect on the reasonableness of the solution obtained, the accuracy of their solution weakened (Butler, 2003; Fuchs & Fuchs, 2005; Krawec et al., 2012; Shing & Bryant, 2015). Students with learning disabilities who are weak predictors of their own knowledge and who overestimate their ability to solve mathematical word problems need to be taught content and context-related strategies to bolster their academic achievement in mathematical-word problem solving. Using think-aloud protocol as a tool to obtain and assess the cognitive and metacognitive processes of students with learning disabilities as they solve mathematical word problems, therefore, is imperative as supported in the literature.

The final section of the literature review examined research on think-aloud protocols. Think-aloud protocols provided substantive information on the metacognitive processes used during a learning task and were powerful predictors of test performance (Shraw, 2010; Veenman & Spaans, 2005). Six empirical studies - Hutchinson, 1993; Maccini and Hughes, 2000; Montague and Applegate, 1993b; Ostad and Sorensen, 2007; Rosenzweig et al., 2011; and Swanson, 1990 - investigated the effects of cognitive and metacognitive strategies, using think-aloud protocols as the dependent measure, on mathematical-problem-solving performance of students. Hutchinson (1993) used a repeated-measures single-subject design to examine the effects of a two-phase cognitive strategy (instruction and representation) on the algebra problem-solving skill of 12 secondary-school students with learning disabilities. Results showed the strategy to be an effective intervention for this sample of students with deficits in algebra problem solving. Maintenance and transfer of the strategy were equally evident. Likewise, Maccini and Hughes (2000) used a multiple baseline across-participant design to measure the effect of a mnemonic *STAR* and a graduated instructional phase of *concrete*, *semiconcrete*, and *abstract* on the word-problem-solving of six secondary-school students with learning disabilities. The results indicated that students with learning disabilities achieved increased performance on the three independent measures: problem representations, problem solution accuracy, and strategy use. Ostad and Sorensen (2007) and Swanson (1990) conducted studies on the relationship between metacognition, strategy use, and academic aptitude with primary-school students. Ostad and Sorensen (2007) found that the mathematical problem-solving ineptitude of students with learning disabilities was due to immature metacognitive skills. Swanson (1990) observed that level of academic aptitude notwithstanding students with high metacognition outperformed students with low levels of metacognition. Montague and Applegate (1993b) and Rosenzweig et al. (2011) investigated the metacognitive abilities of students with varying academic abilities (LA, LD, AA, GA) to investigate if any differences existed in their strategy use, patterns of verbalization, and self-regulation during mathematical problem-solving activities. The results indicated statistically significant differences between the groups in strategy use, self-regulation, and patterns of verbalization.

This study examined the effect of cognitive- and metacognitive-strategy instruction on the mathematical problem-solving performance of seventh- and eighth-grade students with learning disabilities. Because numerous researchers have reported successful outcomes with cognitive and metacognitive interventions, investigating the effect of cognitive-metacognitive strategies with seventh- and eighth-grade students is theoretically sound and appropriate. By conducting this study within an intact resource classroom, this study filled a gap in the research literature. The next chapter informs on the research design of this study including the instructional design, data-gathering methodologies, statistical tests used to analyze quantitative data, and qualitative data-analysis techniques.

CHAPTER III

METHODOLOGY

The purpose of this study was to investigate the effect of implementing a cognitive- and metacognitive-strategy instruction on the word-problem-solving performance and metacognitive experience of seventh- and eighth-grade students with learning disabilities. The previous two chapters provided the background for the study in relation to its aims, context, and theoretical framework. This chapter presents the research paradigm and design adopted to enable the attainment of the aims of the study. Furthermore, more information about the procedures of the study at various stages of the research, the nature and rationale of the research design and the methodology adopted, and the selection of data generation and data-collection techniques were presented. Information about the protection of human subjects and about the reliability and validity evidence, scoring, and administration procedures for the instrumentation were included.

Research Design

This investigation was a mixed-method study that triangulated qualitative data with quantitative data (Creswell & Clark, 2007). Patton (2001) recommended the use of triangulation as a means of strengthening a study by combining methods. Creswell and Clark (2007) argued further that using mixed methods solidifies the strengths of the quantitative and qualitative methodologies in addressing the research problem more thoroughly. For this study, qualitative and quantitative data were collected separately and concurrently to assess the effectiveness of cognitive- and metacognitive-strategy instruction on participants' self-efficacy beliefs and mathematical-problem-solving performance. Qualitative methods, therefore, were used to corroborate and complement quantitative findings in this study.

Think-aloud protocol was the independent variable for this study; six participants' verbalizations were audio recorded, transcribed, and coded. The dependent variables comprised the pre- and postintervention scores on the Metacognitive Experience Survey that prompted participants to rate their perceived confidence in solving two sets of three word-problem probes (see Appendix A), and the students' performance on the six word-problem probes. The first set of three word problems made up of 1-step, 2-step, and 3-step difficulty levels served as pre-intervention probes, and the second set, with identical attributes, served as the postintervention probes. The think-aloud protocols served as the verbal measure of the intervention. This study's methodological protocol is summarized in Table 2.

Table 2
Methodological Protocol

Preintervention (week 1)	Intervention (weeks 2-6)	Postintervention (week 7)
Procedures: Students attempted first set of three word-problem probes Students completed Metacognitive Experience Survey (MES) Researcher introduced cognitive-metacognitive strategy use for effective mathematical problem solving Researcher introduced thinking out loud while solving word problems	Procedures: Researcher provided instruction on cognitive-metacognitive strategy for 2 days per week for 40 mins Researcher modeled thinking aloud using scripted lessons and cue cards Students practiced cognitive and metacognitive strategies using cue cards Students practiced thinking aloud	Procedures: Students attempted second set of three word-problem probes Students completed MES 6 students audio-recorded thinking out loud as they solved the three word-problem probes.
Products: Student folders Student Cue Cards Class Wall Chart	Products: Use 10 word problems from district-approved text for instruction and practice Conduct mastery checks	Products: Think-Aloud Protocols (TAPs) Transcript Audio recordings, transciption, and coding

The researcher gathered qualitative and quantitative data concurrently to examine the efficacy of cognitive- and metacognitive-strategy instruction, assessed via Think-Aloud Protocols, on participants' development of proficiency in mathematical-problem-solving skills and in self-efficacy beliefs. This study used a pre- and postintervention design with an intervention group to investigate the effect of a cognitive and metacognitive strategy on students' mathematical problem-solving performance and perception. Quantitative data were used to analyze the Metacognitive Experience Survey (MES) and to compare students' performance on two sets of three word-problem probes. To explore the role of cognitive- and metacognitive- strategy instruction on the metacognitive experience of students with learning disabilities, a qualitative analysis of the students' verbalization, in the form of think-aloud transcripts, provided information that related students' cognitive and metacognitive processing to the outcome of mathematical problem-solving performance (Stake, 1995, p. 41).

Capitalizing on the strength of a mixed-method design, Lee (1999) suggested that a quantitative aspect of a pre- and postintervention research design provides substantive data on which to hypothesize that a cognitive and metacognitive strategy effects students' word-problem-solving ability after receiving the strategy instruction. Similarly, the qualitative approach (Creswell, 2008) provides the researcher insight on how students' metacognitive decisions during mathematical-word-problem-solving effect how students perform, where Think Aloud Protocol is the vehicle for generating and evaluating qualitative information. According to Creswell (2008), when using qualitative approaches, researchers attend to participants' verbalizations, ask general open-ended questions, and collect data in natural settings as the study develops.

The researcher in this study used narratives to evaluate students' metacognitive experience and subsequent verbalizations as they solved mathematical word problems. A higher metacognitive-experience score indicated a student's perceived confidence in solving the mathematical word-problem probes.

Settings and Participants

Twenty-two seventh- and eighth-grade students from a middle school in a medium-sized suburban school district located in the greater San Francisco Bay Area of Northern California constituted the convenience sample for this study. The participants were composed of students who have met the district's qualifying criteria as students with learning disabilities. Students with learning disabilities present evidence of (a) a disorder in one or more of the basic psychological processes including visual, auditory, or language processes; (b) academic achievement below the student's level of intellectual functioning; (c) learning problems that are not due to other handicapping conditions; and (d) ineffectiveness of general-educational alternatives to meet the student's educational needs.

Due to problems associated with the validity of the learning-disabilities label and the heterogeneity of school-identified learning-disabilities populations (Shephard, Smith, & Vojir, 1983), additional criteria were required for participation. The additional criteria were that students have a full-scale intelligence quotient (IQ) of 85 or higher on the Wechsler Intelligence Scale for Children – Revised (WISC-R; Wechsler, 1974), a Wechsler Individual Achievement Test III (WIAT Edition-III; Wechsler, 2009) problem-solving subtest score of 85 or below, a reading stanine of 3 or higher on the district-administered group test, and an algorithm knowledge for performing the four basic mathematical operations (addition, subtraction, multiplication, and division) as measured by a score of 85 or higher on the WIAT III (Wechsler, 2009) Mathematics Problem Solving and Fluency subtests. Finally, all the students who participated in this study had active Individualized Education Plans (IEP) and received special-education support and services in the area(s) of their identified disabilities. The study was conducted in a single self-contained resource classroom composed of students of diverse demographic status with regard to students' gender, English Language proficiency, and disabilities. Table 3 provides demographic information on the 22 participants involved in the current study.

Table 3
Demographical Characteristics of Study Participants

	7th Grade		8th Grade		Total
Category	Female	Male	Female	Male	*N=22*
Gender #:	5	7	4	6	22
Language:					
ELP	4	5	2	4	15
ELL	1	2	2	2	7
Disability:					
OHI	2	3	2	3	10
SLD/SLI	3	4	2	2	11
Aut.	0	0	0	1	1

Language. ELP (English Language Proficient); ELL (English Language Learner).
Disability. OHI (Other Health Impaired); SLD (Specific Learning Disability)
SLI (Speech and Language Impairment); Aut. (Autism)

Based on Metacognitive Experience Survey scores, 6 of the 22 students were selected to participate in the think-aloud process. A description of each student can be found in the Think-Aloud Protocol section of this chapter.

This study was conducted at a comprehensive middle school with sixth-, seventh-, and eighth-grade students and a population of 672 students. Families report a variety of home languages, and 11% of students receive instruction in English language development. Ethnicity data show the makeup of the school to be as follows: Hispanic American 27%, Asian American 12.0%, African American 4.0%, European American 56%, and declined to state 1.0%. At the site of this study, 28.0% of students participate in the Federal Free and Reduced Lunch Program and 13.0% are identified as students with learning disabilities. Participants in this study ranged in age between 12 and 14 years.

Protection of Human Subjects

The protection of the participants in this study followed the guidelines of the American Psychological Association's (APA, 2012) rule of conduct for research and publication to ensure that the fundamental rights of all participants were preserved. The researcher obtained approval to conduct the study from the Assistant Superintendent of the school district where the study was conducted, the middle-school principal, and the University of San Francisco's Institutional Review Board for the Protection of Human Subjects. In accordance with the district's policy, the informed-consent forms were translated to Spanish, which was the only other native language on the researcher's caseload, and mailed to the parents of the proposed participants by the office staff. The district's Spanish-certificated translator, who translated all district-related documents and communications, conducted the translation. The consent forms informed about the nature and benefits of the study and requested parent's authorization to allow their child's scores to be used anonymously and in a secure manner. In addition, the forms notified parents that participation in the study was voluntary and that students could withdraw consent to have their data included in the study at will and without repercussion.

All students who returned signed approval forms were selected to participate in the study. The researcher subsequently provided consent forms to the selected students during class informing them of the nature and benefits of the study. The forms requested their consent to participate in the study and for their scores to be used anonymously and in a secure manner. In addition, students were informed that participation was voluntary, that withdrawal from the study meant that they would still receive the strategy-instruction used in the study, and that their data would not be included in the study's data analysis.

To ensure anonymity, each participant was assigned a number. The researcher was the only person authorized to access the master list of participants and their assigned numbers. The researcher tracked all testing materials, observation notes, transcripts, and audio recordings by matching the participant to his or her number. All data and recordings gathered in the course of the study were stored in a secure location that was only accessible to the researcher. At the end of the study, the students' names and corresponding numbers were destroyed. The recordings, however, will be stored for 3 years and subsequently destroyed.

Instrumentation

This section focused on the different research instruments that were used to generate, collect, and analyze data. The rationale for the use of the tools, and demographic information on the six participants whose verbalizations generated the think-aloud protocols are included. Two quantitative instruments were used in this study: the Metacognitive Experience Survey (MES; Appendix B) and two sets of three mathematical word-problem probes (Appendix A). The Metacognitive Experience Survey and the six mathematical problem-solving probes were pre- and postintervention measures that assessed participants' metacognitive experience or self-efficacy beliefs and the efficacy of the treatment on students' perception and performance in solving the word problem probes. The Metacognitive Experience Survey was used as a pre-intervention instrument to assess participants' metacognitive experience after viewing the first set of three word-problem probes. The Metacognitive Experience Survey was re-administered postintervention to assess the effectiveness of the cognitive- and metacognitive-strategy instruction on participants' metacognitive experience after viewing the second set of three word problems. The dependent variable was the difference between pre- and postintervention scores.

Metacognitive Experience Survey scores ranged from 15 to 60 possible points. The first set of three word-problem probes was analyzed using the dependent-sample *t* test. The total preintervention scores range from 0 to 6. The 1-step mathematical word problem was scored as incorrect (zero) or correct (one point). The 2-step mathematical word problem was scored as 2-steps incorrect (zero), 1-step correct (one point) or 2-steps correct (two points). The 3-step mathematical word problem was scored as 3-steps incorrect (zero), 1-step correct (one point), 2-steps correct (two points), or 3-steps correct (three points). The second set of three word-problem probes was analyzed using the Cox-Stuart (X^2) test. The postintervention scores ranged from 0 to 6. Error type was analyzed by whether the 1-step, 2-step, and 3-step probes were answered correctly across pre- and postinterventions.

The verbalizations of the 6 students served as a qualitative instrument to observe, record, code, transcribe, and analyze across the mathematical-problem-solving activities. The students' verbalizations allowed for the emergence of the core aspects of the phenomena under study. The think-aloud protocols (TAPs) measured students' knowledge and use of the mathematical problem-solving strategies (Montague, 2003, 2008) and produced information about students' accurate application of specific problem-solving strategies (i.e., reading, paraphrasing, visualizing, hypothesizing, estimating, computing, and checking problems). Scoring, coding, and transcribing of the TAPs, therefore, required interrater agreement.

Metacognitive Experience Survey (MES)

The Metacognitive Experience Survey is designed as a diagnostic tool that elicits information related to students' task-specific self-efficacy and motivational beliefs before and after performing specific mathematical-problem-solving tasks. The MES was administered to obtain information about students' metacognitive experience before completing each of the three word-problem probes for which six students were selected and required to think aloud while solving the probes. The survey explored students' perception of familiarity, knowing, confidence, satisfaction, and difficulty (Efklides, Kiorpelisou, & Kiosseoglou, 2006) of the word problems, and comprised five Likert-type items. Students responded by placing an “X” in the box that best described how each statement applied to them. The selection choices were *Not at all True, Hardly True, Mostly True,* and *Absolutely True,* as listed in Appendix B. Each choice was given a value of 1 through 4; a higher score depicted a higher perceived metacognitive experience or self-efficacy. The researcher used reverse coding (DiStefano, Zhu, & Mîndrilă, 2009) on negatively worded items so that a high value indicated the same type of response on every item. Students, therefore, responded to the five-item questionnaire twice (pre- and postintervention) for each of the three-word problem probes (1-, 2-, 3-step mathematical word problems) yielding a total score of 60 points. Students’ scores on the MES were further categorized as *high* (50 to 60 points), *average* (40 to 49 points), and *low* (below 40 points).

Prior to completing the Metacognitive Experience Survey, participants received practice with the self-efficacy assessment procedure by participating in a similar activity where individuals were required to self-assess their capability of jumping progressively longer distances, from a few inches to several yards (Graham & Harris, 1989). Graham and Harris (1989) verified participants’ proficiency with self-assessment task prior to the initiation of the Metacognitive Experience Survey. Subsequent to reading each of the six word-problem probes, participants rated their perceived confidence of their ability to solve the problems successfully. To assess validity, the six mathematics word problems “passed” the item-analysis test (Kabiszyn & Borich, 2003) depicting the problems as being within the students’ grade level of difficulty. Additional validity evidence is that the word problems selected represent typical tasks in which students are expected to be proficient in the seventh- and eighth-grade curriculum, based on district-adopted

Common Core State Standards (CCSS, 2010) in mathematics to prepare and evaluate students.

Six Mathematical-Problem-Solving Probes

The six mathematical-problem-solving probes were selected from a pool of one-, two-, and three-step mathematical word problems developed by Montague (2002). From this pool, the researcher randomly selected 2 1-step, 2 2-step, and 2 3-step problems. Each problem required knowledge of the four basic arithmetic operations and comprised of whole numbers or decimals. Each word problem was printed on a single sheet of paper to allow room for problem solving and to lessen the amount of text on the paper. The six word problems were the same ones that the students used for the Metacognitive Experience Survey activity. A set of three questions was used for the Metacognitive Experience Survey preintervention, and the second set was used for the Metacognitive Experience Survey postintervention and for the TAPs. Each pair of 1-, 2-, and 3-step problem probes were determined to be equal in difficulty by the district curriculum specialist and by the developers (Montague et al., 1990, 1991).

Think-Aloud Protocols

With students as the unit of analysis, Think-Aloud Protocol is appropriate to the design and purpose of the study. This study used a pre- and postintervention design with an intact group of students with learning disabilities. As a qualitative component, observing, audio recording, transcribing, coding, and analyzing the verbalization of students' mathematical problem-solving endeavor allowed for the emergence of the core aspects of the phenomena under study. Audio-recording students' thinking out loud revealed essential details of the nature and extent of the students' knowledge, use, and control of cognitive and metacognitive strategies during mathematical word-problem-solving event.

Six students were individually audio recorded, in a quiet setting, while solving the second set of the three word-problem probes postintervention. The verbalizations of the six participants (Participants #2, #3, #4, #5, #12, and #13) constituted the transcript for the Think-aloud protocols. Two participants each were selected from the three metacognition categories namely *high metacognition* (HM; Participants #4 and #13), *low metacognition* (LM; Participants #3 and #12), and *average metacognition* (AM; Participants #2 and #5). Next, the demographic information of the six participants who thought out loud as they solved the second set of the three mathematical-word-problem probes is described.

Participant #4 (HM) was a 13-year-old, eighth grade, European American male with the special-education qualifying criteria of Other Health Impaired (OHI). The Individuals with Disabilities Education Act (IDEA, 2004) defined *OHI* as characterized by having limited strength, vitality, or alertness, including a heightened alertness to environmental stimuli, that results in limited alertness with respect to the educational environment, that is due to chronic or acute health problems such as asthma, attention deficit disorder, and adversely affects a child’s educational performance.

Participant #13 (HM) is a 13-year old eighth-grade, Hispanic American male with the special-education qualifying criteria of Other Health Impaired (OHI). Participant #13 was reclassified as English Language Proficient in the 2016-2017 school year.

Participant #2 (AM) was a 12-year old seventh-grade, Hispanic American female with the special-education primary-qualifying criteria of Speech or Language Impairment and a secondary-qualifying disability of Specific Learning Disability. Participant #2 was reclassified as English Language Proficient in the 2017-2018 school year. The Individuals with Disabilities Education Act (IDEA, 2004) defines the following learning disabilities as follows:

Speech Impairment (Sec. 300.8 © (11) (1):

(a) Articulation Disorder:

The pupil displays reduced intelligibility or an inability to use the speech mechanism which significantly interferes with communication and attracts adverse attention. Significantly interferes with communication occurs when the pupil's production of single or multiple speech sounds on a developmental scale of articulation competency is below that expected for his or her chronological age or developmental level and adversely affects educational performance.

(b) Specific Learning Disorder (Sec. 300.8© (10) (1):

Specific Learning Disability (SLD) means a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, that may have manifested itself in the imperfect ability to listen, think, speak, read, write, spell, or do mathematical calculations, including conditions such as perceptual disabilities, brain injury, minimal brain dysfunction, dyslexia, and developmental aphasia. The basic psychological processes include attention, visual processing, auditory processing, sensory motor skills, cognitive abilities including association, conceptualization and expression.

Participant #5 (HM) was a 13-year old, eighth grade, European American male with the special-education qualifying criteria of Autism Spectrum Disorder (ASD) with pragmatic language support needs. The Individuals with Disabilities Education Act (IDEA, 2004) defines Autism as follows:

Autism Spectrum Disorder (Sec. 300.8 © (1) (1):

Autism means a developmental disability significantly affecting verbal and nonverbal communication and social interaction, generally evident before age three, and adversely affecting a child's educational performance. Other characteristics often associated with autism are engagement in repetitive activities and stereotyped movements, resistance to environmental change or change in daily routines, and unusual responses to sensory experiences.

Participant #3 (LM) was a 13-year old, eighth-grade, European American male with the special-education primary-qualifying criteria of Speech or Language Impairment and a secondary-qualifying disability of Specific Learning Disability. Participant #3's special-education placement was changed from the Special Day Class to a Resource Specialist Program during the year when the study was conducted. A Resource Specialist Program (RSP) is a special-education program that provides specially designed academic instruction to students with identified learning disabilities who are assigned to general-education classroom for more than 50% of their school day, whereas a Special Day Class (SDC) is a self-contained special-education classroom that provides services to students with intensive educational needs that cannot be met by the general education, Resource Specialist Program, or the Designated Instructional Support (DIS) program. Students identified for the SDC program are provided academic instruction within the SDC setting for more than 50% of the student's day.

Participant #12 (LM) was a 12-year old, seventh-grade, Hispanic American male with the special-education primary-qualifying criteria of Speech or Language Impairment, and a secondary-qualifying disability of Specific Learning Disability. Participant #12 received English Language support through the English Language Development (ELD) and the Resource Specialist programs.

Transcription and Coding

The audio recordings were transcribed verbatim by the researchers over a 4-week period. Thirty percent of the transcription were cross-checked for accuracy with the original recording and yielded a 100% transcription-accuracy score. The students' verbalizations were coded and analyzed by the researcher and a trained assistant. The researcher used Montague's (2003) model of mathematical problem solving that includes seven cognitive processes (i.e., reading, paraphrasing, visualizing, hypothesizing, estimating, computing, and checking) and three metacognitive strategies (i.e., self-instruction, self-questioning, and self-monitoring) to serve as a base for coding system. Audio recordings and the think-aloud transcripts were validated by means of an interrater reliability protocol.

To determine interrater agreement, the researchers' codings of TAP were compared with established initial agreement. Next, the researchers used an established iterative protocol to negotiate and resolve disagreements. When an agreement was reached about coding, both researchers rerated all protocols based on the agreed-upon criteria. Interrater agreement was calculated using the formula: number of agreements divided by total number for proportion of agreements. The interrater agreement was 96%.

(# of Agreements / # of agreements + disagreements) X 100

Procedure

The setting for this study was the resource classroom where seventh- and eighth-grade students with learning disabilities received one period of academic support from a certificated education specialist and an educational aide. The resource-elective period occurred in a quiet setting that enhanced audio recording as six participants individually thought aloud while solving the mathematical word problems.

Table 4
Timeline for the Cognitive- and Metacognitive-Strategy Instruction Program

Phase 1 Week 1	Phase 2 Weeks 2 to 6	Phase 3 Week 7
<u>Preintervention</u>	<u>Intervention</u>	<u>Postintervention</u>
Administered 3 word problems to all participants Administered MES to all participants Provided students	Provided explicit instruction on cognitive-metacognitive processing Modeled thinking	Administered 3 word problems to all participants Administered MES to all participants Audio recorded 6 students identified through MES scores of

folders with Student Cue Cards and progress charts	aloud Used district-approved mathematics textbook two times a week for 5 weeks; 10 word problems 40-minute periods	High (50-60pts.), Medium (40-49pts.), and Low (Below 40pts.)

Each resource period, comprised of 51-minute sessions of academic remediation and intervention, hosted a maximum of seven students. The researcher, with the assistance of the trained personnel, conducted the study during each of the four resource periods. As illustrated in Table 4, aspects of the study were carried out in three phases.

During the first phase (i.e., Preintervention), the researcher administered individually the Metacognitive Experience Survey, with three word-problem probes to the students. The problems, consisting of 1-step-, 2-step-, and 3-step-difficulty levels, were presented singly on three different sheets of paper. Each student read and subsequently responded to the five-item Metacognitive Experience Survey (Appendix B).

The second phase (i.e., Intervention phase) occurred after the administration of the Metacognitive Experience Survey and the first set of three word-problem probes. The researcher explained the purpose of the study to the students. The researcher informed the students that acquiring and applying cognitive and metacognitive processes and strategies while thinking out loud during mathematical problem-solving events produced effective and efficient problem solving (Montague et al., 2000). The researcher reiterated that this technique was effectively used by expert mathematics word-problem solvers. The researcher taught students how to think-aloud while solving mathematical word problems. The researcher modeled thinking aloud (see Appendix C) using a logical reasoning problem and demonstrated applying the seven cognitive (Read, Paraphrase, Visualize, Hypothesize, Estimate, Compute, Check) and three metacognitive (self-monitoring, self-instruction, and self-questioning) processes, as well as using affective statements that facilitated solving the problem (Appendix C). Other researchers have demonstrated that cognitive and metacognitive skills need to be taught explicitly to enable students with learning disabilities to develop the construct (Desoete et al., 2003, 2008; Elbaum, Vaughn, Hughes, & Moody, 2000; Fuchs & Fuchs, 2005; Montague, 2004).

Students practiced thinking aloud as they applied the cognitive and metacognitive strategies to solve mathematical word problems from the district-approved mathematics textbook. Students were trained to think-aloud as they practiced and solved the mathematical word problems. To ensure consistency, the researcher used scripted lessons (Montague, 1996), as listed in Appendix C for all instructional sessions during the intervention phase. Other interventions materials included a class wall chart that outlined the cognitive processes and metacognitive strategies for mathematical problem solving, a personalized folder for each participant containing a cue card that illustrated the cognitive and metacognitive strategies, a progress chart for recording session events, and a blank notebook for working out word problems from the district-approved grade-level textbook.

During the third phase (i.e., postintervention), the researcher re-administered individually the second set of the three word-problem probes and the Metacognitive Experience Survey to the participants. Following an identical pattern as in the preintervention phase, the problems, made up of 1-step-, 2-step-, and 3-step-difficulty probes, were presented singly on three different sheets of paper. Each participant read and subsequently responded to the five-item Metacognitive Experience Survey (Appendix B) as presented by the researcher.

In a different setting, six participants were audio-recorded thinking out loud while solving the second set of the three word-problem probes during the postintervention phase. Each participant was individually audio recorded. Participant selection for think-aloud audio recording was based on preintervention Metacognitive Experience Survey scores of *high metacognition* (HM) of 50 to 60 points, *average metacognition* (AM) of 40 to 49 points, and *low metacognition* (LM) of below 40 points. Two students were selected from each category of metacognition. Minimal probes and prompts (for instance: "*tell me more, anything else?", "please explain that")* were used as the researcher deemed appropriate. The six participants were directed to work on the word problems one at a time (each problem was on a half sheet of a plain paper). Each participant was encouraged to ask questions or ask for help if he or she was not able to read or understand the words. In addition, if participants remained silent for longer than 30 seconds, they were encouraged to keep verbalizing. Reminders, however, were minimal to circumvent undue interference with students' cognitive and metacognitive processing (Jacobse & Harskamp, 2012). Student verbalizations were recorded using an audio recorder and subsequently coded and transcribed to produce verbal protocols.

Sample Script: Adapted from Montague's (1992) Scripted Lesson Plan.

Researcher: The goal of this study is to have you learn effective strategies used by proficient mathematics word-problem solvers. So, twice a week for the next 5 weeks, I am going to teach you to use a strategy for solving mathematical word problems. First, I will teach you a seven-part strategy for solving mathematical word problems. In the course of each session, we will practice using the strategy on a word problem from your regular mathematics textbook. I also will teach you how to think out loud while solving the word problems. You will use a Cue Card to help you remember the strategy and a progress chart to daily record your progress. All are included in your individual folder for this project. Do you have any questions?

All right. Let us begin.

*Researcher***:** People who are good mathematics problem solvers do several things in their head when they solve problems. They use several processes. Raise your hand if you know what a process is. [*Call on students. Student responses recorded on the Smartboard*]

Researcher: A process is a thinking skill. What is a *process*?

[*Students respond in unison: A process is a thinking skill.]*

*Researcher***:** Good problem solvers tell us they use the following seven processes when they solve mathematical word problems. I have placed these processes on your Student Cue Card in your folders and on these wall charts that we will use in class as we learn the strategy.

[*Show Class Chart (RPV-HECC). 3 wall charts listing (a) the seven cognitive charts only, (b) the three metacognitive strategies only, and (c) the combination of cognitive and metacognitive strategies as depicted in* Figure 2 (see Chapter 1, pg. 21). *Point to each process and read, explain, model, and question*.]

[*The instructional procedure (IP) is as follows: First, the researcher models the response, then asks the question, then students respond in unison. Then the researcher* *models the response again—e.g., "Yes, that's right, a process is a thinking skill." The researcher will ask the same question and call on students individually to respond.*

[*IP*] First, good problem solvers read the problem for understanding.

Why do you *read* mathematical word problems? You *read* for understanding.

Then good problem solvers *paraphrase* the problem in their own words to help them remember the information.

[*IP*] What does *paraphrase* mean? Put the problem in your own words.

The third process is *visualizing*. When people *visualize* word problems, they use objects to show the problem, or they draw a picture or a diagram of the problem on paper, or they make a picture in their head.

[*IP*] How do people *visualize*? They draw a picture or diagram.

Next, good problem solvers *hypothesize*. Raise your hand if you know what *hypothesize* means. [*Call on students*.]

[*IP*] *Hypothesize* means to set up a plan to solve the problem. What does *hypothesize* mean? [*Call on students*.]

Then good problem solvers *estimate* the answer. Raise your hand if you know what estimation is. [*Call on students*.] To *estimate* means to make a good prediction or have a good idea about what the answer might be using the information in the problem. Raise your hand if you know what a prediction is. [*Call on students*.] Good problem solvers *estimate* or predict answers before they do the arithmetic. After they do the arithmetic and get the actual answer, they compare their answer with the estimated answer. This helps them decide if the answer they got is right or if it is too big or too small.

[*IP*] What is estimating? Estimating is predicting the answer.

So, after good problem solvers *estimate* their answers, they do the arithmetic. We call this *computing*.

[*IP*] What is *computing*? Doing the arithmetic.

Finally, good problem solvers *check* to make sure that they have done everything right. That is, they *check* to see if they have used the right operations, completed all the necessary steps, and that their arithmetic is correct. People sometimes use the reverse operation to check their computation. For example, they may use addition to check subtraction problems and use multiplication to check division problems. Use calculators, smartphones, or computers to do the arithmetic and to check computations.

[*IP*] Why do you *check* mathematics word problems? To make sure everything is right.

[*Review Process Only Chart*]

All right, here are the seven processes and the explanations for each one. [Review the chart with the processes.]

[*Transition to SAY, ASK, CHECK Strategies*.]

Researcher: People who are good mathematics problem solvers also do several things in their head when they solve problems. First, they SAY different things to tell themselves what to do. Second, they ASK themselves questions. Third, they CHECK to see that they have done what they needed to do to solve the mathematics problems. I have put each SAY, ASK, CHECK activity with the right process on these charts.

[*Replace Cognitive Processes chart with Cognitive Processes and Metacognitive Process Strategy chart. These charts also will be mounted on the wall for easy viewing*.]

[*Show Student Cue Cards*]

I have these problem-solving processes and strategies written on cue cards for you to keep in your folders and to use when you do mathematical word problems during our sessions for this project.

Now I am going to read the entire mathematical problem-solving routine through once. Then we will read it as a group. Then I will call on each one of you to read the routine.

[*Point to each activity and verbalization as you read and explain it*.]

All right, now I would like you to read through the charts. I will help you with words if you need help. [*Group reading—twice*.]

Now I would like you to read the process and the words SAY, ASK, and CHECK. I will read the activities. [*Group*.]

Now I will read the process and the words SAY, ASK, and CHECK. You will read the activities. [*Group*.]

Now I want you to read everything. [*Individual students*.]

[*Give Student Cue Cards to students*.]

You do not need to memorize the seven processes and the activities, although I want you to know them.

Qualifications of the Researchers

The primary researcher holds a multiple-subject teaching credential, an Education Specialist credential (Mild/Moderate Disabilities), a Masters degree in Special Education, and a Master of Business Administration degree with a focus in International Marketing and Research. She has been teaching in the California public schools for 16 years. She has taught mathematics remediation courses for high-school students with LD, co-taught Algebra for eighth-grade students in the regular education setting, and instructed intern teachers in the Masters' level credential program for 5 years.

The secondary researcher assisted in coding qualitative data. The secondary researcher is a graduate student working on her doctoral degree in education. She has conducted a variety of studies including mixed-methods research and is familiar with coding transcripts. The secondary researcher is a credentialed teacher who has taught for 17 years. Currently, she is enrolled in a mathematics certificate program through the University of Phoenix online program. She has been trained in cognitive- and metacognitive-strategy instruction in the area of mathematical problem solving.

Subjectivity

When conducting a qualitative study, researchers' perceptions may interfere with the research itself (Creswell & Plano, 2007). The researcher in this study is the resource teacher of the participants and teaches intervention strategies, including cognitive and metacognitive strategies, to enable students with learning disabilities to participate successfully in the general-education curriculum. The researcher, consequently, possesses strong opinions regarding the value of cognitive and metacognitive strategies to the academic achievement of students with learning disabilities.

Qualitative data analysis entails coding and tabulating responses with data gathered through observations, think-alouds, and structured interviews. In order to circumvent researcher bias in the data-analysis process, the researcher used treatment validation procedures, interrater reliability testing, and the reporting of disconfirming evidence. For instance, employing standardized protocols for data collection, including training of study personnel, can minimize interrater variability when data was gathered and coded by multiple individuals. Additionally, steps were taken to ensure that the intervention was taught in the manner described in the proposal and prescribed by the developer of the treatment. Accordingly, each lesson was scripted.

Data Analysis

This section contains details on how the data were analyzed in relation to the research questions that guided this study. This investigation addressed the following research questions:

Research Question 1: To what extent are the seventh- and eighth-grade students with learning disabilities using the cognitive and metacognitive strategies solving mathematical word problems? Using the information obtained postintervention on the six students thinking out loud while solving three mathematical word-problem probes, participants' knowledge, use, and control of cognitive and metacognitive processes were presented descriptively for each of the three word-problem probes.

Research Question 2: To what extent does using cognitive and metacognitive strategies improve the mathematical problem-solving performance of seventh- and eighth-grade students with learning disabilities as measured by the change from pre- to postintervention scores on two sets of word-problem-solving probes? Using the three word problems solved by all participants, pre- and postintervention answers correct, a dependent-sample *t* test was used to address the question at the .05 level of significance. Using the three word problems solved, pre- and postintervention error type, a Cox-Stuart test was used to address the question at the .05 level of significance.

Research Question 3: To what extent does cognitive and metacognitive strategies improve the metacognitive experience of seventh- and eighth-grade students with learning disabilities as measured by the Metacognitive Experience Survey (MES)? Using the preintervention and postintervention Metacognitive Experience Survey scores, a dependent-sample *t* test was used to address this question at the .05 level of significance. In this study, the seven cognitive processes (read, paraphrase, visualize, hypothesize, estimate, compute, check) and the three metacognitive strategies (i.e., self-instruction, self-questioning, self-monitoring) served as the basis for the coding system. Verbalizations were coded as a metacognitive process or as belonging to one of the seven cognitive categories (i.e., reading, paraphrasing, visualizing, computing, checking, and so on). This qualitative information was coded using open or emergent coding. The open- or emergent-coding construct allowed the researcher to identify related concepts that emerged per chance during the review of the data. Emergent coding is a qualitative design that reduces immense amount of data by developing themes and core consistencies (Patton, 2002) from the data collected during TAP. Each verbalization obtained through a comment from a participant represented an occurrence. Each occurrence can be linked to more than one code. Once the coding of the qualitative information is complete, the researcher employed an ATLAS.ti (Muhr, 2004) co-occurrence frequency table to identify related concepts. *ATLAS.ti* is a qualitative-research tool that strengthens a study design by providing rich details to convey the findings. Krippendorff (2013) noted that the *ATLAS.ti* software provides systematic ways to manage text thereby eliminating the human tendency to read and recall data selectively. In addition, the software incorporates the respondents' own words in the discussion of results. The derived concepts were grouped into productive and nonproductive metacognitive verbalizations. Productive verbalizations are metacognitive statements that can be identified as self-monitoring, self-instructing, self-correcting, and self-questioning and are related to solving the problem.

Examples of productive verbalizations include "*I think this is a multiplication problem*," and "*This solution is not reasonable.*" In total, the coding system generated seven cognitive and seven metacognitive codes. Nonproductive verbalizations are affective statements that may seemingly relate to the problem but lack the essential metacognitive attributes that enhance finding a solution to the problem. Examples of nonproductive metacognitive statements include, "*I think I need a formula to solve this problem*," "*How do I do this problem*?" and "*This problem looks complicated.*"

Summary

This mixed-methods study examined the effect of cognitive- and metacognitive-strategy instruction, using think-out protocols, on the mathematical problem solving of seventh- and eighth-grade students with learning disabilities. This study used a triangulation-mixed method design where different but complementary data were collected on participants' use of cognitive and metacognitive processes. Quantitative data were used to measure the effect of teaching students how to think out loud while solving mathematics word problems after 5 weeks of cognitive- and metacognitive-strategy instruction. Quantitative data included participants' test scores from three mathematics word problems and scores on a Metacognitive Experience Survey. Qualitative data were gathered concurrently through think-aloud protocols that yielded insights into students' thought processes and mathematical word-problem-solving skills as participants' engaged in mathematical problem-solving tasks. Qualitative data, therefore, corroborated and clarified the quantitative results. In Chapter IV, the results of the quantitative and qualitative data analysis are presented and described, and these findings are discussed in Chapter V.

CHAPTER IV

RESULTS

The purpose of this study was to investigate the effect of implementing a cognitive- and metacognitive-strategy instruction on the word-problem-solving performance and metacognitive experience of seventh- and eighth-grade students with learning disabilities (LD). This chapter contains the results of the statistical analyses that addressed the research questions. The process used to analyze the verbalizations of the six think-aloud participants conducted to uncover codes and themes is described. Coding of the think-aloud protocols was based on Montague's (2003) Model of Effective Mathematical Problem Solving (Figure 2) that featured seven cognitive processes and three metacognitive strategies. Codes and themes are presented in tables, and vignettes from the think- alouds are used to emphasize key themes.

This study investigated the effect of implementing a cognitive- and metacognitive-strategy instruction on the mathematical-problem-solving performance and perception of 22 seventh- and eighth-grade students with learning disabilities. The setting was an intact resource classroom where participants received 51 minutes of academic support with the general- education curriculum. The instruments used to measure intervention outcomes were the Metacognitive Experience Survey, two sets of three mathematical-word-problem probes of varying complexity (1-step, 2-step, 3-step), and Think-Aloud Protocols. For the think-aloud protocols, six students with learning disabilities were selected based on their Metacognitive Experience Survey scores of high metacognition (50 to 60 points), average metacognition (40 to 49 points), and low metacognition (below 40 points). All participants received cognitive- and metacognitive-strategy instruction from the researcher for 5 weeks.

The dependent variables were the preintervention and postintervention scores that represented students' accuracy in solving six mathematical-word-problem probes as well as students' metacognitive experience in relation to their knowledge and ability to solve accurately the mathematical-word-problem probes. Based on the outcomes of previous empirical and theoretical research, the researcher hypothesized that as the mathematics problems increased in complexity, both accuracy and metacognitive experience would be effected negatively (Krawec et al., 2012; Montague, 2008). The researcher also predicted that participants' mathematical-word-problem-solving scores will not align with their MES scores as other studies have indicated that students with learning disabilities characteristically overestimated their ability compared with their peers (Garette, Mazzocco, & Baker, 2006; Montague, 1997, 2003).

Research Question 1

To what extent are the seventh- and eighth-grade students with learning disabilities using the cognitive and metacognitive strategies solving mathematical word problems? To answer this question, the information obtained postintervention on the six students who thought out loud while solving three mathematical word-problem probes was used. Participants' knowledge, use, and control of cognitive and metacognitive processes are presented descriptively for each of the three word-problem probes.

The first research question probed the effect of cognitive- and metacognitive-strategy instruction on the mathematical-problem-solving performance of seventh- and eighth-grade students with learning disabilities as measured by their metacognitive verbalizations collected through think-aloud protocols. The word-problem-solving performance of six participants (Participants #2, #3, #4, #5, #12, and #13) was used to answer the first research question. Two participants each were selected from the three metacognition categories namely *high metacognition* (HM; Participants #4 and #13), *low metacognition* (LM; Participants #3 and #12), and *average metacognition* (AM; Participants #2 and #5). Each participant was audio recorded individually as he or she solved each of the mathematical-word-problem probe of varying complexity. Verbalizations were transcribed and coded using the coding and scoring system as well as the mathematical problem-solving framework developed by Montague (1992). The Think-Aloud Protocol coding and scoring system used to generate the frequency counts and percentages of productive and nonproductive metacognitive verbalizations is illustrated in Appendix D.

Initial coding was based on Montague's (1992) Model of Effective Problem Solving in Mathematics mentioned in chapter I (Figure 2) and used as the theoretical framework for this study. Montague's (1992) model is comprised of seven cognitive processes (***R****ead*, ***P****araphrase*, Visualize, ***H****ypothesize*, ***E****stimate*, ***C****ompute*, and ***C****heck)* and three metacognitive strategies (*self-instruct, self-question, and self-monitor*). Table 5 illustrates the operational definition and coding of the seven cognitive processes as used in this study.

Table 5
Operational Definition and Coding of the Seven Cognitive Processes

Category Cognitive Processes	Operational Definition	Code
Read	Student reads the problem in its entirety	R
Paraphrase	Student restates the problem in his/her own words	P

Visualize	Student uses visual aids (diagrams, pictures, highlighting, mental imagery) to understand task	V
Hypothesize	Student sets up a solution path, identifies operation to use	H
Estimate	Student predicts an answer	E
Compute	Student verbalizes computation	C
Check	Student checks that steps used are sound and correct, computations are accurate	Ch

The metacognitive processes used for coding included *self-correct, self-instruct*, *self-question*, and *self-monitor*. Based on the qualitative analysis conducted by other researchers (Montague et al., 2008; Sweeney, 2010), an adapted think-aloud protocol coding and scoring sheet was used for the qualitative analysis (Appendix D). Metacognitive verbalizations were coded into productive and nonproductive categories. Productive metacognitive (PM) verbalizations operationally were identified as verbalizations that encompassed participants engaging in one or more of the following actions while solving the mathematical-word-problem probe. The participant could *self-correct* (corrects product or process errors), *self-instruct* (makes a statement that indicates control of procedures), *self-monitor* (focused attention on performance and progress), and *self-question* (considers the reasonableness of a problem or a solution path). An example of productive metacognitive verbalization (PMV; *self-question*) is illustrated in the transcript below:

> She could have made more money but she spent $12 so the total of pictures she sold was 9 pictures I think. (Participant #5, Average Metacognition, 2-step problem probe; solved correctly)

Nonproductive metacognitive verbalizations (NMV) is comprised of affective statements that conveyed no objective or practical pathway to facilitate problem solution. Nonproductive metacognitive verbalizations are defined operationally as (a) requesting the use of a calculator, (b) making statements relating to personal functioning while solving problem probes, and (c) making statements (coherent or incoherent) relating to participant's emotional disposition. An example of nonproductive metacognitive verbalization is illustrated in the transcripts: "…6 and no…12 plus 6…hmmm…maybe it's 6 times 18…Oh my gosh, what is it?..36?" (Participant #3, Low Metacognition, 2-step probe; solved incorrectly).

Verbalizations were coded as metacognitive (productive or nonproductive) only because students used their cue cards that listed the cognitive processes during assessment. Participants were not required to memorize the seven cognitive processes; however, they were assessed on their knowledge and usage of the processes. For instance, Participant #4 (High Metacognition; HM) read the 1-step probe (see Appendix E: Transcripts) and stated that "now I think I'll have to use subtraction for this one." This cognitive process of *hypothesizing* was coded as the metacognitive strategy, *self-instruct.*

To answer the first research question, productive metacognitive verbalizations (PMV) and nonproductive metacognitive verbalizations (NMV) were tallied to obtain three frequency counts for the varying problem types (1-step, 2-step, 3-step). Frequency counts were transformed into percentages to derive percentages for each of the two categories (productive metacognitive verbalizations, nonproductve metacognitive verbalizations) within each of the problem types (1-step, 2-step, 3-step). To illustrate,

$$\text{Percentage of PM for 2-step probe} = \frac{\text{frequency count of PMV for 2-step probe}}{\text{total \# of meta verbalizations across categories}} \times 100$$

Table 6
Think-Aloud Protocol for the 1-Step Probe for Participant #4

Participant #: 4	**MES: HM**	**PROBE TYPE: 1-STEP**	**SOLVED: Correct**
R: Tom needs 42 more yards to match the school passing record of 1,493 yards in football. How many yards does Tom have? P: I need 42 more yards to match …need? V: Now, I will visualize some of the key terms: 42 and 1493. H: Now I think I'll have to use subtraction for this one.			

C: Now I need to solve it. Silence… Ch: So now I need to check to make sure everything is right. So I did 1493 minus 42 and I got 1,451. That's 1,451 is how many yards Tom needs to pass the record.
Prometa: 57.2% **NonproMeta: 42.8%**

Table 7
Think-Aloud Protocol for the 1-Step Probe for Participant #13

Participant #: 13	**MES: HM**	**PROBE TYPE: 1-STEP**	**SOLVED: Correct**
R: First you should read the problem. Reads the problem. P: So as soon as you're done reading, you paraphrase in your own words. So Tom needs 42 yards to beat the record of 1493 yards in football. How many yards does Tom have so far? V: After you're done paraphrasing it, you visualize…so you underline the important stuff so 42 more yards, 1,493 yards, and how many yards. So after you done visualizing, you think to yourself, what operation should I use? H: I think we're going to use subtraction because you want to…I think you need to subtract 1493 minus 42 to see how many yards he as so far. So as soon as you are done thinking about it, then you start doing the work. C: So I wrote 1,493 minus 42 and I got 1,541. Ch: That's my answer because it tells you how many yards does he have so I subtracted 1,493 the passing record out of 42 more yards that so much he needs to match it. He has 1451 so far. That's my answer.			
Prometacog: 83.3%			**NonproMeta:16.7%**

Table 8
Think-Aloud Protocol for the 1-Step Probe for Participant #2

Participant #: 2	**MES: AM**	**PROBE TYPE: 1-STEP**	**SOLVED: Correct**
Reads the problem P: I'm going to paraphrase this question. I need 42 more yards to match the school passing record of 1493 yards in volleyball. How many yards does Tom have? V: I'm going to visualize by highlighting the important parts. 42 more yards…I'm going to highlight 1493 yards. H: Now I'm going to using subtraction and now I'm going to solve the answer. C: So I got my answer and the answer is 1451 yards and I've corrected and that's my final answer.			
Prometacog: 100.0%			**NonproMeta: 0.0%**

Table 9
Think-Aoud Protocol for the 1-Step Probe for Participant #5

Participant #: 5	**MES: AM**	**PROBE TYPE: 1-STEP**	**SOLVED: CORRECT**
R: Let's read the problem first. (Reads the problem). So it's basically asking us …ok…we're going to need to understand this problem though. (rereads the problem). P: So let's make this our own words. Tom has 42 more yards to match the school record …and the record is 1,493 yards…and we gonna try to figure out how much more yards Tom actually has at the moment. So let's get started. V: We gonna underline some important words like 42 more yards, 1,493 yards…and I think that's the words we'll underline. H: Let's hypothesize what the problem will do. I think we're going to do subtraction because we need to figure out how much Tom needs or has. C: He needs 42 more yards …so let's do 1.493 minus 42… and three minus two is one, 9 minus 4 is 5, and 4 come down and 1 brought down. So we get 1,451 and I think that's our answer but we got to see if we did the operations in the right order. Ch: Let's check…yay I'm pretty sure we did it right so the answer is…I think 1,451 yards he has at the moment.			
Prometacog: 100.0%			**NonproMetaco: 0.0%**

Table 10
Think-Aloud Protocol for the 1-Step Probe for Participant #3

Participant #: 3	MES: LM	PROBE TYPE: 1-STEP	SOLVED: Correct
R: Reads the problem. H/C: So I'm pretty sure I'm gonna subtract and do 1,493 minus 42. This gives me 1,451. Yep!			
Prometacog: 100.0%			**NonproMeta: 0.0%**

Table 11

Think-Aloud Protocol for the 1-Step Probe for Participant #12

Participant #: 12	MES: LM	PROBE TYPE: 1-STEP	SOLVED: Correct
R: Now I'm going to read the problem and if I don't understand it I will read it again. Reads the problem. P: So what I'm gonna do is paraphrase it in my own words now. So Tom needs 42 yards …42 more yards to beat …to match the school passing record of 1,493 yards in football. How many yards does Tom have so far? H: And now I'm going to hypothesize by planning how to solve my problem. So I'm going to divide 1493 by 42. C: Actually what I'm trying to do now is subtract 42 by 1,493. So Tom needs 1,451 yards . Tom has 1,451 yards right now.			
Prometacog: 83.3%			**NonproMeta:16.7%**

Analyses from the coded Think-aloud Protocols above (Tables 6 to 11) indicated that participants from the three metacognitive categories of *high metacognition* (HM), *average metacognition* (AM), and *low metacognition* (LM), solved the 1-step probe correctly using different metacognitive-verbalization skills. Participant #4 (HM) used approximately 43% of nonproductive metacognitive verbalizations compared with participants #2 and #5 (AM, nonproductivemeta:0%) and participants #3 and #12 (LM, Nonproductivemeta: 0% & 16.7%, respectively) and still solved the problem correctly. In contrast, participants from LM and AM obtained incorrect solutions whenever their nonproductive metacognitive verbalizations were higher than 25%.

Participant #4's preintervention total scores were 55 points out of 60 points on the MES and 1 point out of 6 points correct for the three mathematical problem-solving probes. Participant #4's postintervention total scores were 59 points out of 60 points on the MES and 4 points out of 6 points correct for the three mathematical problem-solving probes. Participant #4, therefore, achieved increased self-efficacy as well as increased problem-solving performance as a measure of the intervention effect.

Participant #4's metacognitive verbalizations on the 1-step probe were 57.2% productive and 42.8% nonproductive. He was successful, in spite of the high nonproductive metacognitive verbalizations, in solving the probe. For participants in the other metacognition categories (AM and LM), nonproductive metacognitive verbalization scores of higher than 25% yielded an incorrect solution. Participant #4, however, solved the 2-step probe incorrectly even with 100% productive metacognitive verbalizations. The researcher hypothesized that attributes of the 2-step probe, that will be examined later in Chapter V, in combination with the characteristics of participants' learning disabilities (discussed in chapter III, p. 86-88) may have contributed to the inability of five out of the six participants to solve the 2-step probe correctly.

Participant #13 (HM) preintervention total scores were 55 points out of 60 points on the Metacognitive Experience Survey and 1 point out of 6 points correct for the three mathematical-word-problem probes. Participant #13's postintervention total scores were 54 points out of 60 points on the Metacognitive Experience Survey and 4 points out of 6 points correct for the three mathematical-word-problem probes. Participant #13 has achieved decreased self-efficacy scores as measured by the Metacognitive Experience Survey and increased word-problem solving performance as a potential measure of the intervention effect. Participant #13's metacognitive verbalizations on the 1-step probe was 83.3% productive and 16.7% nonproductive and that is identical to the metacognitive verbalization percentages of participant #12 (LM) for the 1-step probe as well. This finding is supported by empirical research that demonstrated that students' with learning disabilities are similar in their metacognitive processing when solving relatively easier mathematical word problems (Rosenzweig et al., 2011).

Participant #2 preintervention total scores were 45 points out of 60 points on the Metacognitive Experience Survey and 2 points out of 6 points correct for the three mathematical-word-problem probes. Participant #2's postintervention total scores were 51 points out of 60 points on the Metacognitive Experience Survey and 4 points out of 6 points correct for the three mathematical-word-problem probes. Participant #2 achieved increased self-efficacy scores and increased word-problem-solving performance as measures of the intervention effect. In fact, participant #2 moved from AM (preintervention) to HM (postintervention) equally as a possible intervention effect. Participant #2's metacognitive verbalizations on the 1-step probe was 100.0% productive and 0.0% nonproductive that is identical to the metacognitive verbalization percentages of participant #3 (LM) and #5 (AM) for the 1-step probe. As surmised earlier, students with LD seem to manifest similar metacognitive behaviors when solving easier mathematical word problems.

Participant #5's preintervention total scores were 47 points out of 60 points on the Metacognitive Experience Survey and 5 points out of 6 points correct for the three mathematical-word-problem probes. Participant #5's postintervention total scores were 54 points out of 60 points on the Metacognitive Experience Survey and 6 points out of 6 points correct for the mathematical-word-problem probes. Participant #5 obtained increased self-efficacy scores on the MES and increased word-problem-solving performance. Furthermore, he moved from average metacognition during the preintervention phase into the high metacognition during the postintervention phase. This increment in Metacognitive Experience Survey scores can be attributed to enhancement in his self-efficacy as a result of the intervention. Participant #5's metacognitive verbalizations on the 1-step probe were 100.0% productive and 0.0% nonproductive. In fact, participant #5 correctly solved the three mathematical word-problem probes of varying complexity levels. Additionally, participant #5 had 25.0% nonproductive verbalizations for the 3-step probe and still solved the probe correctly.

Participant #3's preintervention total scores were 37 points out of 60 points on the Metacognitive Experience Survey and 1 point out of 6 points correct for the three mathematical-word-problem probes. Participant #3 achieved no effect on his self-efficacy scores as measured by the MES and a slight increase in word-problem solving performance as a potential measure of the intervention effect. Participant #3's metacognitive verbalizations on the 1-step probe was 100.0% productive and 0.0% nonproductive that is identical to the metacognitive verbalization percentages of participants #2 (AM) and #5 (AM) for the 1-step probe. As indicated earlier, Participant #3 correctly solved the 1-step probe. Again, the 1-step probe was easier than the 2-step probe however Participant #3 used the same verbalization count for both probes. Participant #3's nonproductive verbalizations greatly increased from 0% for the 1-step probe (correct) to 100% for the 2-step probe (incorrect), and 50% for the 3-step probe (incorrect). These increases in nonproductive metacognitive verbalizations for the more complex probes suggest that when faced with a challenging problem, unlike participants in the HM category (#4 and #13) and AM category (#2 and #5), participants in the LM category (#3 and #12) did not have or did not use appropriate strategies which may be indicative of misperception of the problem complexity or a perception that the probe was too hard and consequently triggered a processing meltdown. The literature suggests that when solving a novel or difficult problem, students with low metacognition use more nonproductive metacognitive strategies or verbalizations (Rosenzweig et al., 2011; Veenman & Spaans, 2005) due to limited metacognitive skillfulness. A deficiency in metacognitive skillfulness in participants in the LM category may explain their inability to correctly solve the 2-step and 3-step probes in the current study.

Participant #12's preintervention total scores were 38 points out of 60 points on the MES and 2 points out of 6 points correct for the three mathematical-word-problem probes. Participant #12's postintervention total scores were 40 points out of 60 points on the Metacognitive Experience Survey and 1 point out of 6 points correct for the three mathematical-word-problem probes. Participant #12 achieved increased self-efficacy scores as measured by the Metacognitive Experience Survey but decreased word-problem solving performance from pre- to postintervention. Participant #12's metacognitive verbalizations on the 1-step probe was 83.3% productive and 16.7% nonproductive, and that is identical to the metacognitive verbalization percentages of participant #13 (HM). As stated earlier, the 1-step probe was easier than the 2- and 3-step probes hence all of the participants metacognitively behaved in similar manner. Participant #12 further used the productive metacognitive verbalization coded as *self-correct* (SC) to clarify that he would not use division as he had hypothesized but rather would use subtraction and that was the correct operation needed to solve the problem.

The 2-step word-problem probe constituted a challenge for all the participants except for participants #5, #11, #17, and 22. Here are the wordings of the 2-step problem: *Marcy sold some pictures she had made for $6 each. Her materials cost her $12 and she made $42 profit. How many pictures did she sell?* Participants #4 (HM) hypothesized that he would use …division and probably addition…to solve the problem, and those were the right operations, but participant #4 did not follow through as hypothesized. Participant #13 (HM) hypothesized impulsively that *…we might use division*. Participant #2 (AM) reread the problem because, *…I don't really get it*. She subsequently hypothesized saying, *…I'm going to use division and addition*. Participant #2, however, used division only and obtained an incorrect solution. Participant #3 (LM) used 100.0% nonproductive metacognitive verbalization while solving the 2-step probe and solved the problem incorrectly. Participant #12 (LM) used the highest quantity of nonproductive metacognitive verbalizations while solving the 2-step probe. The researcher later analyzed the wordings and outcomes of the 2-step probe and surmised that the language complexity of the problem may have triggered the impulsive and illogical responses obtained from most of the participants.

For the 3-step probe, the participants in the LM category (#3 and #12) used fewer verbalization frequency count (100 and 125, respectively) with higher nonproductive metacognitive percentages (50.0% and 33.3%, respectively), whereas the participants in the AM category (#2 and #5) used more verbalization frequency counts (128 and 283, respectively) with lower or zero nonproductive metacognitive percentages (0.0% and 25.0%, respectively). Both participants in the LM category solved the 3-step probe incorrectly, whereas both participants in the AM category solved the probe correctly (see Transcript in Appendix E). The behaviors of participants #3 and #12 support the research that when students with learning disabilities, who perceive their metacognition as low, solve mathematical word problems with higher complexity, they use increased quantities of nonproductive verbalizations that do not facilitate successful problem solving compared with students with high and average metacognition (e.g., Participant #4) who tend to be more productive with their metacognitive verbalizations and more efficient in cognitive strategy use.

As illustrated in table 12, a summation of the productive metacognitive strategies used by participants who thought out loud for the three word problem probes revealed that participant #5 was the only student whose productive metacognitive strategy-use indicated elevated proportions between the three probes of varying complexity levels as an awareness of the additional cognitive load imposed by the nature of the 2-step probe used in the current study.

Table 12

Productive Metacognitive Verbalizations Used by Participants to Solve the Probes

	Productive Metacognitive Verbalizations		
Participants	1-Step Probe	2-Step Probe	3-Step Probe
4 (HM)	4	3	5
13 (HM)	5	5	10
2 (AM)	4	4	4
5 (AM)	7	10	6
3 (LM)	1	1	2
12 (LM)	5	5	6

Metacognition Categories: HM (High Metacognition); AM (Average Metacognition);
LM (Low Metacognition)

Research Question 2

To what extent does using cognitive-metacognitive strategies improve the mathematical problem-solving performance of seventh- and eighth-grade students with learning disabilities as measured by the change from pre- to posttest scores on two sets of word-problem-solving probes? Using the three word problems solved by all participants, pre- and postintervention, a dependent-samples *t* test was conducted at the .05 level of statistical significance to compare participants' preintervention scores and postintervention scores on two sets of three word-problem-solving probes of varying complexity. Pre- and postintervention descriptive statistics, the results of the dependent-sample *t* tests, and effect size are found in Table 13. For the 1-step and 3-step questions, there is an increase in the means from pre- to postintervention. The increase for 1-step question only is statistically significant with a medium effect size of .51. The change from pre- to postintervention for the 2-step questions is a decrease in the mean and is not statistically significant (Table 13). The 2- and 3-step problem probes are more complex than the 1-step problem probe. The decrease in the mean of the 2-step probe is related to the additional linguistic complexity of the probe. Further explanation is provided in the next chapter.

Table 13
Means and Standard Deviations on the Pre- and Postintervention Measure of Accuracy in Solving Mathematical-Word- Problem Probes (n = 22)

	Preintervention		Postintervention		*T*	
Ques. Type	*M*	*SD*	*M*	*SD*	*df*=21	*D*
1-step	15.41	2.42	16.77	2.92	2.35*	.51
2-step	16.45	2.92	16.00	3.07	-1.00	
3-step	16.23	3.00	16.82	2.36	1.20	

*Statistically significant at the .05 level.

As the sample size was small, the results of the dependent-sample *t* tests were checked against the nonparametric Wilcoxon test. The results are found to be the same, that is, only the 1-step probe is statistically significant.

The 22 students in this study were administered a sets of three word problems of varying complexity levels (1 step, 2 step, 3 step) pre- and postintervention to assess the effect of the strategy instruction. Students were provided and used cue cards as cognitive prompts as they solved the probes. In line with other studies that had measured the effect of cognition or metacognition on the word-problem solving of students with learning disabilities (Krawec et al., 2012; Montague, 1992, 1997) calculators were not provided, and none of the participants asked to use one. Twelve students demonstrated increased mathematical word-problem-solving performance, seven students demonstrated decreased performance, and three students demonstrated no effect on their mathematical-problem-solving performance. Further analyses revealed that, from pre- to postintervention, 21 students solved the 1-step probe (postintervetion) correctly compared with 12 students (preintervention), and 16 students solved the 3-step probe correctly postintervention compared with 10 students preintervention. Conversely, four students solved the 2-step probe correctly during the postintervention phase compared with 12 students during the preintervention phase. As stated in the *Summary of Findings* section, the wordings of the postintervention 2-step probe posed a challenge for students with learning disabilities who possess cognitive and processing deficits as an underlying manifestation of their different learning disabilities. Accordingly, the 2-step probe that entailed linguistic and mathematical complexities became a daunting task for 18 out of 22 participants. An illustration of the students' performance on the mathematical-word-problem probes is outlined in Table 14.

Table 14
Pre- and Postintervention Performance of Participants' Successful Solving of the Mathematical-Word-Problem Probes

Problem Complexity	Preintervention	Postintervention
1-Step	12	21
2-Step	12	4
3-Step	10	16

Additional analyses on the mathematical-problem-solving probe relating to the change from pre- to postintervention indicated that 12 students demonstrated increased performance, 7 students demonstrated declined performance, and 3 students demonstrated no change in performance. To answer the second research question, therefore, 12 out of the 22 participants or approximately 55% of the participants achieved improved mathematical problem-solving performance as an effect of the cognitive-metacognitive strategy-instruction received during this study. This finding is consistent with the result obtained in Montague's (1992) study that conducted a similar investigation. Additionally, students who thought out loud as they solved the word problems contributed 27.3% to the result, which implies that when students with disabilities received cognitive- and metacognitive-strategy instruction, the positive effect of the strategy-instruction is evident equally whether students verbalized solving the problems or simply solved the problems on paper.

Research Question 3

To what extent does cognitive-metacognitive strategies improve the metacognitive experience of seventh- and eighth-grade students with learning disabilities as measured by the Metacognitive Experience Survey? Using the pre- and postintervention MES scores, a dependent-samples *t* test was used to address this question at the .05 level of significance. Prior to conducting the analysis, the assumption of normally distributed difference scores was examined. The assumption was considered satisfied as the skewness and kurtosis levels were within the range for a normal distribution. The skewness and kurtosis estimates for the difference between preintervention and postintervention scores are 1.29 and 1.77, respectively, which is less than the maximum allowable values for a paired-samples *t* test (i.e., skewness $< |2|$ and kurtosis $< |9.0|$; Posten, 1985).

The third research question relates to the effect of the cognitive-metacognitive-strategy instruction on the metacognitive experience of seventh- and eighth-grade students with learning disabilities as measured by the MES. To assess students' perception of their ability to solve mathematical word problems, a metacognitive experience survey was administered prior to students' solving each problem. Scores derived from the MES were defined operationally as *high metacognition* (50 to 60 points), *average metacognition* (40 to 49 points), and *low metacognition* (below 40 points). The statistical results (see Table 15) indicated an increase in the participants' metacognitive experience means from pre- to postintervention, but the postintervention mean was not statistically significantly different than the preintervention mean.

There was an increase in the Metacognitive Experience Survey means from preintervention to postintervention indicating a possible intervention effect. The postintervention mean, however, was not found to be statistically significantly different than the preintervention mean (Table 15). Nonparametric test was conducted and was not statistically significant, the same result as was obtained from the paired-samples *t* test.

Table 15
Means, Standard Deviations, and Paired-Samples *t*-Test Result for the Metacognitive Experience SURVEY Assessing Participants' Mathematical Word-Problem Solving (*n* = 22)

MES	*M*	*SD*	*t* (*df* = 21)
Preintervention	47.27	7.31	1.98
Postintervention	49.59	6.62	

Table 16
Categories for Metacognitive Experience Scale Assessing Participants' Mathematical Word-Problem Solving (*n* = 22)

MES		MES Postintervention			
Preintervention	Statistic	High	Average	Low	Total
High	*f*	5	3	0	8
	%	22.73	13.64	0.0	36.37
Average	*f*	3	8	1	12
	%	13.64	36.30	4.55	54.49
Low	*f*	0	1	1	2
	%	0.0	4.50	4.55	9.05
Total	*f*	8	12	2	22

When the MES scores were classified as high, average, and low, the change from preintervention to postintervention is revealed (see Table 16).

To answer the third research question descriptively, 14 out of the 22 or 63.6% of students remained in the categories that they were classified originally, and 8 students changed their metacognition experience categories; three students indicated decreased metacognitive experience (two students went from HM to AM, and one student went from AM to LM), and 5 students indicated increased metacognitive experience (four students indicated increased metacognitive experience from AM to HM and one student indicated increased metacognition from LM to AM). No students' experienced metacognitive shifts from high to low or vice versa.

Summary

The results presented in this section addressed the three research questions that were the basis of the current study. A dependent-samples *t* test computed to measure the effect of the strategy instruction on the mathematical word-problem-solving performance of seventh- and eighth-grade students with LD on two sets of three word problems, with varying complexity levels, revealed no strong statistically significant relationships. One weak but statistically significant relationship was depicted on students' performance on the 1-step probe. An examination of means and standard deviations revealed an increase in the means for the 1-step and 3-step probes from pre- to postintervention. For the 2-step probe, however, the change from pre- to postintervention was not statistically significant, and there was a decrease in the mean. Another dependent-samples t test was conducted to measure the effect of the strategy instruction on the metacognitive experience of students with learning disabilities on their mathematical word-problem-solving perception. An examination of means indicated an increase in the MES means from pre- to postintervention, but the mean difference was not statistically significant.

Qualitative analysis using Think-aloud Protocols (TAPs) revealed four emerging themes in students' metacognitive verbalizations. In Theme 1, students with high metacognition (HM) were more successful in performing tasks correctly even when their nonproductive metacognitive verbalizations were considered high (above 25.0%). In Theme 2, students in the high and average metacognition (HM, AM, respectively) categories successfully solved the 3-step probe, whereas students in the low-metacognition category were not successful in solving the 3-step probe. In Theme 3, on the average, students in the LM category used less productive metacognitive verbalizations as the complexity level of the probe increased. In Theme 4, during the postintervention phase, students from all the metacognitive categories (HM, AM, and LM) used cognitive processes and metacognitive strategies extensively compared with preintervention observations.

The findings presented in Chapter IV, whether disputing or confirming previous studies, contribute to the reasoning for the results obtained. The meaning of these findings, including the nonsignificant self-efficacy and mathematical problem-solving scores, are further explored in chapter V.

CHAPTER V

SUMMARY OF FINDINGS, LIMITATIONS, AND IMPLICATIONS

This chapter is comprised of a summary of the study, a presentation of the research findings, contributions of this study to educational theory and practice, recommendations for future research, and conclusions. The results obtained in chapter IV, whether disputing or confirming prior studies, compel further clarification as to why these results occurred. This chapter will integrate discussions about the perceptions and performance of students with learning disabilities toward mathematical problem solving, the underlying uniqueness of learning disabilities, practical scaffolds to enable metacognitive reasoning for students with learning disabilities, and critical curricular accommodations and modifications that afford equitable learning to students with learning disabilities in the general-education classrooms.

The purpose of this study was to examine the effect of cognitive- and metacognitive- strategy instruction on the mathematical problem-solving performance and metacognitive experience of seventh- and eighth-grade students with learning disabilities. This study used think-aloud protocols to assess six participants' cognitive and metacognitive processing as they solved three mathematical word probes of varying complexity level. The presentation and description of the quasi-experimental data and think-aloud protocol in chapter IV provided insights on the effect of cognitive and metacognitive strategy instruction on the performance and perception of students with learning disabilities in mathematical problem solving.

Twenty-two seventh- and eighth-grade students with learning disabilities received cognitive and metacognitive strategy instruction over a 5-week period. The purpose was to examine the effect of the strategy instruction on their mathematical-problem-solving performance and perception. To assess the effect of the strategy instruction on their word-problem-solving performance, participants attempted three word problems of different complexity level (one step, two step, and three step). All students performed the tasks pre- and postintervention. Six students subsequently were selected, based on their scores on the preintervention Metacognitive Experience Survey, to think aloud as they solved three word problems of varying difficulty levels.

This study was influenced by the lag students with learning disabilities' experience in achievement in the mathematics cognitive and metacognitive domains. Ample research has established that cognitive and metacognitive domains were difficult for students with learning disabilities (Fuchs & Fuchs, 2002; Geary, 2003; Hanich, Jordan, Kaplan, & Dick, 2001; Montague & Applegate, 1993). Researchers agreed that when solving mathematical word problems, students with LD responded impulsively, used trial and error, and failed to verify solution path more than their typically-achieving peers (Bryant, Bryant, & Hammermill, 2000; Fuchs & Fuchs, 2002; Gonzalez & Espinel, 2002; Geary, 2004; Johnson, Humphrey, Mellard, Woods, & Swanson, 2010). Yet, students with learning disabilities characteristically overestimated their ability to solve mathematical word problems compared with their peers (Garette, Mazzocco, & Baker, 2006; Montague, 1997).

Each participant in this study received cognitive and metacognitive strategy instruction prior to completing a self-efficacy survey and solving three word problems of different complexity levels. Scant studies have examined the effect of cognitive and metacognitive-strategy instruction on the mathematical problem-solving performance and perception of students with learning disabilities. Research has shown, however, the importance of metacognition to academic success (Krawec & Montague, 2012; Meijer & Veenmann, 2006; Nota & Zimmerman, 2004; Trainin & Swanson, 2005; Wong, Harris, Graham, & Butler, 2006), as well as to successful mathematical-word-problem solving (Montague, 2000). At the end of the study, six participants took part in individually audio-recorded sessions where each thought aloud as he or she solved three mathematical word problems. Participants were instructed to solve the probes and to verbalize their problem solving thereby generating verbal protocols (Greene, Robertson, & Croker Costa, 2011). Thinking out loud is considered an appropriate representation of self-regulatory actions and metacognitive processes (Ericsson & Simon, 1993; Greene et al., 2011).

Metacognitive strategies that include self-instruction, self-questioning, and self-monitor are critical attributes for monitoring and evaluating cognitive progress during task execution (Montague, 2008; Veenman & Spaans, 2005). Nonetheless, sparse research has combined and measured the resulting effect of implementing cognitive- and metacognitive-strategy instruction on the mathematical-problem-solving performance and perception of seventh- and eighth-grade students with learning disabilities.

Summary of Findings

Key findings based on the three research questions suggest that cognitive- and metacognitive-strategy instruction enabled students with learning disabilities to know and use more cognitive strategies and more productive metacognitive verbalizations during mathematical-word-problem-solving event. Other themes that emerged from this qualitative investigations include the following: Theme 1: students with high metacognition (HM) were more successful in performing tasks correctly even when their nonproductive metacognitive verbalizations were considered above 25.0%. Theme 2: students in the high- and average- metacognition (HM, AM, respectively) categories successfully solved the 3-step probe, whereas students in the low-metacognition category were not successful in solving the 3-step probe. Theme 3: on the average, students in the low-metacognition (LM) category used less-productive metacognitive verbalizations as the complexity level of the probe increased. Theme 4: students from all the metacognitive categories (HM, AM, LM) used cognitive processes and metacognitive strategies extensively compared with preintervention observations that indicated the knowledge and use of less strategies.

There is statistically significant evidence that cognitive- and metacognitive-strategy instruction benefit students with learning disabilities' performance on the 1-step mathematical probe, but there is no statistically significant evidence that cognitive and metacognitive strategy instruction benefit their performance on the 2- or 3-step probes.

Finally, there is no statistical significant evidence that cognitive and metacognitive strategy instruction benefitted the metacognitive experience of students with learning disabilities in relation to their self-efficacy with mathematical-word-problem solving.

Limitations

Limitations to this study's design included construct validity (Robson, 1993; Yin, 1989), internal and external validity (Cohen, Manion, & Morrison, 2000; Vellutino & Schatschneider, 2003), and reliability (Cohen et al., 2000). Construct validity relates to how the researcher established correct operational measures for the constructs under study (Wainer & Braun, 1998). Yin (1989) noted that researchers can achieve construct validity by demonstrating that the measures employed in studies are the correct ones to measure the items or behaviors being studied. For this study, the students' word-problem-solving self-efficacy as measured by the Metacognitive Experience Survey (MES), the students' word-problem-solving performance as measured by the six word-problem probes, and the perception and performance of the six students on three word problems as measured by think-aloud protocols, served as good indicators of the effect of cognitive and metacognitive strategy instruction on the mathematical word-problem-solving skills of students with learning disabilities.

Think-aloud Protocol (TAP) data-collection process, however, is time and labor-intensive (Veenman et al., 2006). Moreover, given the richness of data and the possible multiple ways of analyzing, it is not unlikely that the interpretation, analysis, and reporting would be challenging and even subjective. As such, a researcher's attempt to deduce solely the underlying motive of certain behaviors may be constrained (Schellings & Broekkamp, 2011; Wolters, Bezon, & Arroyo-Giner, 2011), which is the case in this study where the researcher did not ask the participants to explain his or her thoughts or motives in order to avoid reactivity (Ericsson & Simon, 1993). Students with disabilities seldom spontaneously verbalize the motives for their actions (Vandevelde et al., 2015). For example, when participant #3 during TAP verbalizes, "then I am going to divide the hotel bill by 4 to see how much", the researcher coded this statement as an instructing activity; however, this statement did not match the actions that participant #3 carried out on paper.

Internal validity threat that may limit this study pertained to the nonrandom assignment of the participants in the study. Some threats that are inherent in the nonrandom assignment include *selection bias and history effects*. In this study, all participants received the given experimental treatment. Participants, however, may have been different at the start of the experiment in ways that might be attributed, inaccurately, to the treatment; *history effects* refer to the fact that all participants were receiving ongoing mathematics instruction in their general-education classrooms during the experiment that may have specific effects in addition to the effect of the treatment because the students were not receiving general-education mathematics instruction from the same teacher.

Another limitation to this study was external validity (Cohen et al., 2000). External validity pertains to whether the results of the study can be generalized to the larger population of students' with LD. This study used a small sample size for the think-aloud protocols as a result of the time and labor intensity of the data gathering and analysis process that is in line with previous studies using think-aloud protocols (Bannert & Mengelkamp, 2008; Schellings & Broekkamp, 2011; Stromso, Braten, & Samuelstuen, 2003; Swanson, 1990). Using a small sample size, however, limits the generalization of the results. Additionally, an implication of small sample size is the likelihood of being underpowered, which may explain the lack of statistically significant results obtained for this study. Furthermore, two out of three instruments used in this study (i.e., Metacognitive Experience Survey and the Think-Aloud Protocols) are measures that may be subject to researcher or participant bias. For instance, using the Metacognitive Experience Survey is dependent on a participant's accurate assessment of his or her self-efficacy measures. Students with learning disabilities notably may not self-assess precisely. Additionally, using the think-aloud protocols as a means of collecting data presumes that participants are capable of thinking out loud while engaged in task completion thereby enabling the researcher access to their metacognitive processing. Meijer, Veenman, and Van Hout-Wolters (2006) observed that students with learning disabilities do not articulate thinking and regulation thereby incorporating doubt toward the completeness or wholeness of their think- aloud protocols. This study, however, used Think-Aloud Protocols because other studies established construct validity using think-aloud protocols to measure metacognitive behaviors and processes (Ericsson & Simon, 1993; Greene & Gihooly, 1996). Ericsson and Simon (1993), cognitive psychologists, asserted that think-aloud methods draw on thoughts in the short-term memory, the domain of cognitive processes, which implies that the conscious thoughts of the individual can be reported at the time they are processed. The researchers posited, in addition, that cognitive processes that generate verbalizations are part of the cognitive processes that generate behavior. Consequently, think-aloud protocols are appropriate and valid method for collection and measurement of metacognitive data (Ericsson &

Simon, 1993; Fonteyn, Kuipers, & Grope, 1993). Other researchers recommended using retrospective data (e.g., surveys and questionnaires) to mitigate and provide corroborating and clarifying information to TAPs (Branch, 2000; Desoete, 2008; Veenman et al., 2005) and not using think-aloud procedures for studies with large samples (Azevedo & Cromley, 2004; Schellings, 2011, 2013). In this study, the Metacognitive Experience Survey (MES) and 2-set of three word-problem probes were used to collect data as a triangulation of the data collected through think-aloud protocols (Creswell & Clark, 2007).

Joppe (2000) indicated that reliability deals with the reproducibility of the results of a study under a similar methodology. Reliability is a measure of the consistency of a procedure to produce comparable results under uniform conditions, over time, and over similar samples (Cohen et al., 2000). Embedded in the definition is the concept that reliability serves to minimize biases and errors inherent in a study. For this study, the convenience sampling and the use of think-aloud protocols limit the reproducibility of the results. For instance, as a data- collection mechanism, TAP assumes that students with learning disabilities are capable of thinking aloud as they complete a task and that students' will verbalize the totality of their metacognitive activity, thereby, affording the researcher access to their metacognitive processes (Rosenzweig, Krawec, & Montague, 2011).

Another limiting factor is the duration of the study. Typically, students with learning disabilities possess one or more aspects of cognitive-processing deficit, including challenges with memory, attention, and the metacognitive domains of generating, selecting, monitoring, and applying learning strategies. Students with learning disabilities are as smart or smarter than their peers but the processing deficiencies mentioned earlier manifest as difficulty in reading, writing, spelling, reasoning, recalling, or organizing information. Considering the extent and effect of a learning disability on academic achievement, effective strategy instruction will need to be provided over longer periods. Vaughn and Wanzek (2014) contended that students with learning disabilities need more time to learn and practice new skills. Torgesen (2000) argued similarly that increasing instructional time has been shown to be an effective way to help students with learning disabilities learn advanced content and skills because the additional time affords them ample opportunity to master cognitively complex tasks. Research recommends that strategy instruction can be intensified by providing intervention every day rather than two or three times a week (Wanzek & Vaughn, 2008). Otherwise and depending on the attention and focus of students with learning disabilities, strategy instruction could be provided in longer stretches or by increasing the duration of the intervention (e.g., from 5 weeks to 30 weeks). As a consequence that this study was conducted in a school setting where students have to meet their mandated state and districtwide curricular requirements, and the researcher had limited time allotted for the study, the study was implemented twice a week for 5 weeks only.

Discussion of Findings

This section focuses on the results presented in chapter IV and, subsequently, expands on those findings. The three research questions that guided this study were as follows:

1. To what extent are the seventh- and eighth-grade students with LD using the cognitive and metacognitive strategies solving mathematical word problems?

2. To what extent does using cognitive and metacognitive strategies improve the mathematical problem-solving performance of seventh- and eighth-grade students with learning disabilities as measured by the change from pre- to posttest scores on two sets of word-problem-solving probes?
3. To what extent does cognitive and metacognitive strategies improve the metacognitive experience of seventh- and eighth-grade students with learning disabilities as measured by the Metacognitive Experience Survey (MES)?

Research Question 1

The first research question probed the effect of cognitive- and metacognitive-strategy instruction on the mathematical-problem-solving performance of seventh- and eighth-grade students with learning disabilities as measured by their metacognitive verbalizations collected through think-aloud protocols. Based on the scores of the MES administered preintervention, participants #4, #13, #2, #5, #3, and #12 were selected and audio recorded postintervention as they thought aloud while solving each of the three word-problem probes of varying complexity levels. Participants #4 and #13 were categorized as high metacognition (scoring between 50 to 60 points on the MES), participants #2 and #5 were categorized as average metacognition (scoring between 40 to 49 points on the MES), and participants #3 and #12 were categorized as low metacognition (scoring below 40 points on the MES). The three word-problem probes used postintervention in this study are outlined below.

1-step Probe: Tom needs 42 yards to match the school passing record of 1,493 yards in football. How many yards does Tom have?
2-step Probe: Marcy sold some pictures she had made for $6 each. Her materials cost $12. She made $42 profit. How many pictures did she sell?
3-step Probe: On a 4-day trip, a family spends $25 per day on gas and $35 per day on food. Their total hotel bill was $200. How much did the trip cost?

Characteristically, the attributes of the 1-step probe was challenging mathematically but easier than the 2- and 3-step probes. As noted in the Results section in chapter IV, all of the participants solved the 1-step probe correctly notwithstanding their metacognition category. Further analysis revealed that the productive metacognitive verbalizations (as measured by the frequency count) used by the students in the high-metacognition category, participants #4 and #13, were 4 and 5, respectively; used by the students in the average-metacognition category, participants #2 and #5, were 4 and 7, respectively, and used by the students in the low- metacognition category, participants #3 and #12, were 1 and 5, respectively. In terms of productive metacognitive verbalizations, therefore, at least one participant from each category used similar quantities of productive verbalizations to solve the 1-step probe. Furthermore, no students made an affective comment (i.e., *this problem is too hard for me*) or asked to use a calculator. Additionally, participant #3 (LM) used only one (*self-instruct*) out of the four productive metacognitive strategies (*self-correct, self-monitor, self-question, and self-check*), and participant #12 (LM) was the only student who used the productive metacognitive strategy operationally defined as *self-correct* for the 1-step probe. In other words, participants #4, #13, #2, #5, and #3 did not use *self-correct* to solve the 1-step probe, and still all the participants correctly solved the 1-step probe, which may be attributed to the ease of the 1-step probe as well as to the effect of the metacognitive-strategy instruction taking into consideration that the participants' average productive metacognitive verbalizations on the 1-step probe were 87.3%.

The researcher observed that, on the average, all of the 22 participants used an increased number of verbalizations on the 1-step probe postintervention compared with preintervention. When compared with the 16 participants who solved the 1-step probe without thinking out loud, the participants who thought out loud had a probe-solving success rate of 100.0% and the participants who did not think out loud indicated a probe-solving success rate of 93.8%. The researcher theorizes that this finding supports Montague's (2003) finding that the 1-step probe was not cognitively challenging on the participants, hence, concurrently thinking out loud while solving the mathematical word problem did not create extra-cognitive load on the participants who engaged in the think-aloud model. As noted earlier, the participants who thought out loud as they solved the 1-step probe outperformed their peers who did not think aloud as they solved the probes; still both groups indicated mostly similar metacognitive problem-solving behaviors while solving the 1-step probe (i.e., both groups used all the metacognitive processes except for *self-correct*). Results of the current study suggest that when one does not discriminate between the types of metacognitive verbalizations (productive or nonproductive), students with learning disabilities present relatively equivalent amounts of verbalizations irrespective of the problem difficulty. This means that students with learning disabilities are successful in using cognitive and metacognitive strategies to solve successfully mathematical word problems that are easier in complexity. Furthermore, instructions on how to solve mathematical word problems, for students with learning disabilities, could use the 1-step problem-type as a foundation to build on prior to introducing more complex problem types.

The 2-step probe posed a challenge for all the participants who thought out loud as well as for the participants who did not. As stated in chapter IV, the 2-step probe was more difficult in mathematical complexity than the 1-step probe and harder to interpret due to linguistic complexity than the 3-step probe. Table 14 in chapter IV (Results section) illustrates that amongst the student who thought out loud, participant #5 was the only student whose productive metacognitive-strategy use indicated elevated proportions between the three probes of varying complexity levels indicating an awareness of the additional cognitive load imposed by the nature of the 2-step probe used in the current study; participant #5 solved the 2-step probe correctly. The inability of other participants to adjust and adapt their strategy implementation to align with the complexity of the probe can be attributed to the fact that students with learning disabilities do not change their reasoning or strategies as the complexity of the problems change (Kraai, 2011; Larson & Gerber, 2002; Rosenzweig, Krawec, & Montague, 2011).

Table 17

Think-Aloud Protocols of Participants #5 and #13 Illustrating the Different Cognitive and Metacognitive Processes, Skills, and Strategies Used by Each Student for the 2-step Probe

Participant #: 13	**MES: HM**	**PROBE TYPE: 2-STEP**	**SOLVED: Incorrect**
R: First you read the problem. Reads the problem. So when you are done reading it, you paraphrase it into your own words. P: So Marcy sold pictures that she had made for $6 each. The materials she used cost $12. The profit she had made was $42. How many pictures did she sell? After you done paraphrasing, you visualize. V: So you underline or highlight the important stuff. The $6 each, $12, $42, and how many. After you done visualizing, you see what type of operation you would use. H: So you might use…we might use division, then after you have a hypothesis you start solving. C: So now that we're done, I believe check our answers. Ch: So it said that she made …she made 6…she made pictures for $6 each and her profit was $42. How many pictures did she sell? So that told me is that you divide $42 because that is the profit and then you divide them by 6 that's how much she sold them each and then if you divide 42 and 6, you get 7 and that's how many pictures she made because if you multiply 6 and 7, you get 42. So I checked the answer and I got 6 pictures that she sold.			
Prometacog: 100.0%			**NonproMeta:0.0%**

Participant #: 5	**MES: AM**	**PROBE TYPE: 2-STEP**	**SOLVED: Correct**
R: Read the problem for understanding it. (Reads the problem). P: So she's selling it for $6 each and her materials had cost $12. E: So that means he made more than $40. He made like $12 more if you think about it. So, we're going to underline, so we'll be paraphrasing it so it's $6 each Marcy made some pictures in. She had materials cost $12 and we want to know her profit and she made $42 that's probably not the answer. We're going to try to figure out how much …how many pictures did she sell. V: So let's see, we're visualizing it and we're going to underline $6 each, $12, and $42 profit. So let's do $42…we're doing addition here.			

C: We have to hypothesize and add $12 and that's $54. So 6 divided by 54 is 9. 6 divided by 54 is 9. And that's basically like 9 pictures in total because she had of profit. She could have made more money but she spent $12 so the total of pictures she sold was 9 pictures, I think. Ch: Let me get a look at this and I'm going to get to check to see if I'm right here. Ok, $6 each and she made a profit. So she sold … was $42…I think it's 9 pictures.
Prometacog: 91.0% **NonproMeta: 9.0%**

The think-aloud protocols (TAPS) of participant #5 and #13 are presented in Table 17 to illustrate the different cognitive and metacognitive processes, skills, and strategies used by each student for the 2-step probe. Previous research on cognitive load, linguistic complexity, and mathematics difficulty (Barbu & Beal, 2010) informed that the mathematical performance of students was poorer for word problems written in more complex language compared with the same problems in easier text, and the weakest performance was observed for problems that were both linguistically and mathematically challenging. According to Barbu and Beal (2010), linguistic complexity has an important influence on students' perceptions of the difficulty of mathematical- word problems (Barbu & Beale, 2010). In the current study, students' self-rating on the MES indicated an average of 16.8 points on the 1-step probe, 16.5 points on the 2-step probe, and 15.8 points on the 3-step probe. The Metacognitive Experience Survey scores represent participants' perception of their ability to solve each probe after reading the word problem. The participants' average Metacognitive Experience Survey scores for the three word problems (1-step, 2-step, and 3-step) are consistent with the researcher's hypothesis as well as research findings (Krawec, Huang, Montague, Kressler, & de Alba, 2012; Monatgue, 2008) that as the problem complexity increases, the performance of students with learning disabilities often decreases. In contrast to proficient problem solvers who typically use more cognitive and metacognitive strategies when solving mathematical word problems of varying complexity levels, students with learning disabilities fail to adjust and adapt their reasoning to align with the increased problem complexity (Bryant, Bryant, & Hammil, 2000; Kraai, 2011).

The 3-step probe required additional steps compared with the 1- and 2-step probes but used more predictable language in the wordings of the problem than was evident in the 2-step probe. The productive metacognitive verbalizations (as measured by the frequency count) used by the students in the high-metacognition category, participants #4 and #13, were 5 and 10, respectively; used by the students in the average-metacognition category, participants #2 and #5, were 4 and 6, respectively, and used by the students in the low-metacognition category, participants #3 and #12, were 2 and 56 respectively. When compared with the 16 participants who solved the 3-step probe without thinking out loud, the participants who thought out loud had a success rate of 66.7% (4 out of 6 students derived the correct solution), whereas the participants who did not think out loud indicated a success rate of 75% (12 out of 16 students derived the correct solution). The researcher observed that the lower success percentages earned by the participants on the 3-step probe aligns with the performance of students with learning disabilities on mathematical problems with increased levels of difficulty (Montague & Applegate, 1993). Although not as linguistically complex as the 2-step probe, the 3-step probe was mathematically more complex than both the 1- and 2-step probes and, therefore, required the implementation of additional steps to solve the probe. This additional step triggered a cognitive challenge for the participants, six of whom were required to think out loud concurrently while solving the 3-step mathematical word-problem probe, and 16 of whom made more nonproductive metacognitive verbalizations from the 2-stepto the 3-steps probes. This behavior pattern is supported in the research (Montague & Applegate, 1993; Rosenzweig et al., 2012) for students with learning disabilities and mathematical-word-problem solving with varying complexity levels. In this study, the researcher concluded that students with learning disabilities' use of increased nonproductive metacognitive verbalizations denote their increased frustration with the problem suggesting that when faced with difficult mathematical word problem, students with learning disabilities did not use appropriate resources to enhance solving the problem. Therefore, special-education and general-education teachers, who provide mathematical-problem-solving instructions to students with learning disabilities, should scaffold instructions related to solving multiple-step problems or solving problems that require increased cognitive and metacognitive processing to enable students' performance in mathematical word-problem solving.

Research Question 2

To what extent does using cognitive and metacognitive strategies improve the mathematical problem-solving performance of seventh- and eighth-grade students with learning disabilities as measured by the change from pre- to posttest scores on two sets of word-problem-solving probes? The second research question relates to the effect of using cognitive-metacognitive processes on the mathematical problem-solving performance of students with learning disabilities. To measure the effect of the strategy instruction, changes in mathematical performance from preintervention to postintervention were compared. For obvious reasons and as stated in the analysis of the findings, there was a statistically significant difference in student's mathematical performance on the 1-step probe from preintervention to postintervention. The participants' performance on the 1-step probe is consistent with previous research

(Rosenzweig et al., 2011) and can be attributed to the fact that the complexity of this problem-type was cognitively less demanding than the complexity of the 2- and 3-step probes. It should be noted that 100% of the participant who though out loud while solving the 1-step probe answered the question correctly and approximately 94% of the students who did not think out loud while solving the probe answered it correctly. One can infer that the characteristic complexity of the 1-step probe allowed participants to solve the problem correctly either by thinking out loud or by writing it out.

For the 2-step probe, the difference in the mean from pre- to postintervention was decreased and not statistically significant. Additional analysis of the 2-step probe indicated that a problem and language complexity confounded the performance of the participants. Ten participants correctly solved the probe preintervention compared with four participants postintervention. Of the six students who thought aloud, only participant #5 (1 out of 6 or 16.7%) solved the probe successfully. Participant #5 was the only student who used the cognitive process of *estimation* ("she had material cost of $12 and we want to know her profit and she made $42 that's probably not the answer. We're going to try to figure out how much…how many pictures did she sell"), and the productive metacognitive strategy of *self-question* ("we want to know her profit and she made $42 that's probably not the answer"*)*. Participant #5 was equally metacognitive in using the cognitive process of *visualization*, "we're going to underline $6 each, $12, and $42 profit". Participant #13, in comparison, did not use the metacognitive strategy of *self-questioning* or the cognitive process of *estimation*. Participant #13 was not metacognitively efficient in using the cognitive process *visualization* ("so you underline or highlight the important stuff, the $6 each, $12, $42, and how many"). Participant #13 used rote processing to verbalize the indicated amounts and the contents without authentic substantiation of the significance of the values. Of the students who did not think out loud as they solved the 2-step probe, only participants #11, #17, and #22 (3 out of 16 or 18.8%) solved the 2-step probe correctly. It is noteworthy that the metacognitive experience average score of the 4 students who answered the 2-step probe correctly was 17.8 points, and for the 18 students who answered incorrectly, it was 17.4 points.

This similarity in the perception of the students signify a lack of discernment of knowledge and ability because students had to first read the probe, then take the metacognitive experience survey, and finally solve the problem. The data indicate that participants who could not solve the problem rated their ability similarly to participants who could; This finding is consistent with the research that students with learning disabilities tend to overestimate their mathematics ability (Garnett, Mazzocco, & Baker, 2006; Montague, 1997). Participants' difficulty with solving the 2- and 3-step probes compared with the 1-step probe supports similar findings from other studies that students with learning disabilities have difficulty with multistep mathematical word problems (Bryant et al., 2000).

For the 3-step probe, 16 participants correctly solved the problem postintervention compared with 10 participants preintervention. Four out of the six students (66.7%) who thought aloud as they solved the 3-step probe answered the problem correctly, whereas 12 out of the 16 students (75%) who did not think aloud as they solved the probe answered the problem correctly. Further analysis indicated that the 3-step probe was more difficult than the 1- and 2-step probes by its characteristics of requiring additional steps to solve the problem, but unlike the 2-step probe, the language used was straightforward.

Research Question 3

The third research question relates to the effect of the cognitive- and metacognitive-strategy instruction on the metacognitive experience of seventh- and eighth-grade students with learning disabilities. The result of the current study suggests that when one does not discriminate between the types of metacognitive verbalizations (productive or nonproductive), students with learning disabilities present relatively equivalent amounts of verbalizations irrespective of the problem difficulty. Patterns of metacognitive activities, however, were different for the different metacognition categories when type of metacognitive verbalization and problem complexity were analyzed. On the 1-step probe, students' metacognitive patterns were similar and productive. On the 2- and 3-step probes, however, students had a different but similar metacognitive behavior pattern that was aligned to their metacognition categories. The researcher hypothesized that the metacognitive behavior patterns as presented for the probes with higher complexity denotes students' perception of the problem type and their ability to solve the probe. For example, participants in the low-metacognition category made more metacognitive verbalizations than participants in the average- and high-metacognition categories but the verbalizations were mostly nonproductive. This finding is consistent with others in the research (Rosenzweig et al., 2011).

Accordingly, of the 12 students who had increased scores for mathematical problem-solving performance, one student was from the LM category, five students were from the AM category, and six students were from the HM category. It appears that when students with learning disabilities perceive a problem as possessing higher complexity, they do not activate appropriate (productive) metacognitive resources as efficiently as they do with easier problems. In the current study, for instance, students were using cognitive processes in a rote manner without engaging the productive metacognitive strategies that facilitates problem solving. The literature recommends that when solving a complex mathematical word problem, students can rely on their metacognitive acuity if the cognitive skills are not available (Veenman & Spaans, 2005). Educators currently focus more on teaching cognitive skills but the findings in this study and others (Crowley, Shrager, & Siegler, 1997; Rosenzweig et al., 2011) support that the objective of conscious and intentional metacognitive-strategy instruction is to facilitate the application of cognitive processes until automaticity is developed. Rosenzweig et al. (2011) asserted that students with learning disabilities need a conscious and deliberate explicit instruction in metacognitive strategies that is anchored in developmentally appropriate cognitive processes and skills. To illustrate the need for metacognitive strategy that is anchored in the cognitive processes as evidenced in the current study, participant #13 (HM) made zero nonproductive metacognitive verbalization on the 3-step probe and solved the problem incorrectly, which affirms that students with learning disabilities use random metacognitive strategies that are not grounded in the cognitive skills and processes required to solve mathematical word problems of varying complexities. The problem-solving activity consequently fails to yield the correct solution. Mitigation strategy entails direct explicit instruction on cognitive and metacognitive processes to ensure that students with LD acquire the requisite mathematical problem-solving skills, processes, and strategies for successful problem solving.

Ancillary Analysis

Methodologically, the 5-week duration of the intervention was inadequate. Considering that the main characteristics of learning disabilities include processing deficits, memory deficits, and attention deficits, it is imperative that substantive instruction for students with learning disabilities be direct, explicit, purposeful, and long term. For the current study, data were collected over 5 weeks, but the researcher continued to work with nine of the students after the study was completed, and the researcher observed differences in these students' mathematical problem-solving behaviors in approximately 12 weeks. Currently eighth graders, 9 students who were study participants as seventh graders continued to work with the researcher. Accordingly, the researcher's resource classroom comprised 9 students who participated in the study and 13 students who did not participate in the study. The researcher observed that the students who were involved in the study experienced a lasting effect of the think-aloud metacognitive processing. The students who had been taught to use metacognitive strategies and to think out loud while solving mathematical word problems continued to use the cognitive- and metacognitive-strategy instruction model to solve mathematical word problems.

During the researcher's mathematics resource period, the researcher continued to model thinking aloud while solving mathematical word problems and to provide cognitive- and metacognitive-strategy instruction. The researcher observed that when given six mathematical word problems from the district-assigned textbook, the students who received cognitive- and metacognitive-strategy instruction solved an average of 4 problems correctly, whereas their peers who did not participate in strategy instruction averaged 2.5 problems correct after 12 weeks of intervention. After 24 weeks, the students who had received cognitive- and metacognitive- strategy instruction solved an average of 5 word problems correctly whereas the students who did not receive strategy instruction solved an average of 3.5 problems correctly. The researcher deduced that future study design should allocate a minimum of 12 weeks of intervention in order to detect and derive valid intervention effect. In spite of the limitations outlined in this section, there are some obvious implications for instruction.

Implications for Educational Practice

Brennan et al. (2010) noted that students do not arrive at a solution simply by talking about the problem; verbalizations must be productive. Montague and Applegate's (1993) study that examined the verbalizations of middle-school students as they thought aloud while solving mathematical word problems found that students who failed to solve the problem correctly used more nonproductive metacognitive verbalizations than did students who solved problems correctly. For example, in the current study, participant #12 with low metacognition used the highest verbalization count (264), mostly nonproductive, for the 2-step probe and solved the problem incorrectly, whereas participant #5 with average metacognition used a lower verbalization count (239) and solved the problem correctly, which supports Brennan et al's (2010) conclusion that more verbalizations do not necessarily lead to solving the problem correctly. High metacognition, however, facilitates the problem-solving performance irrespective of the use of productive or nonproductive verbalizations as indicated in the current study. For example, participant #4 (HM) used the highest count of nonproductive metacognitive verbalizations for the 1-step probe and solved the problem correctly, whereas participant #3 (LM) used the highest count of nonproductive verbalizations for the 2-step probe and solved the problem incorrectly. Additionally, Participant #4 (HM) used the highest count of nonproductive metacognitive verbalizations for the 3-step probe and also solved the problem correctly. Participant #3 (LM), moreover, demonstrated no variation in metacognitive verbalizations from the 1-step probe (39 verbalizations) to the 2-step probe (39 verbalizations), which demonstrated that as a problem's difficulty increased, participant #3 did not discriminate but rather used the same resources regardless of problem difficulty. It is obvious, therefore, that students with higher-metacognitive processing can adapt and adjust their strategy use as deemed necessary, whereas students with lower metacognition are not able to do likewise.

Accordingly, education practitioners should provide cognitive- and metacognitive- strategy instruction to boost the skills and abilities of all students, but specifically the skills of student with learning disabilities who predominantly manifest lower-metacognitive reasoning. Metacognitive-strategy instruction should be consistent, contextual, and explicit; metacognitive instruction should not be a single snapshot or an isolated event but rather a functional self-help tool that students with learning disabilities can acquire and use when they encounter difficult or novel tasks.

Furthermore, metacognitive research has focused mainly on examining the metacognitive differences between students with learning disabilities and average-performing students or high-achieving students. The current study, however, focused entirely on students with learning disabilities thus making it easier to assess more subtle differences between students with learning disabilities that tend to be oversimplified or ignored when this population of students are examined as a group or with other groups. Educators need to be aware of the unique and individualized needs of each student with learning disability. The special-education mantra asserts that *when you see one student with learning disability, you have seen one student with learning disability.* This statement represents the uniqueness of learning disabilities; even when multiple students have identical special-education qualifying criteria or similar academic behavior patterns, the special-education program and related services obligatorily should be individualized.

An illustration of the uniqueness of learning disabilities can be drawn from the mathematical problem-solving behaviors (perception and performance) of participants #4 and #13, both of whom were classified operationally as high metacognition in the current study. The special-education qualifying criteria for participant #4 and #13 was Other Health Impaired (see definition in Chapter III under Think Aloud Protocol). Both students were eighth graders at the time of the study and both received the cognitive- and metacognitive-strategy instruction from the researcher during the same resource period. Both participants #4 and #13, additionally, earned standard scores within the above-average range on the Weschler Individual Academic Tests' Mathematics Problem Solving subtest (WIAT III; Weschler, 2009) used as part of the assessment tools to qualify students for special-education support and services. For the current study, participants #4 and #13 manifested different metacognitive experience and performance in relation to their mathematical-problem-solving behaviors.

The following example compares participant #4 and participant #13 in relation to their performance in the current study. For the 1-step probe, participant #4's frequency count for the productive-metacognitive-strategy use was four whereas participant #13's was five. Both solved the probe correctly. For the 2-step probe, participant #4's frequency count for productive-metacognitive-strategy use was three whereas participant #13's was five. Both solved the probe incorrectly. The 2-step probe was the mathematical word problem that comprised both problem and language complexity and was, therefore, the most challenging of the three probes. Proficient word-problem solvers would use increased productive metacognitive strategies to adapt to an increased mathematical-word-problem complexity. Participant #4, however, used a decreased number of productive metacognitive strategies and participant #13 used the same number of productive metacognitive strategies compared with the numbers each participant used for the 1-step probe. In this instance, the mathematical-problem-solving behavior was different although similar in its ineffectiveness. For the 3-step probe, participant #4's frequency count for productive-metacognitive-strategy use was five, whereas participant #13's was 10. Both solved the probe correctly. In this instance, as opposed to the 2-step probe event, the mathematical- problem-solving behavior (perception and performance) was different although similar in its effectiveness. To summarize, educators need to be aware that the processing patterns, problem productivity, and verbalizations of thoughts differ by students rather than by learning disabilities. Individualized instruction, therefore, is mandatory for optimal mathematical-problem-solving perception and performance for students with learning disabilities.

The preceding analyses that compared the mathematical problem-solving behavior of two participants with seemingly identical characteristics emphasize the individuality of learning disabilities and, therefore, the need for teachers to individualize, not generalize, mathematical- problem-solving instruction. Based on the researcher's 17 years of experience working with students with learning disabilities, individualizing mathematical-problem-solving instruction by fostering cognitive and metacognitive processes and skills enables equitable curricular access in mathematical-word-problem solving for students with learning disabilities.

The current study also provided a description of the similarities and differences between the metacognition of students with learning disabilities through Think-Aloud Protocols, through the varying special-education-qualifying criteria and through the English Language Proficiency levels of the students. This comprehensive aggregation of the critical educational information that pertains to students with learning disabilities reveal subtle processing differences that may not be apparent when one solely focuses on students' disability labels. Accordingly, education practitioners are encouraged to customize cognitive- and metacognitive-strategy instruction by intentionally designing instruction that encompasses multiple processing abilities. Educators can customize their instruction to encompass multiple abilities and skills through scripted lessons as was used in this study. Scripted lessons are used as scaffolds to aid teachers to adhere to the topics and learning objectives thereby creating a learning environment that facilitates appropriate instruction individualized to the needs of each learner (Guccione, 2011). Reeves (2010) contended that scripted teaching uses repetition to reinforce the concepts that students are learning.

Additionally, educators can use think-aloud procedure to aid in assessing specific areas of weakness in the processing skills, error type, and strategy implementation of students with learning disabilities' during mathematical-problem-solving activity. Think-aloud data provides educators with invaluable information about students' problem-solving-behavior patterns that is inherently inaccessible through paper-and-pencil performance measures. Educators can use the information obtained through having students think out aloud to adapt and differentiate instruction on mathematical-word problem solving in order to help students with learning disabilities make adequate progress on the mathematics curriculum as mandated by the Individuals with Disabilities Education Act (IDEA, 2004) and the No Child Left Behind Act (NCLB, 2001). The current study demonstrated that educators need to model thinking out loud with content and contexts to help students with learning disabilities acquire the metacognitive essence that thinking out loud elicits. Considering the individualization of instruction to students with learning disabilities, think-aloud sessions can be provided directly to small groups of three to five students, or to an individual student who requires increment in their cognitive and metacognitive processing to be able to make progress on the mathematical problem-solving curriculum. Resource classrooms typically is comprised of an education specialist and a paraeducator with a maximum of 10 students per period; an environment suited for think-aloud activities.

Implications for Educational Research

A major contribution of this study was that it qualitatively delineated two types of metacognitive verbalization strategies (productive and nonproductive) against whether the solution was correct or incorrect thus helping researchers and practitioners to understand what students know about effective strategies and how students apply the strategies they know (Krawec & Montague, 2012). Think-Aloud Protocol was the instrument used to access and assess students' knowledge and application of the cognitive and metacognitive strategies. The six participants who thought out loud as they solved mathematical word problems consistently were knowledgeable about and used at least six (Read, Paraphrase, Visualize, Hypothesize, Compute, Check) out of the seven cognitive strategies taught to solve the three probes. On average, for the mathematical-word-problem types, 1-step, 2-step, and 3-step, participants' productive metacognitive verbalizations were 87.3%, 81.8%, and 79.2%, respectively. On average for the metacognition category types, HM, AM, and LM, participants' productive metacognitive verbalizations were 70.3%, 100.0%, and 91.7%, respectively for the 1-step probe, 100.0%, 95.5%, and 50.0%, respectively for the 2-step probe, and 91.7%, 87.5%, and 58.4%, respectively for the 3-step probe. These findings unequivocally emphasize the fundamental need to continue the investigation on ways to increase the metacognition of students with learning disabilities as the LM group, on the average, used the least amount of productive metacognitive verbalizations on the 2- and 3-step mathematical-problem-solving probes, both of which they solved incorrectly.

The current study contributes to the relatively small body of educational research concerning the cognitive and metacognitive performance of students with learning disabilities as they engaged in mathematical problem solving. Think-aloud protocols enabled the understanding of students' processing patterns when actually engaged in problem solving rather than retrospectively reporting how they solved a problem. Having students with learning disabilities think out loud while solving mathematical word problems is an important determinant of the type of remediation teachers need to adopt.

This study used a more descriptive nature and recommends future larger scale research to augment the research findings. Additionally, more research is needed to examine the interactive and recursive nature of mathematical-word problem solving and the effect of cognitive, metacognitive and affective factors on the development of problem-solving ability by students with learning disabilities. Future research using both qualitative and quantitative measures could investigate the variability within the learning disabilities' group and between the learning disabilities' categories. It is likely that this approach will reveal subsets of students with specific mathematical problem-solving-behavior patterns thereby enabling targeted strategy-instruction as a function of each student's deficit-specific attribute. For instance, this study found that as mathematical word problems increased in complexity, students with low metacognition tended to use more nonproductive metacognitive strategies and failed to adjust their productive metacognitive strategies accordingly. Students with learning disabilities who also possess low metacognition may have either a deficiency in productive metacognitive strategies or simply do not implement the effective and efficient strategies that they have. It is critical consequently that remediation focus on explicitly teaching students metacognitive strategies that will help them be successful when engaged in mathematical problem-solving tasks.

Another noteworthy methodological implication pertains to the underlying purpose of the study. The purpose of the current study was to examine the effect of cognitive- and metacognitive-strategy instruction on the perception (metacognitive processing) and performance (mathematical word-problem solving) of seventh- and eighth-grade students with learning disabilities. Considering the characteristics of the population under study, in relation to deficits in processing, memory, and attention, the researcher recommends that future research focuses initially on helping students to develop cognitive and metacognitive mindset by focusing solely on the process and not on the product. Using this method, educators would provide instruction on acquiring the knowledge and experience of the seven cognitive processes and the three metacognitive strategies (see Figures 1 and 2 in chapter I) to solve mathematical word problems, and students with learning disabilities would have reduced anxiety as they do not have to worry about knowing the cognitive and metacognitive processes as well as implementing the knowledge to solve the word problems correctly within a short time span. As a special-education practitioner, the researcher is aware that students with learning disabilities need extra time to acquire proficiency in academic tasks more than their average-performing peers. Students with learning disabilities, therefore, require scaffolded instruction initially and exclusively on how to cognitively and metacognitively process a mathematical word problem. Separating the focus on process-proficiency from product-accuracy allows students to focus on acquiring one skill at a time thereby reducing cognitive load. Cognitive load is a potential effect of an expectation for students with learning disabilities to be proficient in implementing the computational (process) skills to setup and solve the word problem (product) correctly while thinking and verbalizing about the word problem as was done in the current study.

Finally, based on the varying performance and perception of students with different and unique types of learning disabilities as evidenced in this study, future educational research may consider investigating the effect of a learning disability (e.g., Other Health Impaired or Speech and Language Impairment) on students' perception of and performance on mathematical word-problems, which may, in turn, help educators to better customize strategy instruction to learning-disability type thereby increasing the chances of obtaining more statistically significant outcomes. The following section points out recommendations for future research based on the insights garnered from the current study.

Recommendations for Future Research

Four main areas have been identified as critical for extending the findings of the current study. A key methodological issue demonstrated by the current study is the fact that 5 weeks did not afford enough time for the intervention and its effect to be validated. The researcher observed that students demonstrated intervention effects substantively after 12 weeks. It is recommended, therefore, that future research allot a minimum of 12 weeks when designing the methodological protocol.

In the current study, additionally, students got a score of one point for strategy knowledge even when they do not effectively apply the strategy by solving the probe correctly. This implies that the measure was more quantitative than qualitative. The concern with focusing on the

quantitative aspect relates to the fact that students with learning disabilities struggle with cognitive- and metacognitive-strategy awareness and application (Roberts et al., 2008; Schmitt & Sha, 2009); either because they are unaware of effective strategy to use or they are not conversant with effective strategy use. Since knowing the effective strategy to use is a prerequisite to knowing the effective way to use the strategy, and observed results from the current study support increased students' strategy awareness, future research can focus on the qualitative aspects of cognitive- and metacognitive-strategy use. The focus, essentially, will be on how students with learning disabilities know and control cognitive and metacognitive strategies when solving mathematical word problems, and whether they are able to apply successfully the cognitive and metacognitive strategies to other curricular content areas. The proficiency of students with learning disabilities at knowing effective strategies and having the ability to successfully apply the strategies during task performance affirms that the students grasp the essence of cognitive and metacognitive endeavors.

Finally, an aspect that was germane, but not assessed, in the current study was the participants' developmental differences, academic achievement in both reading and mathematics, and age, since these variables have been shown to effect performance variances. For example, it is prevalent that students who struggle academically are held back in the early primary grades. Consequently, students with learning disabilities may be older than their grade-level peers. It is recommended, therefore, that future research on the effect of cognitive- and metacognitive-strategy instruction on the mathematical-problem-solving performance and perception of students with learning disabilities delineate and assess the effect of these variables where applicable.

Conclusion

The current study was grounded on the hypothesis that cognitive- and metacognitive- strategy instruction would increase the mathematical word-problem-solving performance and self-efficacy profiles of students with learning disabilities thereby enabling them to make adequate progress in the regular-education mathematics curriculum. Although the findings of this study did not reveal statistically significant differences in students' performance and perception pre- and postintervention, there were nevertheless qualitatively different outcomes as seventh- and eighth-grade students with learning disabilities achieved and presented increased problem-solving performance and self-efficacy strategies observed through think-aloud protocols. The educational implications of these findings lend strong evidence to the importance of cognitive- and metacognitive-strategy instruction in the inclusive classroom. Future research should examine how a specific learning disability effect student's perception and performance of mathematical-problem solving as measured by think-aloud protocols and paper-and-pencil assessments. In order to strengthen study outcomes, it is recommended that this investigation be carried out over a minimum of 12 weeks to enable ample time for the intervention effects to be discernible.

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APPENDICES

APPENDIX A

THREE WORD-PROBLEM PROBES (SET 1 & 2)

THREE WORD-PROBLEM PROBES (SET 1)

PREINTERVENTION

Name____

1. Two schools plan a trip to the science museum. There are 1,044 people. Each bus holds 58 people. How many buses are needed?
2. In one week, the local newspaper printed 762,954 copies. From Monday to Saturday, a total of 255,960 morning papers were printed and 396, 475 evening papers were printed. How many copies of the Sunday paper were printed?
3. Two cantaloupes and three honeydew melons cost a total of $1.75. The cantaloupes cost $.50 each. How much did each honeydew melon cost?

THREE WORD-PROBLEM PROBES (SET 2)

POSTINTERVENTION

Name____

1. Tom needs 42 yards to match the school passing record of 1,493 yards in Football. How many yards does Tom have?
2. Marcy sold some pictures she had made for $6 each. Her materials cost $12. She made $42 profit. How many pictures did she sell?
3. On a four-day trip, a family spends $25 per day on gas and $35 per day on food. Their total hotel bill was $200. How much did the trip cost?

APPENDIX B

Metacognitive Experience Survey (MES)

Metacognitive Experience Survey (MES)

Probes 1, 2, 3

PREINTERVENTION

	Directions: Read the following questions carefully and place an (X) in the box that best describes how each statement relates to you	Not at all True	Hardly True	Mostly True	Absolutely True
	1. I have seen this type of problem before	1	2	3	4
	2. I understand what the problem ask me to do	1	2	3	4
	3. The problem is going to be difficult to solve	4	3	2	1
	4. I will need to use a lot of effort to solve the problem	4	3	2	1
	5. I am confident that I will solve this problem correctly	1	2	3	4

Metacognitive Experience Survey (MES)

<u>Probes 1, 2, 3</u>

POSTINTERVENTION

	Directions: Read the following questions carefully and place an (X) in the box that best describes how each statement relates to you	Not at all True	Hardly True	Mostly True	Absolutely True
	1. I have seen this type of problem before		2	3	4
	2. I understand what the problem ask me to do		2	3	4
	3. The problem is going to be difficult to solve	4	3	2	1
	4. I will need to use a lot of effort to solve the problem	4	3	2	1
	5. I am confident that I will solve this problem correctly		2	3	4

APPENDIX C

SCRIPTED LESSON

Sample Script for (Day 1)

Researcher: The goal of this study is to have you learn effective strategies used by proficient mathematics word-problem solvers. So, twice a week for the next five weeks, I am going to teach you to use a strategy for solving mathematical word problems. First, I will teach you a seven-part strategy for solving mathematical word problems. In the course of each session, we will practice using the strategy on a word problem from your regular mathematics textbook. I will also teach you how to think out loud while solving the word problems. You will use a Cue Card to help you remember the strategy and a

progress chart to daily record your progress. All are included in your individual folder for this project. Do you have any questions?
All right. Let's begin.
Researcher: People who are good mathematics problem solvers do several things in their head when they solve problems. They use several processes. Raise your hand if you know what a process is. [Call on students. Student responses will be recorded on the Smartboard]
Researcher: A process is a thinking skill. What is a process? [Students respond in unison]
Researcher: Good problem solvers tell us they use the following seven processes when they solve mathematical word problems. I have placed these processes on your Student Cue Card in your folders and on this big wall chart that we will use in class as we learn the strategy.
[Show Class Chart (RPV-HECC). Show chart with only names of processes to students. Point to each process and read, explain, model, and question.]
[The instructional procedure (IP) is as follows: First, the researcher models the response, then asks the question, then students respond in unison. Then the researcher models the response again—e.g., "Yes, that's right, a process is a thinking skill." The researcher will ask the same question and call on students individually to respond.
[IP] First, good problem solvers read the problem for understanding.
Why do you read mathematical word problems? You read for understanding.
Then good problem solvers paraphrase the problem in their own words to help them remember the information.
[IP] What does paraphrase mean? Put the problem in your own words.
The third process is visualizing. When people visualize word problems, they use objects to show the problem, or they draw a picture or a diagram of the problem on paper, or they make a picture in their head.
[IP] How do people visualize? They draw a picture or diagram.
Next, good problem solvers hypothesize. Raise your hand if you know what hypothesize means. [Call on students.]
[IP] Hypothesize means to set up a plan to solve the problem. What does hypothesize mean? [Call on students.]
Then good problem solvers estimate the answer. Raise your hand if you know what estimation is. [Call on students.]
To estimate means to make a good prediction or have a good idea about what the answer might be using the information in the problem. Raise your hand if you know what a prediction is. [Call on students.] Good problem solvers estimate or predict answers before they do the arithmetic. After they do the arithmetic and get the actual answer, they compare their answer with the estimated answer. This helps them decide if the answer they got is right or if it is too big or too small.
[IP] What is estimating? Estimating is predicting the answer.
So, after good problem solvers estimate their answers, they do the arithmetic. We call this computing.
[IP] What is computing? Doing the arithmetic.
Finally, good problem solvers check to make sure that they have done everything right. That is, they check to see if they have used the right operations, completed all the

necessary steps, and that their arithmetic is correct. People sometimes use the reverse operation to check their computation. For example, they may use addition to check subtraction problems and use multiplication to check division problems. Use calculators, smartphones, or computers to do the arithmetic and to check computations.

[IP] Why do you check mathematics word problems? To make sure everything is right.

[Review Process Only Chart]

All right, here are the seven processes and the explanations for each one. [Review the chart with the processes.]

[Transition to SAY, ASK, CHECK Strategies.]

Researcher: People who are good mathematics problem solvers also do several things in their head when they solve problems. First, they SAY different things to tell themselves what to do. Second, they ASK themselves questions. Third, they CHECK to see that they have done what they needed to do to solve the mathematics problems. I have put each SAY, ASK, CHECK activity with the right process on these charts.

[Replace Cognitive Processes chart with Cognitive Processes and Metacognitive Process Strategy chart. These charts also will be mounted on the wall for easy viewing.]

[Show Student Cue Cards]

I have these problem-solving processes and strategies written on cue cards for you to keep in your folders and use when you do mathematical word problems during our sessions for this project.

Now I am going to read the entire mathematical problem-solving routine through once. Then we will read it as a group. Then I will call on each one of you to read the routine.

[Point to each activity and verbalization as you read and explain it.]

All right, now I would like you to read through the charts. I will help you with words if you need help. [Group reading—twice.]

Now I would like you to read the process and the words SAY, ASK, and CHECK. I will read the activities. [Group.]

Now I will read the process and the words SAY, ASK, and CHECK. You will read the activities. [Group.]

Now I want you to read everything. [Individual students.]

[Give Student Cue Cards to students.]

You do not need to memorize the seven processes and the activities, although I want you to know them.

APPENDIX D

Think-Aloud Protocol Coding/Scoring Sheet

Think-Aloud Protocol Coding/Scoring Sheet

Think-Aloud Protocol Coding/Scoring Sheet: Participant #4 (HM)

Metacognitive								
Category	Operational Definition	Code	P1 (V=93)		P2 (V=113)		P3 (V=165)	
			F	%	*f*	%	*f*	%

Nonproductive								
Calculator	Request use of calculator	Cal	0	0.00	0	0.00	0	0.00
Comment	Statement of personal function	Com	3	42.80	0	0.00	1	16.70
Affect	Statement of emotional kind	AF	0	0.00	0	0.00	0	0.00
Total			3	42.80	0	0.00	1	16.70
Productive								
Self-Correct	Corrects process or product errors	SC	0	0.00	0	0.00	0	0.00
Self-Instruct	Statement about control of procedure	SI	1	14.30	1	33.33	2	33.30
Self-Monitor	Attends to performance and progress	SM	2	28.60	1	33.33	2	33.30
Self-Question	Considers problem and solution path	SQ	1	14.30	1	33.34	1	16.70
Total			4	57.20	3	100.0	5	83.30
Grand Total			7	100.0	3	100.0	6	100.0

Think-Aloud Protocol Coding/Scoring Sheet: Participant #13 (HM)

Metacognitive								
Category	Operational Definition	Code	P1 (V=196)		P2 V=232)		P3 (V=232)	
			F	%	*f*	%	*f*	%
Nonproductive								
Calculator	Request use of calculator	Cal	0	0.00	0	0.00	0	0.00
Comment	Statement of personal function	Com	1	16.70	1	12.50	0	0.00
Affect	Statement of emotional kind	AF	0	0.00	0	0.00	0	0.00
Total			1	16.70	1	12.50	0	0.00
Productive								
Self-Correct	Corrects process or product errors	SC	0	0.00	0	0.00	0	0.00
Self-Instruct	Statement about control of procedure	SI	2	33.30	3	27.50	3	30.00
Self-Monitor	Attends to performance and progress	SM	1	16.70	2	25.00	4	40.00
Self-Question	Considers problem and solution path	SQ	2	33.30	2	27.50	3	30.00
Total			5	83.30	7	87.50	10	100.0
Grand Total			6	100.0	8	100.0	10	100.0

Think-Aloud Protocol Coding/Scoring Sheet: Participant #2 (AM)

Metacognitive								
Category	Operational Definition	Code	P1 (V=101)		P2 (V=142)		P3 (V=123)	
			F	%	*f*	%	*f*	%
Nonproductive								
Calculator	Request use of calculator	Cal	0	0.00	0	0.00	0	0.00
Comment	Statement of personal	Com	0	0.00	0	0.00	0	0.00

	function							
Affect	Statement of emotional kind	AF	0	0.00	0	0.00	0	0.00
Total			0	0.00	0	0.00	0	0.00
Productive								
Self-Correct	Corrects process or product errors	SC	0	0.00	1	25.00	0	0.00
Self-Instruct	Statement about control of procedure	SI	1	25.00	1	25.00	1	25.00
Self-Monitor	Attends to performance and progress	SM	2	50.00	1	25.00	2	50.00
Self-Question	Considers problem and solution path	SQ	1	25.00	1	25.00	1	25.00
Total			4	100.0	4	100.0	4	100.0
Grand Total			4	100.0	4	100.0	4	100.0

Think-Aloud Protocol Coding/Scoring Sheet: Participant #5 (AM)

Metacognitive								
Category	Operational Definition		P1 (V=126)		P2 (V=239)		P3 (V=283)	
			F	%	*f*	%	*f*	%
Nonproductive								
Calculator	Request use of calculator	Cal	0	0.00	0	0.00	0	0.00
Comment	Statement of personal function	Com	0	0.00	1	9.00	1	12.50
Affect	Statement of emotional kind	AF	0	0.00	0	0.00	1	12.50
Total			0	0.00	1	9.00	2	25.00
Productive								
Self-Correct	Corrects process or product errors	SC	0	0.00	0	0.00	1	12.50
Self-Instruct	Statement about control of procedure	SI	2	28.60	4	36.40	3	35.70
Self-Monitor	Attends to performance and progress	SM	3	42.80	4	36.40	1	12.50
Self-Question	Considers problem and solution path	SQ	2	28.60	2	18.20	1	12.50
Total			7	100.0	10	91.00	6	75.00
Grand Total			7	100.0	11	100.0	8	100.0

Think-Aloud Protocol Coding/Scoring Sheet: Participant #3 (LM)

Metacognitive								
Category	Operational Definition	Code	P1 (V=39)		P2 (V=39)		P3 (V=100)	
			F	%	*f*	%	*f*	%
Nonproductive								
Calculator	Request use of calculator	Cal	0	0.00	0	0.00	0	0.00
Comment	Statement of personal	Com	0	0.00	4	66.70	2	50.00

	function							
Affect	Statement of emotional kind	AF	0	0.00	2	33.30	0	0.00
Total			0	0.00	6	100.0	2	50.00
Productive								
Self-Correct	Corrects process or product errors	SC	0	0.00	0	0.00	0	0.00
Self-Instruct	Statement about control of procedure	SI	1	100.0	0	0.00	1	25.00
Self-Monitor	Attends to performance and progress	SM	0	0.00	0	0.00	1	25.00
Self-Question	Considers problem and solution path	SQ	0	0.00	0	0.00	0	0.00
Total			1	100.0	0	100.0	2	50.00
Grand Total			1	100.0	6	100.0	4	100.0

Think-Aloud Protocol Coding/Scoring Sheet: Participant #12 (LM)

Metacognitive								
Category	Operational Definition	Code	P1 (V=125)		P2 (V=264)		P3 (V=125)	
			F	%	*f*	%	*f*	%
Nonproductive								
Calculator	Request use of calculator	Cal	0	0.00	0	0.00	0	0.00
Comment	Statement of personal function	Com	1	16.70	0	0.00	2	22.20
Affect	Statement of emotional kind	AF	0	0.00	0	0.00	1	11.10
Total			1	16.70	0	0.00	3	33.30
Productive								
Self-Correct	Corrects process or product errors	SC	2	33.30	0	0.00	2	22.20
Self-Instruct	Statement about control of procedure	SI	2	33.30	1	0.38	1	11.10
Self-Monitor	Attends to performance and progress	SM	1	16.70	1	0.38	1	11.10
Self-Question	Considers problem and solution path	SQ	0	0.00	1	0.38	2	22.20
Total			5	83.30	3	1.14	6	66.70
Grand Total			6	100.0	3	1.14	9	100.0

APPENDIX E

Six Coded Think-Aloud Protocols of Students with Learning Disabilities

Six Coded Think-Aloud Protocols of Students with Learning Disabilities

Participant #: 4	**MES: HM**	**PROBE TYPE: 1-STEP**	**SOLVED: Correct**
R: Tom needs 42 more yards to match the school passing record of 1,493 yards in football. How many yards does Tom have? P: I need 42 more yards to match …need? V: Now, I will visualize some of the key terms: 42 and 1493. H: Now I think I'll have to use subtraction for this one. C: Now I need to solve it. Silence…			

Ch: So now I need to check to make sure everything is right. So I did 1493 minus 42 and I got 1,451. That's 1,451 is how many yards Tom needs to pass the record.	
Prometa: 57.2%	**NonproMeta: 42.8%**

Participant #: 4	**MES: HM**	**PROBE TYPE: 2-STEP**	**SOLVED: Incorrect**
R: Marcy sold some pictures she had made for $6 each. Her materials cost her $12 and she made $42 profit. How many pictures did she sell? P: Now to paraphrase…ehm… I made some picture that I sell for $6 each. My materials cost me $12 and I made $42 of profit. How many pictures did I sell? V: Now I have to visualize the key terms. The $6 each, $12 and $42 profit. Now I think H: I'll have to use… ehm…division and probably addition. C: Now I have to compute it. (works silently)…. Ch: Now I have to check to make sure everything is right. So 42 divided by 6 is 7. So she sold 7 pictures.			
Prometa: 100.0%			**NonproMeta:0.0%**

Participant #: 4	**MES: HM**	**PROBE TYPE: 3-STEP**	**SOLVED: Correct**
R: First I will read the problem. (*reads the problem*). On a four-day trip, a family spends $25 per day on gas and $35 per day on food. Their total hotel bill was $200. How much did the trip cost? P: Now I will paraphrase it. I went on a four-day trip and spent $25 per day on gas and $35 per day on food. The total amount…the total amount I spent was $200. How much did the trip cost? V: Ehm, now next I visualize. Some of the key words, four-day trip, $25 per day and $35 per day and $200. H: Next I have to (pauses) next, I think I will have to use multiplication and addition to solve this and then I will compute it. C: works silenly…. Ch: Now I have to check to make sure everything is right. I got $440 from multiplying 25 and 4 and 35 times 4and I did $140 together and I get 240 and I added 240 plus 200 to get $440.			
Prometa:83.3%			**NonproMeta:16.7 %**

Participant #: 2	**MES: AM**	**PROBE TYPE: 1-STEP**	**SOLVED: Correct**
Reads the problem P: I'm going to paraphrase this question. I need 42 more yards to match the school passing record of 1493 yards in volleyball. How many yards does Tom have? V: I'm going to visualize by highlighting the important parts. 42 more yards…I'm going to highlight 1493 yards. H: Now I'm going to using subtraction and now I'm going to solve the answer. C: So I got my answer and the answer is 1451 yards and I've corrected and that's my final answer.			
Prometacog: 100.0%			**NonproMeta: 0.0%**

Participant #: 2	**MES: AM**	**PROBE TYPE: 2-STEP**	**SOLVED: Incorrect**
R: Reads the problem. I am going to reread the question because I don't really get it(rereads). P: So now I am going to paraphrase these in my own words. Marcy sold some signs for $6 each. Her materials cost her $12. She made $42 profit. How many signs did she sell? V: So now I'm going to visualize by underlining $6 each, $12, and $42 profit. H: So I think I'm going to be using addition and I'm going to solve the problem. C: works silently… Ch: So I checked the answer and I got 6 pictures that she sold.			
Prometacog: 100.0%			**NonproMeta: 0.0 %**

Participant #: 2	**MES: AM**	**PROBE TYPE: 3-STEP**	**SOLVED: Correct**
R: Reads the problem.			

P: So now I am going to paraphrase using my own words. On a four-day trip, my family spends $25 per day on clothes and $35 per day on gas. The total hotel bill was $200. How much did the trip cost? V: So I am going to highlight $25 per day and $35 per day and $200. H: I think I'm going to be using addition and multiplication. C: And now I'm going to solve the problem. Works silently… Ch: Ok, so my answer is $440 is how much the trip cost and I checked and everything.
Prometacog: 100.0% **NonproMeta: 0.0%**

Participant #: 13	**MES: HM**	**PROBE TYPE: 1-STEP**	**SOLVED: Correct**
R: First you should read the problem. Reads the problem. P: So as soon as you're done reading, you paraphrase in your own words. So Tom needs 42 yards to beat the record of 1493 yards in football. How many yards does Tom have so far? V: After you're done paraphrasing it, you visualize…so you underline the important stuff so 42 more yards, 1,493 yards, and how many yards. So after you done visualizing, you think to yourself, what operation should I use? H: I think we're going to use subtraction because you want to…I think you need to subtract 1493 minus 42 to see how many yards he as so far. So as soon as you are done thinking about it, then you start doing the work. C: So I wrote 1,493 minus 42 and I got 1,541. Ch: That's my answer because it tells you how many yards does he have so I subtracted 1,493 the passing record out of 42 more yards that so much he needs to match it. He has 1451 so far. That's my answer.			
Prometacog: 83.3% **NonproMeta:16.7%**			

Participant #: 13	**MES: HM**	**PROBE TYPE: 2-STEP**	**SOLVED: Incorrect**
R: First you read the problem. Reads the problem. So when you are done reading it, you paraphrase it into your own words. P: So Marcy sold pictures that she had made for $6 each. The materials she used cost $12. The profit she had made was $42. How many pictures did she sell? After you done paraphrasing, you visualize. V: So you underline or highlight the important stuff. The $6 each, $12, $42, and how many. After you done visualizing, you see what type of operation you would use. H: So you might use…we might use division, then after you have a hypothesis you start solving. C: So now that we're done, I believe check our answers. Ch: So it said that she made …she made 6…she made pictures for $6 each and her profit was $42. How many pictures did she sell? So that told me is that you divide $42 because that is the profit and then you divide them by 6 that's how much she sold them each and then if you divide 42 and 6, you get 7 and that's how many pictures she made because if you multiply 6 and 7, you get 42. So I checked the answer and I got 6 pictures that she sold.			
Prometacog: 100.0% **NonproMeta:0.0%**			

Participant #: 13	**MES: HM**	**PROBE TYPE:3-STEP**	**SOLVED: Correct**
Reads the problem. P: So after I'm done reading you paraphrase it and put it in your own words. A family goes on a four-day trip, and they spent $25 on gas and they spent $35 on food a day. Their hotel bill was $200 in total. How much money was the whole trip? So after you done paraphrasing you underline the important stuff… V: so you underline 25 per day and 35 per day, $200 and how much. So after you're done visualizing it, you'll think about the operations you'll use. H: I think we'll use multiplication and addition. Then you'll start solving the problem. C: Silence while solving… Then after you're done solving the problem you go back and see what you've done. Ch: So I wrote $25 because that's what they spent every day for the 4 days they were on the trip times 4 and you'll get a hundred dollars. Then that's for gas. Then on food, it's $35 per day multiply by 4, you get $140 and then the hotel bill…it was $200 and you add $100 and $140 to the $200 and your total will be $440. Ok, so my answer is $440 is how much the trip cost and I checked and everything.			
Prometacog: 100.0% **NonproMeta:0.0%**			

Participant #: 5	MES: AM	PROBE TYPE: 1-STEP	SOLVED: CORRECT
R: Let's read the problem first. (Reads the problem). So it's basically asking us …ok…we're going to need to understand this problem though. (rereads the problem). P: So let's make this our own words. Tom has 42 more yards to match the school record …and the record is 1,493 yards…and we gonna try to figure out how much more yards Tom actually has at the moment. So let's get started. V: We gonna underline some important words like 42 more yards, 1,493 yards…and I think that's the words we'll underline. H: Let's hypothesize what the problem will do. I think we're going to do subtraction because we need to figure out how much Tom needs or has. C: He needs 42 more yards …so let's do 1.493 minus 42… and three minus two is one, 9 minus 4 is 5, and 4 come down and 1 brought down. So we get 1,451 and I think that's our answer but we got to see if we did the operations in the right order. Ch: Let's check…yay I'm pretty sure we did it right so the answer is…I think 1,451 yards he has at the moment.			
Prometacog: 100.0%			NonproMetaco: 0.0%

Participant #: 5	MES: AM	PROBE TYPE: 2-STEP	SOLVED: Correct
R: Read the problem for understanding it. (Reads the problem). P: So she's selling it for $6 each and her materials had cost $12. E: So that means he made more than $40. He made like $12 more if you think about it. So, we're going to underline, so we'll be paraphrasing it so it's $6 each Marcy made some pictures in. She had materials cost $12 and we want to know her profit and she made $42 that's probably not the answer. We're going to try to figure out how much …how many pictures did she sell. V: So let's see, we're visualizing it and we're going to underline $6 each, $12, and $42 profit. So let's do $42…we're doing addition here. C: We have to hypothesize and add $12 and that's $54. So 6 divided by 54 is 9. 6 divided by 54 is 9. And that's basically like 9 pictures in total because she had of profit. She could have made more money but she spent $12 so the total of pictures she sold was 9 pictures, I think. Ch: Let me get a look at this and I'm going to get to check to see if I'm right here. Ok, $6 each and she made a profit. So she sold … was $42…I think it's 9 pictures.			
Prometacog: 91.0%			NonproMeta: 9.0%

Participant #: 5	MES: AM	PROBE TYPE: 3-STEP	SOLVED: Correct
R: Reads the problem. Now I'm going to reread the question for more understanding. (rereads the question). P: Now I'm going to say it in my own words. So paraphrasing… so trying to make it easier. It's a 4-day trip. They spent $25 per day on gas and $35 per day on food...and the total hotel bill was $200. So we're trying to figure out how much the whole trip cost. V: Now I'm going to visualize it which is like …kind of like… try to like do some key stuff, key words or anything or something like that. So, I'm going to highlight some key words. $25 per day will be one of them. $35 per day will be one of them. 4-day trip will be one of them and $200 will be one of them. H: …and the 4-day trip was $25 per day so we are going to do $25 times 4. So for hypothesize, we are going to do times or addition. I guess I am going to do times. C: Alright, 5 times 4 equals 20 and 4 times 25 equals 100. 4 times 35, I got $140. So 140 plus 100…240 plus 200… and ok I did the operations and… Ch: I am checking this…yes…the answer should be $440 in how much the trip actually cost. My answer is $440.			
Prometacog: 75.0%			NonproMeta: 25.0%

Participant #: 3	MES: LM	PROBE TYPE: 1-STEP	SOLVED: Correct
R: Reads the problem. H/C: So I'm pretty sure I'm gonna subtract and do 1,493 minus 42. This gives me 1,451. Yep!			
Prometacog: 100.0%			NonproMeta: 0.0%

Participant #: 3	MES: LM	PROBE TYPE: 2-STEP	SOLVED: Incorrect
R: Reads the problem incomprehensibly. Struggles with sounding out words.			

H: Ok and so how many …I think I may have to add. C: 6 and no…12 plus 6…hmm…maybe it's 6 times 18. Oh my gosh, what is it? 36?… (yawns)…so…got it. 33…6..5…and 7.
Prometa: 0.0% **NonproMeta: 100.0%**

Participant #: 3	**MES: LM**	**PROBE TYPE: 3-STEP**	**SOLVED: Incorrect**
R: Reads the problem. Rereads. H: Ok, so I think I might have to multiply it. C: So…so…35 times 25 equals 85 times 200 equals…so the family cost the trip cost a total of $17,000 and I am pretty sure it's right.			
Prometacog: 50.0%			**NonproMeta: 50.0%**

Participant #: 12	**MES: LM**	**PROBE TYPE: 1-STEP**	**SOLVED: Correct**
R: Now I'm going to read the problem and if I don't understand it I will read it again. Reads the problem. P: So what I'm gonna do is paraphrase it in my own words now. So Tom needs 42 yards …42 more yards to beat …to match the school passing record of 1,493 yards in football. How many yards does Tom have so far? H: And now I'm going to hypothesize by planning how to solve my problem. So I'm going to divide 1493 by 42. C: Actually what I'm trying to do now is subtract 42 by 1,493. So Tom needs 1,451 yards . Tom has 1,451 yards right now.			
Prometacog: 83.3%			**NonproMeta:16.7%**

Participant #: 12	**MES: LM**	**PROBE TYPE: 2-STEP**	**SOLVED: Incorrect**
R: So what I'm gonna do is read the problem so that I can understand it. Reads the problem. P: So I'm gonna try to paraphrase it in my own words. So Marcy sold pictures she had…and she had made for $6 each. Her materials had costed her $12. She made $42. How many pictures did she sell? H/C: So what I did was…ehm…what I'm doing is I'm going to divide 42 by 6. And then I get 7. So her materials had costed her $12 and now what I'm going to do is multiply 7 by 12 and then I get $84 in total. So she sold 7 pictures and she had …she had made 7 pictures and she had…for $12 each so it costed her $84 and since she made $42 profit, she …you have to…you have to see how much …how much money she made by either…by trying to subtract your profit from the amount of money she spent. So she has $42. She didn't make money because she has $42 and her paintings costed $6 which if we divide 6 by …if we divide 42 by 6, we get 7 and if we multiply 12 by 7 we get 84 and then if we subtract 42 from 84 we get 42. So she should have a total of $42 and she sold 7 pictures.			
Prometa: 100.0%			**NonproMeta: 0.0%**

Participant #: 12	**MES: LM**	**PROBE TYPE: 3-STEP**	**SOLVED: Incorrect**
R: So first I'm going to read the problem. Reads the problem. P: Then I'm going to paraphrase it in my own words. My family goes on a four-day trip and we spend $25 per day on gas and $35 per day on food. My total hotel bill was $200. How much did I spend on the trip? H: Next I'm going to hypothesize which I'm going to multiply my problems so I'm going to do 25 times 35 (works silently), actually I'm going to multiply 25 and 35 by 4 because there's 4 days during the trip. And then I'm going to multiply 35 by 4 and now I have my total is 240 and then I'm going to divide the hotel bill by 4 to see how much it E: is…how much… it's going to be 50. And I think the answer is going to be somewhere in the $200 and $100. So the trip costed $420 in total which I estimated wrong. C/Ch:So I'm going to check my answer by dividing it by 25 and 35 (works silently). So when I divided 420 by 35, I got 12 and so I spent a total of $420 during the entire trip.			
Prometa: 66.7%			**NonproMeta:33.3%**

APPENDIX F

Aggregated Participants' Preintervention Scores on the Metacognitive Experience Survey

(MES) and on the Mathematical-Problem-Solving Probes (MPS)

Aggregated Participants' Preintervention Scores on the Metacognitive Experience Survey (MES) and Mathematical-Problem-Solving Probes (MPS)

Participants	MPS Probe 1-step	MES	MPS Probe 2-step	MES	MPS Probe 3-step	MES	Total MPS Probe 6pts	Total MES 60pts
#1	0	15	2	11	0	15	2	41-A
#2	1	13	0	13	0	19	1	45-A
#3	0	11	0	14	0	12	0	37-L
#4	1	18	0	20	0	17	1	55-H
#5	0	13	2	17	3	17	5	47-A
#6	1	17	0	20	3	19	4	56-H
#7	0	12	2	14	3	14	5	40-A
#8	0	15	2	14	3	15	5	44-A
#9	1	14	1	15	0	13	2	42-A
#10	1	18	0	20	0	11	1	49-A
#11	0	15	0	17	0	17	0	49-A
#12	1	14	0	12	1	12	2	38-L
#13	1	16	0	19	0	20	1	55-H
#14	1	17	0	13	0	11	1	41-A
#15	1	18	1	15	0	15	2	48-A
#16	0	14	0	16	0	12	0	42-A
#17	1	20	2	20	3	20	6	60-H
#18	0	20	2	20	1	20	3	60-H
#19	0	18	2	18	1	18	3	54-H
#20	1	18	1	12	3	12	5	42-A
#21	0	16	2	12	1	12	3	40-A
#22	1	20	2	18	0	14	3	52-H

Preintervention Phase: Participants' self-efficacy scores as measured by the Metacognitive Experience Survey (MES) and mathematical-problem-solving performance as measured by the different question types (1-step, 2-step, 3-step).

APPENDIX G

Aggregated Participants' Postintervention Scores on the Metacognitive Experience Survey

and Mathematical-Problem-Solving Probes

Aggregated Participants' Postintervention Scores on the Metacognitive Experience Survey (MES) and Mathematical-Problem-Solving Probes (MPS)

Participants	MPS Probe 1-step	MES	MPS Probe 2-step	MES	MPS Probe 3-step	MES	Total MPS Probe 6pts	Total MES 60pts
#1	1	17	0	11	3	15	4	43-A
#2	1	19	0	13	3	19	4	51-H
#3	1	11	0	15	0	11	1	37-L
#4	1	20	0	20	3	19	4	59-H
#5	1	20	2	17	3	17	6	54-H
#6	0	12	0	20	0	19	0	51-H
#7	1	15	0	15	3	14	4	44-A
#8	1	17	0	17	3	20	4	54-H
#9	1	14	0	15	3	16	4	45-A
#10	1	20	0	20	3	17	4	57-H
#11	1	15	2	18	3	15	6	48-A
#12	1	13	0	13	0	14	1	40-A
#13	1	18	0	18	3	18	4	54-H
#14	1	16	0	14	0	13	1	43-A
#15	1	14	0	13	0	12	1	39-L
#16	1	16	0	12	3	17	4	45-A
#17	1	20	2	20	3	20	6	60-H
#18	1	20	0	13	0	16	1	49-A
#19	1	20	0	20	3	20	4	60-A
#20	1	17	0	17	3	11	4	45-A
#21	1	16	0	14	2	11	3	41-A

#22	1	20	1	16	2	14	4	50-H

Postintervention Phase: Participants' self-efficacy scores as measured by the Metacognitive Experience Survey (MES), and mathematical-problem-solving performance as measured by the score on the different Question Types (1-step, 2-step, 3-step).

APPENDIX H

Change in the Pre- and Postintervention Scores on the Metacognitive Experience Survey

and Mathematical-Problem-Solving Probes

Change in the Pre- and Postintervention Scores on the Metacognitive Experience Survey and Mathematical-Problem-Solving Probes

	MES		Change	MPS		Change
Participants	Pretest	Posttest		Pretest	Posttest	
1	41	43	+2	2	4	+2
2	45	51	+6	1	4	+3
3	37	37	+0	0	1	+1
4	55	59	+4	1	4	+3
5	47	54	+7	5	6	+1
6	56	51	-5	4	0	- 4
7	40	44	+4	5	4	- 1
8	44	54	+10	5	4	- 1
9	42	45	+3	2	4	+2
10	49	57	+8	1	4	+3
11	49	48	+1	0	6	+6
12	38	40	+2	2	1	- 1

13	55	54	-1	1	4	+3
14	41	43	+2	1	1	+0
15	48	39	-9	2	1	- 1
16	42	45	+3	0	4	+4
17	60	60	+0	6	6	+0
18	60	49	-11	3	1	- 2
19	54	60	+6	3	4	+1
20	42	45	+3	5	4	- 1
21	40	41	+2	3	3	+0
22	52	50	-2	3	4	+1