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The relationship among working memory, mathematics anxiety, and mathematics achievement in developmental mathematics courses in community college

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THE RELATIONSHIP AMONG WORKING MEMORY, MATHEMATICS ANXIETY, AND MATHEMATICS ACHIEVEMENT IN DEVELOPMENTAL MATHEMATICS COURSES IN COMMUNITY COLLEGE

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INTRODUCTION

Statement of the Problem

Students in community colleges often have difficulties fulfilling the mathematics competencies required for an associate's degree or transfer to a 4-year institution (Brown & Niemi, 2007). Students, therefore, cannot be awarded an associate's degree or transfer to a 4-year college until the mathematics requirement is completed. California's 109 community colleges serve 2.5 million students annually, with 70% of those students placing in remedial mathematics courses (Research and Planning Group for California Community Colleges, 2005). Nationally, a review of 50,298 students attending community college revealed a referral of 36,246 for developmental mathematics courses for a cohort of students at 35 institutions (Clery, 2006). Figure 1 displays the percentage of students referred for developmental mathematics courses by ethnicity. Of

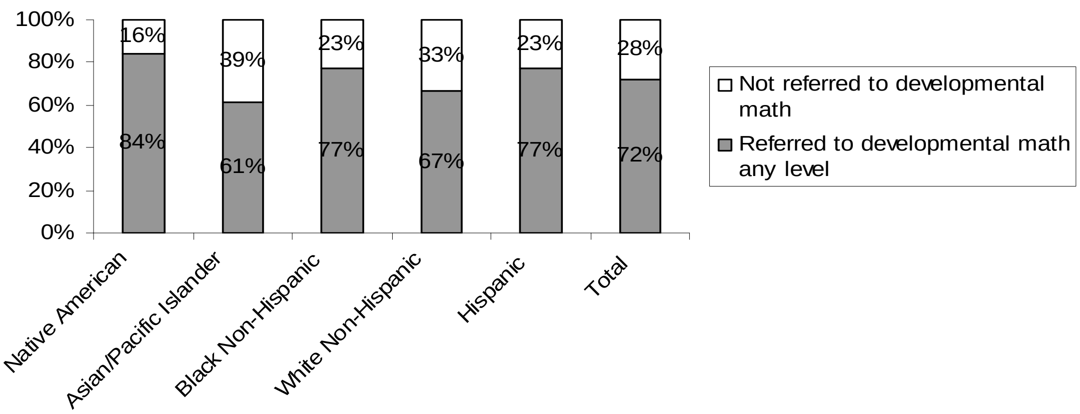

Figure 1. Students Referred to Developmental Mathematics Courses by Race/Ethnicity (Clery, 2006).

the students referred, only 18 % completed developmental mathematics within their first year of college. Many students chose not to enroll in suggested courses or delayed entry into mathematics courses. When this trend was analyzed by Clery, the following numbers were identified:

> 1. 36,246 students were referred to developmental math from the 2002 cohort across 35 institutions.
> 2. Of those students, 6,353 completed developmental math in their first academic year.
> 3. Of those students, 4,077 attempted college-level math by the end of the third academic year.
> 4. Of those students, 3,041 completed college-level math by the end of the third academic year. (Clery, 2006, p.3)

Students who begin the basic skills mathematics sequence at the arithmetic level have a 10% probability of attempting transfer level mathematics courses in California (Research and Planning Group for California Community Colleges, 2005, p. 6).

Is this because a number of these students have mathematics disabilities? Accurate numbers of students in community colleges with disabilities are not known because postsecondary services for students with disabilities require self-reporting (Henderson, 2001). This means that the current seven to nine percent estimate of students with disabilities attending postsecondary institutions may underrepresent the community-college population. In 1999-2000, about nine percent of college students reported having a disability. Of that nine percent of students attending college, about 50% of the students reporting a disability attended a 2-year community college compared with 26% attending 4-year colleges (NCES, 2003).

Working memory deficits consistently are found in children and adults with mathematics learning disabilities (Ashcraft & Kirk, 2001; Geary, Hamson, & Hoard, 2000; McGlaughlin, Knoop. & Holliday, 2005; Swanson & Beebe-Frankenberger, 2004). Working memory is defined as the capacity involved in the preservation of information while simultaneously processing the same or other information (Swanson, 2000).

Students often are frustrated by their inability to hold recently learned information in working memory and recall information from long-term memory when needed to solve a newly presented problem. As a result, students may not achieve a mathematics level that would be expected given their ability level.

One component of a mathematics disability is the discrepancy between potential, measured by an abilities test, and achievement, measured by a standardized achievement test or performance-based assessment (Swanson & Jerman, 2006). Students diagnosed with a specific learning disability exhibit this discrepancy in one or more areas including reading, written language, or mathematics. An indicator of learning disabilities is a processing deficit evidenced by discrepancies between two ability measures. For example, the Wechsler Adult Intelligence Scale (WAISIII) has specific ability measures including working memory, processing speed, and verbal comprehension. When the scores on the working memory measure are substantially different than the verbal comprehension measure, a working memory deficit is identified. The working memory discrepancy is one processing deficit found in research to be associated with children with mathematics disabilities (Geary, Hamson, & Hoard, 2000).

Students with learning disabilities who seek assistance at the postsecondary level often have a processing deficit related to working memory manifested in difficulties in successfully completing mathematics courses. This study, although not designed to diagnose students with learning disabilities, investigated the relationship between working memory and mathematics achievement for students who are placed at the basic-skills level in community college and have difficulties with calculation and problem solving. Students with a processing deficit related to working memory may benefit from strategies developed for students with learning disabilities in order to be more successful in basic mathematics courses.

Purpose of the Study

The primary purpose of this study was to examine the relationship of working memory and mathematics achievement for students in developmental mathematics courses in community college. In addition, one variable that may influence accessibility to working memory and mathematics achievement was considered. That factor, mathematics anxiety, has been shown in previous research to negatively affect working memory in college students (Ashcraft & Kirk, 2001). In this study, mathematics achievement was measured by a standardized assessment, the Woodcock-Johnson Tests of Achievement-Revised (WJ-R) using the Calculation and Applied Problems subtests. Working Memory was assessed using the subtests Arithmetic, Digit Span, and Letter-Number Sequencing that produce the Working Memory Index of the Wechsler Adult Intelligence Scale III (WAISIII). The three subtests measure the verbal working memory component of Baddeley’s (1992) theory including executive processing and phonological loop because all subtests are orally administered and require auditory attention and manipulation of information.

Students in community college form a diverse group regarding age, gender, English Proficiency, Learning Disability status, and ethnicity. A demographic questionnaire was given to the students regarding these factors, with the answers used to describe the sample in this study.

Theoretical Rationale

Two theories provide the rationale for this study. The first is the Working Memory theory proposed by Baddeley (1986, 1992) and second the Cognitive Load Theory introduced by Sweller (1988). In the first theory, Baddeley defined working memory as the collection of structures and processes within the brain used temporarily to store and manipulate information. Working memory consists of both memory for items that currently are being processed and components governing attention and directing the process itself. In other words, to process information, working memory uses both newly required information, as well as long-term memory to process information.

Originally, Baddeley (1986) proposed that this working-memory system consisted of three separate components that interacted to allow processing of information. These components were the central executive that was essentially the “director” of the process responsible for attention and control, the phonological loop that stores and rehearses speech-based information, and the visuospatial sketchpad that manipulates and stores visual information. Because the phonological loop and visuospatial sketchpad are storage components, Baddeley referred to them as “slave” systems controlled by the central executive. In a later revision of his theory, Baddeley (2000) added the concept of an episodic buffer that temporarily stores and retrieves features of short-term memory. The episodic buffer has a limited capacity for chunks of information that can be held simultaneously and requires that episodes be bound together for the most efficient retrieval of information (Allen, Baddeley, & Hitch, 2006).

The second theory relevant to this study is Cognitive Load Theory (CLT) introduced by Sweller (1988). Sweller proposed that there are three types of “load” that affect working memory: extraneous load, intrinsic load, and germane load. Extraneous load often encompasses the way tasks are presented. (van Merrienboer & Ayres, 2005). Extraneous load may be influenced by distractions or using resources to search for information necessary to complete a learning task within instructional materials**.** Intrinsic load is the "mental work imposed by the complexity of the content" (Clark, Nguyen, & Sweller, 2006, p. 9). Element interactivity is a key concept of intrinsic load that is determined by the extent to which individual elements can be learned in isolation rather than interacting with other elements. If element interactivity is high, there is a high working memory demand because many elements must be considered simultaneously (Pawley, Ayres, Cooper, & Sweller, 2005). Intrinsic load is determined by the level of expertise of the learner combined with the type of materials that need to be learned (van Merrienboer & Ayres, 2005). Intrinsic load requires a learner to develop schemas to comprehend interactive elements of materials that are difficult to understand. Germane load is the amount of resources a person willingly allocates to tasks that require the development of schemas and automaticity. Genuine learning is promoted because a person invests effort in schema construction. The key to efficient use of working memory in this theory requires that the learner develop schemas that are processed as single units rather than small bits of unorganized information. Because the effects of intrinsic, extraneous, and germane load are additive, it is necessary to lower extraneous load if intrinsic load is high. According to van Merrienboer and Ayres, "the main instructional principle of CLT is to decrease extraneous cognitive load and to increase germane cognitive load, within the limits of totally available processing capacity (i.e., prevent cognitive overload)" (p. 8). Because the learner's level of expertise with a complex task contributes to intrinsic cognitive load, this additive effect must be taken into account when designing instruction or determining the perspective cognitive overload of a task (Clark, et al., 2006). Overloading working memory inhibits learning, according to cognitive load theorists, and "instructional procedures are most effective when unnecessary cognitive load is kept to a minimum" (Clarke, Ayres & Sweller, 2005, p. 16). The additive effects of extraneous and intrinsic load may prohibit germane load needed to learn the task well. In other words "cognitive load depends on the interaction among the expertise of the learner, the complexity of the content, and the instructional methods used in the training environment" (Clark et al., 2006, p. 23).

In their review of cognitive load theory, van Merrienboer and Ayres (2005) proposed that instruction for learners acquiring novel information should be changing continually to reflect the learner's level of expertise. According to van Merrienboer and Ayres, "novel information must be processed in working memory in order to construct the schemas after which they may be automated if they are repeatedly and successfully applied" (p. 11). The implication for instruction should be to move forward only when correct independent answers are given. As a result, cognitive overload is avoided, and opportunities to fully develop schemas that will assist in problem solving are promoted. This aspect of the theory is relevant to the current study because it is hypothesized that the participants in the study are not able to develop schemas in mathematics because of factors that contribute to extraneous load as well as previous mathematics courses that did not manage intrinsic load effectively.

Individual student factors, therefore, may contribute to cognitive load that in turn affects working memory. If a person has not used working memory efficiently, not as much information will be stored in long-term memory (Clark, et al., 2005). For example, one student factor is mathematics anxiety. Ashcraft and Kirk’s study (2001) suggests that mathematics anxiety negatively influences online working memory. Ashcraft and Kirk (2001) proposed that intrusive thoughts and worry regarding mathematics are located in the central executive component of working memory. “Transient working memory disruption” is the result of the negative effects of mathematics anxiety (Ashcraft & Kirk, 2001, p. 236). This disruption may prevent knowledge from reaching long-term memory to be stored for later use.

Another factor, second-language learning has shown a similar effect on mathematics achievement and influence on working memory availability. Working memory and calculation were found to be statistically significantly different for a person’s first language than the second in bilingual adults. (Ardila et al., 2000). The differences between first and second languages used in word problem solving were found to affect reasoning with the participants’ first language producing increased reasoning skills (Ardila et al., 2000; Bernardo & Calleja, 2005). Brown (2005) conducted a study of students’ performance on high-stakes tests of literacy-based performance assessments (LBPA) in mathematics. The findings of the examination of 984 students’ scores suggest that students who are English-language learners, regardless of socioeconomic (SES) status are at a disadvantage for the performance assessments because of lack of background knowledge as well as academic language.

Other student factors such as ethnicity, socioeconomic status, and learning disability status may contribute to extraneous load. These factors, although not modifiable in the learning environment, may help to explain interference with mathematics achievement. Number of mathematics courses may be a component of the student's level of expertise and opportunity to learn, that are recognized as a facet of intrinsic load.

Learners form new schemas in working memory by integrating incoming information with prior knowledge that also will require resources for maintaining information in long-term memory (Clark, Nguyen, & Sweller, 2006). As more schemas are stored in long-term memory that has a large storage capacity, there is less risk of cognitive load interfering with working memory. Minimizing extraneous load is important when learning tasks are complex. Mathematics is complex for some students in developmental classes, and most students would be considered novice learners who are overwhelmed by tasks that require integration of information in working memory.

Background and Need

Competition in industry and education has necessitated stricter standards and competencies for students in the area of mathematics. Students in high school are required to pass exit examinations in a number of states in order to graduate, receive a diploma, or both. The California Department of Education (2007) defined the mathematics competencies tested on the California High School Exit Examination (CAHSEE) as

> The mathematics part of the CAHSEE addresses state mathematics content standards through the first part of Algebra. It includes statistics, data analysis and probability, number sense, measurement and geometry, algebra and functions, mathematical reasoning, and Algebra I. Students must demonstrate computational skills and a foundation in arithmetic, including working with decimals, fractions, and percentages. The math part of the exam is composed entirely of multiple-choice questions (California Department of Education, 2007, Mathematics Section).

In a review of the standards assessed on the exit examinations across 26 states, the Center on Education Policy (Appendix A) identified grade-area alignment for competencies in mathematics ranging from 6th grade (California) to 11th grade (Minnesota, Georgia, and New Jersey). A variety of other grade levels were given for the remainder of the states, with many of the states (including California) also testing students at the Algebra I level. Students are often surprised to learn that, although they successfully completed mathematics courses in high school equal to Algebra II, they are placed at the pre-Algebra level in college. Nationally 35% to 40% of first-year students enter college remediation courses (Bettinger & Long, 2006). Over 70% of students entering California community colleges are placed in remedial-level mathematics courses following the recommendation of placement tests (Research and Planning Group for California Community Colleges, 2005). In this study, verbal working memory using tasks that require auditory attention, memory and sequencing in two different levels of developmental mathematics courses were examined. Further, mathematics achievement using both calculation and problem solving tasks were measured to view the relationship between working memory and achievement for the two groups.

Students attending postsecondary institutions, particularly community colleges, often do not receive a degree because they cannot pass the required mathematics courses (Research and Planning Group for California Community Colleges, 2005). Results of recommended placement tests for students entering community college often suggest developmental-level courses, that is, remedial courses designed to prepare students for college-level instruction. For students enrolled at this developmental level, the number of mathematics courses required may be two to three times than those students placed higher in the mathematics sequence (college level). For example, a student who placed at the developmental level at a California community college would need to complete a series that includes Basic Arithmetic, Pre-Algebra, Elementary Algebra, and Intermediate Algebra before attempting the first class that may be transferred to a 4-year college, Elementary Statistics. Some majors require mathematics beyond the statistics level. Students who are seeking an Associate of Arts degree are awarded that degree only if they successfully complete Intermediate Algebra. “At the national level, 44% of those who started in community colleges in the 1990s did not reach Algebra 2 in high school, compared to 11% of those who entered four-year colleges. In addition, 55% of these students must take two or more remedial courses and 72% of those who take two or more remedial courses earn no credential whatsoever” (Research and Planning Group for California Community Colleges, 2005, p.5). If a student begins at the developmental level, regardless of the presence of an identified disability, there is only a 10% success rate for students to complete the series and enter the first transferable course, elementary statistics (Brown & Niemi, 2007).

Student Factors

Students who experience failure in mathematics classes often have a working memory deficit. A working-memory deficit prevents students from integrating new knowledge from college courses into their existing mathematics skills that have been developed inconsistently throughout their kindergarten to high school (K-12) experience. Research in the general area of working memory suggests that individual factors influence accessibility to working memory, including prior knowledge (Clark et al., 2006). Because working memory remains relatively stable beyond the adolescent years, other factors combine in some students to interfere with their ability to use working memory efficiently for mathematics' tasks. Wilson and Swanson (2001) were interested in asking two specific research questions in a study designed to investigate age-related factors in mathematics disabilities. The first was whether the relationship between mathematics disabilities and working memory weakened with age. The researchers wanted to investigate if those deficits were domain specific (for example phonological processing, central executive) or were related to a domain general system.

The participants included 98 children and adults with a mean chronological age of 19.95, range of 11.10 to 52.00. The sample represented a variety of ethnic groups including European American, African American, and Hispanic American. Before reporting results, Wilson and Swanson (2001) divided the participants into 3 age categories: age range 11 to14, 14 to 19, and 21 to 52, The researchers used this quadratic age variable to interpret their findings in order to discover if there was a relationship between mathematics and working memory as a function of age. Wilson and Swanson found no statistically significant correlations between the factors as a function of age or ability group. Results also suggested that there was not a statistically significant pattern of the magnitude of correlations between mathematics and working memory that are stronger in younger children than adults. The last result reported that raw scores for mathematics were statistically significantly related to the verbal working-memory composite score for the group of subjects in the mathematics disability group but not for the nonmathematics disability group.

Wilson and Swanson (2001) used a regression analysis to investigate the unique variance in computational ability to address two questions. The first question investigated whether working memory moderated age-related changes in computation ability. The second question was used to examine whether verbal and visual-spatial working memory contributed independent variance to mathematics performance. Wilson and Swanson looked at four issues to determine the amount of variance: quadratic and linear age-related variables alone; age-related performance after working memory was partialed out, age-related contrast after working memory and reading were partialed out, and age-related contrast after working memory, reading, and gender were partialed out (p. 13).

A general result of this analysis was that, regardless of presentation order, verbal working memory (22%), visual spatial working memory (14%), reading ability (4%), and gender (3%) contributed the variation in mathematics achievement when entered in the above order in the analysis. Wilson and Swanson (2001) concluded that the regression analysis resulted in an independent contribution of reading ability, gender, verbal working memory, and visual-spatial working memory to the participants' mathematics computation skills. They interpreted that verbal and visual-spatial working memory contributed statistically significant variance irrespective of their order of entry in the regression analysis.

Wilson and Swanson (2001) cited two findings in this study and their interpretation of results. The first was that there is a relationship between working memory and mathematics ability but that it is no higher in children than adults. The second finding was that groups without mathematics' disabilities scored higher on factor scores that included variance in domain-general, verbal working memory, and visual-spatial working memory. The researchers also concluded that both verbal and visual-spatial working memory scores predicted mathematics performance and that these results support other studies that suggest that the working-memory system plays a critical role in mathematics' development.

The results of Wilson and Swanson's (2001) study provided evidence for working memory remaining relatively stable throughout the lifespan after adolescence. Individual factors in the community-college setting, however, such as second language learning, socioeconomic status, mathematics anxiety, and learning disabilities may influence accessibility to working memory. Community-college developmental mathematics courses have students that are representative of all of these factors.

Students in the lowest developmental mathematics courses are overrepresented by African American and Hispanic American students who also have the lowest success rates in basic skills mathematics courses (Brown & Niemi, 2007). Hispanic American students in elementary algebra courses at California community colleges had a success rate of 46.9% and African American students a success rate of 40.2 % (Research and Planning Group for California Community Colleges, 2005). Opportunity to learn in early-elementary grades often results in lower enrollment in secondary programs with students consequently starting college less prepared than non-Hispanic White peers (Lindholm-Leary & Barsato, 2005). In addition, there are a number of students with identified learning disabilities who may or may not receive services through the Disability Resource Center. Students with learning disabilities make up 40 % of students self-reporting disabilities in postsecondary institutions with an estimated seven to nine percent of the freshman population reporting disabilities (Henderson, 2001). Estimates of students with learning disabilities are difficult to obtain because, unlike Kindergarten to 12th grade services, students are required to self-report a learning disability in community college if they want to receive disability services. Some of the students in the developmental courses previously have received disability services in high school, but have chosen not to self-disclose their disability for eligibility for college services (Hartman-Hall & Haaga, 2002). Access to accommodations such as increased testing times, tutoring, and distraction-reduced settings are not offered to students generally in a mathematics course. Students who had received these accommodations in high school would qualify for those services only if their learning disability was disclosed in college.

Problems to be Studied

Working-memory deficits appear in research as a factor in mathematics difficulties. Research has supported the role of working memory in mathematics achievement particularly in children (Fuchs et al., 2005; Swanson, 2006). A relationship between working memory and mathematics achievement has also been shown in limited studies with students in postsecondary settings (Ashcraft & Kirk, 2001; McGlaughlin, Knoop, & Holliday, 2005). The need for research regarding working memory and adults with learning disabilities has increased as the population of students with learning disabilities and ethnic and socioeconomic diversity has increased in postsecondary institutions (Henderson, 2001).

Student factors such as English as a Second Language (ESL), ethnicity, mathematics anxiety, qualification for learning disability services, and socioeconomic status may contribute to cognitive load that in turn has an effect on the accessibility of working memory resources. The purpose of this study was to view the relationship between working memory and mathematics achievement in developmental mathematics courses. Factors that influence this relationship may provide additional information needed for intervention strategies to increase success rates of students at the developmental level in mathematics.

Instructional Factors

Instructional factors are reviewed in order to help explain a portion of the difficulties students have in mathematics. Although standards in mathematics have increased in complexity and requirements, little has changed in methodology for teaching mathematics (Golfin, Jordan, Hull, & Ruffin, 2005). In some classrooms, instructors of mathematics deliver instruction in lecture format with little emphasis on mastery of a concept before moving on to another concept. Summative assessments, rather than formative, do not encourage students to revise their thinking or provide opportunity for higher order thinking (Golfin et al., 2005). Number sense and automaticity of basic facts (addition, subtraction, multiplication, and division) generally are not taught after elementary school. These retrieval and storage skills are necessary for tasks in mathematics to reduce intrinsic load, which in turn allows students to use working memory for increasingly complex tasks. Students often “learn” the material but are later unable to retrieve that information efficiently in an assessment situation. Factors such as incomplete concept development, application to word problems, and anxiety in a testing situation (Ashcraft & Kirk, 2001) are contributors to failing test grades. In some instances, professors and teachers will move their instruction to the next concept after an examination, leaving students with incomplete skill sets.

Postsecondary students in basic skills mathematics courses have a quarter or semester to develop mathematics skills that traditionally are taught over a series of years in elementary or middle schools. One of the reasons that this skill development is extremely difficult for students is incomplete automaticity of number facts and poor number sense (Gersten & Chard, 1999). Students entering college without developing this skill are required to learn their facts in the first 2 weeks of the quarter when basic arithmetic operations are reviewed. For students who have working-memory deficits, this task may be impossible because of the need to hold the information in working memory and rehearse the answers for the number of times needed to transfer facts to long-term memory. Students who previously had been unsuccessful at this task would need a much longer time and an alternative strategy to accomplish memorizing facts.

Some instructors have implemented changes in traditional mathematics instruction that include a variety of student-centered practices. In a literature review conducted by Golfin et al. (2005), the researchers examined a number of studies related to developmental mathematics in postsecondary settings. Their findings regarding instruction supported greater use of technology, integration of classroom and laboratory instruction, use of multiple approaches to problem solving, and use of project-based instruction. Further suggestions included allowing students to select among different instructional methods, having low student-to-faculty ratios, and integrating counseling into the courses (Golfin et al., p. 3). From this review, it would appear important to provide a variety of instructional options for students to access strategies that may improve performance. Further study of these options regarding individual student factors could then be conducted to identify a menu of strategies to improve developmental mathematics success rates.

Researchers have found that knowledge of strategies has increased access to working memory (Keeler & Swanson, 2001). In their study, Keeler and Swanson found that a stable strategy choice of rehearsal, chunking, association, or elaboration influenced working memory, although one specific strategy was not more effective than another**.** In other words, choosing any strategy and using it consistently produced results that improved working memory. Ayres (2006) investigated whether reducing mathematics task complexity would provide a strategy that would reduce intrinsic cognitive load. He used three strategies: one that had elements isolated from each other (part-task), one where all elements were fully integrated, and a third group that had a mixed strategy from part to whole tasks. In two experiments designed to measure error rates and cognitive load, the isolated strategy had statistically significant less error rates than the other strategies (Cohen's d=.80) and less cognitive load (d=.70) that are considered medium to high effect sizes. Allsopp, Minskoff, and Bolt (2005) used specific and individualized strategy instruction with students in 4-year and community colleges in an attempt to improve student grades and retention in classes. This limited study used personal strategy training delivered on an individual basis, as well as individual choice of strategy. The researchers used grade point average (GPA) to investigate the statistical significance of the strategy instruction. Students who were in the study for one semester improved their GPA from 2.02 to 2.23 with a medium effect size (Cohen's d) of .55. Students in the study for two semesters improved their GPA from 2.91 to 2.37, a large effect size of .95. Although this was a very limited study (46 participants), there is some evidence that individual strategy training may increase academic performance.

Working-Memory Factors in Mathematics

Although most studies have found that working memory remains stable after adolescence, factors that influence efficient use of working memory continue to be a contributor to success in mathematics classes in postsecondary settings. The executive processing component of working memory was found to have an important role in mathematics calculation and word problem-solving skills (Ashcraft & Kirk, 2001; Swanson, 2006; Swanson & Beebe-Frankenberger, 2004). Mathematics anxiety has been found to disrupt working memory when increasingly complex tasks are presented or in situations that require dual processes (Ashcraft & Kirk, 2001). Therefore, working memory, although relatively stable, can be affected by both strategy knowledge and anxiety. Verbal working memory in particular with its reliance on the phonological loop and executive processes has been shown to contribute to word problem-solving skills (Swanson & Beebe-Frankenberger, 2004).

Swanson and Beebe-Frankenberger (2004) assessed problem solving in first-, second- and third-grade children. In the introduction to the study, Swanson and Beebe-Frankenberger identified components of memory needed to solve word problems. The components included the ability to access information accurately from long-term memory to solve a word problem that is introduced into working memory. At that point, the contents of working memory are compared with other information in long-term memory to find a match for a possible action. Then working memory is updated to find a solution to the word problem.

The study examined the relationship between working memory and problem solving for 351 first-, second-, and third-grade children in a Southern California public and private school district. A battery of tests was administered to the children including assessments and tasks identified as classification, criterion, and predictor variables. Classification measures were fluid intelligence, mental computation of word problems, and digit-naming speed. Criterion variables included word problem solving and arithmetic calculation. Predictor variables were reading and phonological processing, short-term memory, working memory, inhibition and updating measures, and semantic processing and vocabulary. Children in this study were defined as at-risk for serious mathematics disabilities (SMD) if they had normal intelligence (IQ standard score >85) but performance below the 25th percentile on problem solving and digit-naming fluency.

Three research questions were investigated. The first question was whether working memory predicted problem solving. The second question was if there was a relationship between problem solving and working memory as a function of age. The third question concerned the cognitive processes that mediate the relationship between working memory and problem solving.

The results showed a statistically significant relationship between working memory and problem solving (r=.54). When the influence of phonological processing, inhibition, speed, mathematics calculation, and reading skill was partialed from the analysis, there continued to be a statistically significant correlation (r=.31**).** Working memory also was found to be as an important part of problem-solving as phonological processing and short-term memory development.

In a study of postsecondary students, McGlaughlin, Knoop, and Holliday (2005) used measures similar to those proposed in the current study (Wechsler Adult Intelligence Scale, Woodcock-Johnson Tests of Achievement) to measure working memory and mathematics achievement for 76 students with and without mathematics disability. Results of their study revealed that college students with mathematics disabilities demonstrated weaknesses in reading comprehension, nonverbal reasoning, working memory, and mathematics fluency when compared with students without mathematics disabilities. Results of analysis of variance (ANOVA) for working memory (effect size, indicated by partial eta squared= .18) and mathematics fluency (effect size, indicated by partial eta squared= .31) suggested these two factors revealed the greatest weakness for students with mathematics disabilities. Attention, a factor that has been associated with studies of elementary students, was not found to be a statistically significant factor.

Significance of the Problem

The current study may extend the results of previous studies regarding the relationship between working memory and mathematics achievement to include community-college students in developmental mathematics courses. This study included students enrolled in two developmental mathematics course levels who were advised to take the courses after completing a college placement examination. The study was based on the need to find factors that influence mathematics achievement at the community-college level in order to design and implement instructional and study strategies that will increase success rates. Success rates for students in developmental mathematics courses are extremely low. Along with additional funding allocated for assessment of current instructional practices at the community-college level (Center for Student Success, 2007), information regarding individual student factors may offer a comprehensive package for intervention, allowing and encouraging more students to complete mathematics sequences successfully and obtain an associate of arts degree or transfer to a 4-year college.

Research Questions

The following research questions were addressed in this study:

1. What are the characteristics of students enrolled in two levels of community-college developmental mathematics courses?
2. Is there a difference between working memory measures in students in two levels of community-college developmental mathematics courses?
3. Is there a difference between mathematics anxiety in students in two levels of community-college developmental mathematics courses?
4. What is the relationship between mathematics achievement and working memory in students in community-college developmental mathematics courses?
5. Is the relationship between working memory and Applied Problems stronger than the relationship between working memory and Calculation for students in community-college developmental mathematics courses?
6. What is the relationship between mathematics achievement and mathematics anxiety in students in community-college developmental mathematics courses?
7. What is the relationship between mathematics anxiety and working memory in students in community-college developmental mathematics courses?

Definition of Key Terms

The following terms are defined in relationship to their use in this study. There may be other definitions commonly associated with these terms.

Applied Problems: The Applied Problems subtest of the Woodcock-Johnson Tests of Achievement (Revised), also known as word problems "measures the subject's skill in analyzing and solving practical problems in mathematics" (Woodcock & Mather, 1989, p. 14).

Automaticity: Skills that are coded into long-term memory and can be exercised with little or no resources from working memory are characterized as reaching a level of automaticity (Gersten & Chard, 2001).

CAHSEE: California High School Exit Examination is a competency-based examination administered in secondary schools beginning at grade 10 (California Department of Education, 2007).

Calculation: Skills that are measured by a subtest of the Woodcock-Johnson Tests of Achievement-Revised include addition, subtraction, multiplication and division of whole numbers, fractions and decimals (Woodcock & Mather, 1989).

Central executive: An attentional-controlling system in working memory that directs available resources to problem solving in mathematics (Baddeley, 1986).

Cognitive Load Theory: A universal set of instructional principles and evidence-based guidelines that offer the most efficient methods to design and deliver instructional environments in ways that best utilize the limited capacity of working memory is the basis of cognitive load theory (Clark et al., 2006, p. 342).

Developmental level: Classes at a postsecondary institution that emphasize basic skills in English and mathematics necessary for successful completion of college-level courses are considered at the developmental level (Boswell & Wilson, 2004).

Episodic buffer: This aspect of working memory is a temporary storage system that holds episodes that are integrated across time and controlled by the central executive (Baddeley, 2000).

Extraneous Load: This work imposed on working memory uses mental capacity but does not contribute to learning (Clark et al., 2006, p. 346).

Germane Load: The work imposed on working memory uses mental capacity in ways that contribute to learning (Clark et al., 2006, p. 346).

Intrinsic Load: The work imposed on working memory is a result of the amount of element interactivity of the content to be learned (Clark et al., 2006, p. 347).

Long-term memory: The permanent storage of knowledge and skills is formed on the basis of schemas. Regarding mathematics concepts long-term memory stores the knowledge of specific mathematical relations and general problem-solving strategies (Swanson, 2006).

Mathematics Anxiety: This is an emotional response to mathematics tasks that may interfere with a person's attention and memory and lead to avoidance of mathematics (Ashcraft & Kirk, 2001). In this study mathematics anxiety was measured by the Mathematics Anxiety Rating Scale-Abbreviated (Alexander and Martray , 1989).

Number sense: This term refers to a person's knowledge, fluidity, and flexibility with numbers, the sense of what numbers mean and an ability to perform mental mathematics (Gersten & Chard, 1999).

Phonological loop: Speech-based information is stored and rehearsed in this area of working memory necessary for reading and solving mathematical word problems (Baddeley, 1992).

Schema: A schema is a cognitive construct that allows a person to organize multiple elements of information into one single element to bring into working memory from long-term memory (Pawley, Ayres, Cooper, & Sweller, 2005).

Visuospatial sketchpad: In working memory, this is the storage area in which visual information is manipulated (Baddeley, 1986).

Working memory: This storage and processing system is composed of executive processing, phonological loop, visuospatial sketchpad, and episodic buffer (Baddeley, 2000).

Summary

This study examined working memory and mathematics achievement for students enrolled in developmental mathematics courses at community colleges. Mathematics anxiety has been shown in previous research (Ashcraft & Kirk, 2001) to disrupt efficient use of working memory in college students and therefore a measure was included to view its relationship to mathematics achievement. Gender, age, ethnicity, socioeconomic status, mathematics courses completed in high school, English language proficiency, and eligibility for learning disabilities services are further factors believed to influence mathematics achievement. The questionnaire in this study helped to describe the sample of students in developmental mathematics classes regarding those factors. Efficient use of working memory is necessary for mathematics tasks and progression in a series of courses required for demonstration of mathematics competency. Success rates in developmental courses in community college that lead to completion of a degree or transfer to a 4-year college are less than ten percent (Brown & Niemi, 2007). This study attempts to show the relationship between working memory and mathematics achievement in developmental courses in community college. The educational significance may be subsequent design of strategies for increased success in developmental mathematics. These strategies may include ways to decrease mathematics anxiety, reduce cognitive load, increase accessibility of working memory, and study techniques for more efficient use of working memory.

CHAPTER II

REVIEW OF THE LITERATURE

It is important to consider a wide range of factors when evaluating working memory and its effect on mathematics related to successful course completion at the community-college level. The purpose of this study was to explore the relationship between working memory and mathematics achievement for students in community colleges. Students who attend community colleges form a diverse group regarding age, ethnicity, disabilities, and second-language status. The focus of the literature review concentrated on five areas that are relevant for the research questions and variables that will be examined. The first section explored the components of working memory including changes in the construct proposed by Baddeley (1986). The second section provided a review of studies of working memory and mathematics at a variety of ages. In the third section, studies specifically highlighting working memory and college-age students are reviewed. The fourth section contains studies of working memory and students with learning disabilities. In the fifth section, studies of the effects of second-language learning on working memory are presented.

Components of Working Memory

Alan Baddeley (1986) in his book *Working Memory*, reviewed 30 years of research regarding memory, including short-term memory and the new construct working memory. His model of working memory involved three systems. The first was the phonological loop that is the speech system required to repeat auditory information. The next is the visuospatial sketchpad for information that cannot be verbalized and is the storage component for both visual and spatial features. The third is the central executive that is responsible for attentional control, problem solving, and cognitive processing. Baddeley (2000) extended his working memory theory to include an additional component, the episodic buffer, that is a system that integrates temporary memories (episodes) with other components of working memory.

Mathematics achievement requires efficiency in the four systems of working memory. A number of studies have been conducted regarding working memory including the more specific studies that have examined the effects of poor working memory on mathematics. Most research has involved elementary-aged children (Geary, Hoard, and Hamsom, 1999, Keeler & Swanson 2001) with some research for college-level students and adults (McGlaughlin, Knoop, & Holliday, 2005)

The information processing model is the theory relevant to working memory. In the information processing model, memory has different levels. Information enters through the senses and is filtered through the sensory register. Some information is lost at this stage because a person is not able to pay attention to or encode all sensory input. Retained information is stored briefly in short-term memory, but again a portion is lost because of attention or relevance to the task at hand. With increased practice or rehearsal, the information is stored in long-term memory. When new information enters the system that requires some level of manipulation, working memory retrieves stored knowledge from long-term memory and uses that knowledge to integrate the new short-term memory to perform some action.

In mathematics, it is necessary to have working-memory skills that work effectively to integrate new information with that stored in long-term memory. For example, when learning the algorithm for long division, a student must hold the algorithm in working memory and retrieve learned multiplication and subtraction facts from long-term memory to solve the problem correctly. Students who have difficulty with working memory have problems with retrieval from long-term memory or inability to hold enough information in short-term memory selectively to attend to relevant information.

Baddely (1992) characterized the phonological loop and visuospatial sketchpad as “slave” systems within working memory. The phonological loop itself has two systems, a storage component and an articulatory control that allows a person to maintain information by subvocalization. The visuospatial sketchpad also has a dichotomous role of imaging visual features and visual memory for spatial tasks. The central executive coordinates the information from these slave systems. In complex tasks, especially those in mathematics, the central executive would be responsible for maintaining attention to the visual aspects of problem solving: numbers, symbols and so on, while coordinating the phonological components of comprehension of verbal or written problems, as well as directing the retrieval of long-term memory components.

A study conducted by Gathercole, Pickering, Ambridge, and Wearing (2004) investigated the development of working memory in children from 4 to 15 years of age. The sample in this study of 700 children in Southwest England consisted of 43 four-year olds, 101 five-year olds, 91 six-year olds, 96 seven-year olds, 63 eight-year olds, 98 nine-year olds, 101 ten-year olds, 37 eleven-year olds, 45 thirteen-year olds, 14 fourteen-year olds, and 47 fifteen-year olds chosen on the basis of parental permission with no exclusionary criteria. No other demographic information was given for these students other than ages. The researchers administered the Visual Patterns Test and eight subtests of the Working Memory Test Battery for Children: digit recall, word list recall, and nonword list recall for phonological loop; block recall, Visual Patterns Test, and mazes memory for visuospatial sketchpad; and backward digit recall, listening recall, and counting recall for complex memory span or central executive.

To analyze the results, the researchers first eliminated 18 children with scores more than three standard deviations from the mean in their age group (4 to 5 years, 6 to 7 years, 8 to 9 years, 10 to 11 years, and 13 to 15 years). Multivariate analyses of variance (MANOVAs) were performed on each set of measures as a function of age and gender. The three verbal measures showed a statistically significant effect for age but not gender or interaction between age and gender. A statistically significant gender effect for boys was found in the visuospatial measures in the older age groups (above age 6). The analysis of the measures of complex memory span revealed a statistically significant age effect but no statistically significant effect of gender. Performance on all nine tests showed increasing performance from 4 to14 years, but a leveling off of performance between 14 and 15 years (p. 180).

In a correlational analysis, correlations were computed for five age groups of over 100 children in each group. The three verbal measures (phonological loop) had high partial correlations with one another (r=.45); moderate partial correlations, verbal with complex memory span (r=.30); and weak correlation between the verbal and visuospatial measures (r=.11). Partial correlations computed between the three visuospatial measures were moderate, with a mean partial r=.37, and a weak correlation (r=.26) with complex memory span measures. The three complex memory span measures had low to moderate correlations with a mean partial r=.30.

A confirmatory factor analysis was used to examine if the children's scores corresponded to the three-factor model of working memory. The researchers concluded that the analysis confirmed that the phonological loop and central executive had very close associations, as well as the visuospatial and central executive. There was a weak association between the visuospatial and phonological loop measures, suggesting that these three factors are needed to provide an efficient working-memory system across all the different age groups. Gathercole et al. (2004) did not assess the episodic buffer as a component of working memory, but did conclude that the three main components of working memory are present by age 6 and increase linearly throughout childhood to adolescence. The three-factor model of working memory (central executive, visuospatial and phonological loop) provides the theoretical rationale for the current study, and Gathercole's study helped to confirm the presence of the working-memory system in college-age students.

In a study designed to include the episodic buffer, Alloway, Gathercole, Willis, and Adams (2004) tested 633 children on measures of verbal short-term memory, complex memory span, sentence repetition, phonological awareness, and nonverbal ability. The participants (mean chronological age of 59.5 months, SD=3.7, range=51 to 68 months) were selected from 26 state primary schools in Northern England and represented a range of low-, middle-, and high-socioeconomic status as suggested by school meal rates. The following tests were used for each working memory component: Complex Memory Span: backwards digits and counting recall from the Working Memory Test Battery for Children (WMTB-C) and sentence completion and recall task; for Phonological short-term memory the digit recall and the word recall test of the WMTB-C were administered as well as the Children's Test of Nonword Repetition; Sentence Repetition tasks designed to measure the episodic buffer; Phonological Abilities Test for phonological awareness; and block design and object assembly subtests from the Wechsler Preschool and Primary Scale of Intelligence-Revised for nonverbal ability measures.

Correlational coefficients for measures that shared specific cognitive functions were in the weak to moderate range with complex memory span ranging from .43 to .45, phonological loop .37 to .55, sentence repetition tasks .53, phonological awareness .33, and nonverbal measures .43. Alloway et al. (2004) used a confirmatory factor analysis to test the adequacy of Baddeley's working memory model. The goodness-of-fit root mean square error of approximation (RMSEA) was .05 for the model that includes central executive, phonological loop, phonological awareness, nonverbal abilities, and the episodic buffer. Comparisons with the other models yielded the best results for this five-component model. The researchers used this information to contend that this structure is in place in 4- to 6-year-old children. They further concluded that the episodic buffer (sentence repetition) integrates representations from working memory, long-term memory, and language processing.

The findings from this study are important to the current investigation because problem solving requires students in mathematics courses to use the episodic buffer to access and store the meaningful information as in the sentence repetition tasks. Evaluating comprehension of verbally presented problems may provide valuable information regarding this component of working memory and its relationship with mathematics achievement.

Bayliss, Jarrold, Gunn, and Baddeley (2003) were interested in the extent to which processing speed and storage factors influence performance. They designed two experiments to examine if complex span performance required an executive coordinating component that was independent of the speed and storage factors. In the first experiment, 75 children in year 3 (n=30, mean age = 8.00, range = 7.08 to 8.06) and year 4 (n=45, mean age = 9.01, range = 8.07 to 9.07) of a primary school in Bristol England were participants.

The researchers administered four complex span tasks developed to include two types of processing: verbal and visuospatial. In the verbal task, children were required to make associations between verbal object names and the colors usually associated with the object. They were then required to find the circle on a computer screen that was the correct color and remember the number contained in the circle for recall at the end of the trial. In the visuospatial task, the children were required to point to the circle and remember the location of the circle until the end of the trial. Span length for the trials increased from two to six processing and storage episodes, with oral responses required for the verbal tasks and pointing responses required for the visuospatial tasks.

Results using a repeated measures analysis of variance (ANOVA) revealed a main effect of task (partial effect size of .78) with slower reaction times for the verbal processing task. The researchers suggest that the two tasks (verbal and visuospatial) involve different processing requirements. The ANOVA for the storage tasks was not statistically significant, therefore, the researchers interpreted these results as performance being comparable across both tasks.

The second experiment included 48 undergraduate students from Bristol England. The participants were administered the verbal and visuospatial processing tasks followed by two complex span tasks, as well as the verbal and visuospatial storage tasks. The measures were modified from those of the children's task to make them more appropriate for adults.

Two-way repeated measures ANOVA revealed a main effect of processing (effect size=.27) with a poorer performance on verbal processing and main effect of storage (effect size=.28). There was a statistically significant Processing by Storage interaction (effect size=.39), as well as a statistically significant effect of storage combined with verbal processing (effect size=.52). There was no statistically significant effect of storage combined with visuospatial processing, but processing was statistically significant when combined with verbal storage (effect size=.57) but not visuospatial storage.

The researchers interpreted their findings to show that individual differences in processing efficiency and storage contribute to complex span performance. They also concluded that complex span performance was domain general and storage constraints were domain specific (visuospatial and verbal). This finding was evident in both children and adults, with an additional ability to coordinate the storage functions influencing reading and mathematics performance. Because verbal storage and processing are important components of the working memory measure used in the current study, Bayliss et al. (2003) study was relevant for describing the interaction of processing and storage of verbal information.

Another study by Bayliss, Jarrold, Baddeley, Gunn, and Leigh (2005) used similar measures to investigate the extent to which developmental improvements in complex span performance were driven by age-related changes. There were 120 (57 boys, 63 girls) children from three age groups (6, 8, and 10 year olds) from a primary school in Bristol, United Kingdom. The students were selected from a 185 children pool based on standardized scores within one standard deviation of the mean (M=100, SD=15) on the British Picture Vocabulary Scale (BPVS). The four complex span measures from the previous experiment were used.

Bayliss et al. (2005) made the point that the findings in this study are inconsistent with previous findings by Fry and Hale (1996). Fry and Hale (1996) proposed that processing speed and age-related changes mediate most of the developmental increases in working memory. Bayliss and the other researchers instead contend that there is a separate age-related constraint with storage ability.

Summary

This series of studies has reinforced the concept of the different components of working memory. They also have shown that working memory develops in a linear way throughout childhood and early adolescence. It would be important, therefore, to examine working memory in young adults to learn if this trend continues or has leveled off as in the studies by Gathercole et al. (2004). By investigating working memory in community-college students, there will be a range of ages (new and returning students) to contribute to diversity in the sample.

Working Memory and Mathematics

There have been a number of studies that have investigated working memory as a function of reading or mathematics deficits (Cormoldi, Rigoni, Tressoldi, & Vio, 1999; Keeler & Swanson, 2001; Swanson, 2006; Wilson & Swanson, 2001). These studies included not only assessment of working memory but also strategies that were examined to show effects on improvement of working memory.

Swanson (2006) conducted a study of the factors that influence mathematical problem solving across groups of individuals varying in terms of ages and incremental changes in working memory. The sample included 353 children (167 girls and 186 boys) in grades 1, 2, and 3 for the first wave and 320 children (166 girls and 154 boys) for the second wave of the study that was conducted one year later. The students in the sample were representative of Southern California in terms of ethnicity with the majority of students European American or Hispanic American (163 European American, 147 Hispanic American, 25 African American, 14 Asian American, and 4 Other). The researchers used a variety of measures that included specific components of the Wechsler Intelligence Scale for Children (WISC III), Wide Range Achievement Test (WRAT III), Wechsler Individual Achievement Test (WIAT), Test of Word Reading Efficiency (TOWRE), Comprehensive Test of Phonological Processing (CTOPP), as well as researcher-developed assessments for word problem-solving processes, short-term memory, executive processing, visualspatial sketchpad, and fluency.

Students' scores on these measures were analyzed in the first and second waves to compute correlations between the measures when first administered and then a year later. Swanson (2006) interpreted the results of this analysis to express three findings: problem solving in Wave 2 was correlated with problem solving, mathematics calculation, reading, phonological knowledge, executive processing, and vocabulary in Wave 1 measures; and phonological processes are related to most of the other measures and executive processing correlates with arithmetic calculation (r=.56), reading (r=.60), and vocabulary (r=.56) in Wave 2. When a hierarchical regression analysis was used, the findings suggested that the executive processing component of working memory and reading in Wave 1 contributed variance to problem solving in Wave 2 and the visuospatial working memory component was related to mathematics calculation (p. 275). Swanson (2006) further suggested that the results of this study are evidence that the monitoring components of working memory play a more important role in problem solving than long-term memory skills. The monitoring components of working memory play an important role in the achievement of students in a community-college mathematics course. Students are required to hold basic mathematics components in memory and know when to apply those skills in higher-order problems.

A similar finding was expressed regarding working memory and story-problem accuracy. Fuchs et al. (2005) conducted a study of first-grade students in a Southeastern metropolitan school district who were at-risk (AR) and not at-risk (NAR) of mathematics difficulty to investigate cognitive factors that affected identification and prevention. Of 319 children identified as the lowest scoring on measures of Curriculum-Based Measurement (CBM) Computation, addition fact fluency, subtraction fact fluency, and CBM concept and applications, 139 of the lowest performing students were identified as AR. These AR students were placed into two groups: 69 AR control students and 70 AR tutored students. A total of 437 were NAR students, 145 of whom were individually and group tested, and 292 who were group tested only. Reading skill was measured using the word identification subtest of the Woodcock Reading Mastery Test-Revised (WRMT-R), and intelligence was measured using the Wechsler Abbreviated Scale of Intelligence (WASI), with other measures for phonological processing, listening comprehension, nonverbal problem solving, processing speed, and induction ability. For working memory, the researchers used the Working Memory Test Battery for Children-Listening Recall (WMTB).

One-way analyses of variance (ANOVAs) were applied to the seven different mathematics-dependent variables at pretest and posttest as well as improvement scores. The results indicated that the NAR students' performance exceeded that of both AR groups in the pretest condition. The NAR students' performance on the posttest exceeded AR tutored students that in turn was higher than the AR control students on the Woodcock-Johnson III Tests of Achievement (WJIII) for Calculation skills. Effect sizes of 1.14, .41, and -.64, First-Grade Concepts and Applications, effect sizes of 2.04, 1.35, and -.46, and Story Problems, effect sizes of 1.56, 1.10, and -.51 were reported. In order to determine the relationship among seven cognitive abilities and early mathematics development, the researchers used a multiple regression with six variables entered simultaneously as the first step and the remaining variable entered in the second step so that a different cognitive variable was entered alone each time in the second step. Results of this analysis suggested that attention and phonological processing were unique predictors of basic fact fluency. Working memory, nonverbal problem solving, and attention were three predictors of story-problem skills. The researchers estimated that working memory contributed 1.40 % of unique variance to story problem performance. Correlations between working memory and the mathematics measures were WJIII calculation, .31, CBM computation, .33, Grade 1 Concepts and Applications, .51, Story Problems, .45, and Basic facts, .30.

The addition of attention and phonological processing as predictors of basic fact fluency points to the need for strategies that include increased attention in the mathematics classroom. Problem-solving, however, also needs to include strategies that increase working memory and attention.

Number sense and estimation are skills needed for effective problem solving that require extensive working memory (Gersten & Chard, 2001; Seethaler & Fuchs, 2006). In a study of estimation skills of third-grade students, Seethaler and Fuchs (2006) investigated cognitive variables involved in estimation skills including working memory. The researchers also were interested in whether estimation in addition is a function of mathematics computation performance. Participants in the study were 315 third-grade students (166 female, 149 male; 132 African American, 139 European American, 19 Hispanic American, 9 Kurdish, and 16 other) randomly selected from 494 children in 30 classrooms in a Southeastern metropolitan school district. Of those in the sample, 128 received free or reduced lunch and 26 received special-education services.

Students were assessed in 30- to 60-minute group sessions and then tested individually for 45-minute sessions by research assistants. Whole class assessments were mathematics fact fluency and double-digit addition and subtraction battery. The list of individually administered assessment subtests and constructs tested included Woodcock Diagnostic Reading Battery (WDRB), listening comprehension (Language), Test of Language Development (TOLD), grammatic closure (Language), Wechsler Abbreviated Scale of Intelligence (WASI) vocabulary (Language) and matrix reasoning (Nonverbal Reasoning), Woodcock-Johnson Psycho-Educational Battery (WJ-III), concept formation, numbers reversed (Working Memory), visual matching (Processing Speed) and retrieval fluency (Long-Term Memory), Woodcock Memory Test Battery-Children (WMTB-C), listening recall (Working Memory), Woodcock Reading Mastery Test-Revised (WRMT-R), word identification and word attack, and Test of Word Reading Efficiency (TOWRE), sight word efficiency. Further, a measure of inattentive behavior, the SWAN, an 18-item 7-point teacher rating scale was completed by each student's teacher.

Results were reported from an ANOVA that examined estimation skill using computation performance as the between-groups factor. The students were placed into groups based on their computation performance (low=less than one SD below the mean, average=within one SD of the mean, and high=greater than one SD above the mean) on a second-grade timed measure. The results of the ANOVA showed a statistically significant difference among groups, $F(2, 312) = 40.89$. An effect size from post-hoc analyses using Cohen's d were moderate, .55 for low versus average, high, .72 for average versus high and very high, 1.34 for low versus high (p. 238). In a correlation and multiple regression analysis, 10 predictor variables were used: language, listening comprehension, nonverbal reasoning, concept formation, processing speed, long-term memory, working memory, inattentive behavior, basic reading skills, arithmetic number combinations, and double-digit computation. Correlations for these variables were highest for the arithmetic number combination ($r=.62$), double-digit computation ($r=.55$), inattentive behavior ($r=.51$), concept formation ($r=.49$), visual matching ($r=.43$), nonverbal reasoning ($r=.47$), and working memory ($r=.38$). All of the preceding correlations were statistically significant and ranged from weak to moderately strong. These same 10 variables were entered into a simultaneous regression analysis with 5 statistically significant predictors emerging that accounted for 50% of the variance: arithmetic number combination, nonverbal reasoning, concept formation, inattentive behavior, and working memory. The unique variance was found by entering all but one of the variables and then noting the change in R^2, repeating this process 10 times to estimate the unique variance for each of the 10 predictors.

Computation skills were a statistically significant predictor of estimation skills. According to the researchers, computation skills accounted for the differential outcomes of students who were placed into the low, average, and high groups, rather than a function of developmental level. Even the students in the high-level group, however, achieved a little over 50% accuracy on the estimation tasks with most (78%) scoring 37% or less correct. Seethaler and Fuchs (2006) questioned whether the accuracy on estimation tasks is due to estimation being a developmental task or the lack of instruction in estimation skills at the third-grade level. One limitation of this study is its assessment of students at the beginning (in the Fall) of their third-grade year in school. Students in the Fall may not have had access to instruction in this highly cognitive demanding skill that could be a reason for overall lack of accuracy. Other variables that were statistically significant were nonverbal reasoning, concept formation, working memory, and inattentive behavior.

This study is relevant for community-college students in that working memory is necessary to hold computational information in memory in order to make decisions regarding appropriateness of a response. As the results of the Seethaler and Fuchs (2006) study have shown, it is first necessary to have developmentally appropriate computation skills before becoming proficient at estimation. The relevance for college-age students is that some students entering community college lack the computation and number fluency skills necessary for the decision-making demands of algebra.

When strategies are introduced as a measure of improvement for working memory in mathematics, different findings are presented. For example, Keeler and Swanson (2001) suggested that one approach to improving mathematics achievement is to understand working-memory deficits and develop methods to increase a student's knowledge about effective strategies. Van Merrienboer, Kirschneer, and Kester (2003) proposed that working memory is "the most central aspect of human cognitive architecture" (p. 5). For this reason, they investigated instructional designs that would decrease cognitive load and thus allow working memory to be activated more efficiently. Using simple-to-complex sequencing of learning tasks, using partially completed tasks that require active participation, and using just-in-time presentation of new information were three of the suggestions the researchers used to demonstrate effective learning strategies.

Keeler and Swanson (2001) investigated the effect of strategy knowledge on working memory. The researchers conducted two quasi-experimental studies with students with mathematics disabilities (MD) under three conditions: without probes (initial), with probes (gain), and without probes on the highest level achieved under gain (maintenance) on measures of verbal and visual-spatial working-memory tasks. The participants in the first experiment were 54 children (36 boys, 18 girls) with a mean age of 10.8 years (SD=2.8 years) from a private school for children with learning disabilities. All of the children in the study were reported to have average ability (Mean IQ=99.54, SD=20.75) measured by the Wechsler Intelligence Scale for Children-Revised (WISC-

R); the SD, however, puts the range of the students' abilities outside the "average range." Standard scores on the WISC were available, however, for only 24 of the 54 participants. Mathematics achievement was measured by the Wide Range Achievement Test-Revised (WRAT-R). The researchers included students with standard scores below 90 (Mean=75.5, SD=15.3) on the mathematics section for the designation of mathematics disability.

Two tasks were used to measure working memory (WM) selected from the Swanson-Cognitive Processing Test, one for verbal and one for visual-spatial processing, that required the "maintenance of some information during the processing of other information" (Keeler & Swanson, 2001, p. 420). The two measures used were Digit Sentence Span Task in which numerical information is embedded in a sentence and the Mapping and Directions Task that has a sequence of directions on a map without labels. The presentation of each of these tasks followed a procedure that included stimulus presentation, processing question, strategy selection, and recall. Before the stimulus presentation, strategies for recalling the information were displayed for the participant. The strategies for the verbal task were rehearsal, chunking, association, and elaboration. For the visual-spatial task, the strategies were elemental, global, sectional, and backward processing. For each task, all items were administered individually to the participant until the probes did not improve performance. The results using a multivariate analysis of variance (MANOVA) showed that the students' strategy choice had a statistically significant effect on initial, gain, and maintenance for the Digit Sentence Span task (Wilks's lambda=.47, $F(9,98)=3.97$;) and Mapping and Directions task (Wilks's lambda=.65, $F(6,82)=3.24$) (p.423). Although the researchers interpreted the results to show that stable strategy choices are related to working memory, they were not able to demonstrate which strategy provided the most influence on performance.

In the second experiment, a group of 20 students (7 girls, 13 boys) with mathematics disabilities (MD) was compared with 18 chronologically matched students (7 girls, 11 boys) and a group of 19 younger students (5 girls, 14 boys) matched on mathematics-achievement level on the same working-memory tasks as in the first experiment. According to the researchers, analysis of the data revealed that working-memory strategy stability contributed to mathematics performance.

Does consistency of strategies mean that working memory is accessed under all conditions in the same manner? Hecht (2006) examined strategies regarding selection and use of simple arithmetic calculation tasks in groups of undergraduate students. Hecht (2006) investigated whether the hypothesis of retrieval of facts from long-term memory for adults was the exclusive method used by undergraduate students. There were 78 psychology students in the first experiment and 85 psychology students in the second experiment. No demographic information was given about these students so it is difficult to generalize the results to populations outside the university where the study took place. In the first experiment, each student was shown a computer screen with a simple arithmetic problem for the first task and a multiplication problem for the second task and was required to provide a verbal answer to the problem. The third task measured was the calculation subtest from the Woodcock-Johnson Psycho-Educational Battery-Revised (WJ-R). The working-memory task was the counting-span task, and the last task was number-naming speed. In the simple arithmetic tasks, the participants were prompted both verbally and written on a computer screen to either use only a retrieval strategy (no choice) or to use whatever strategy was necessary to solve the problem (choice). After these measures were administered, the participants were divided into three groups: perfectionists, good retrievers, and not-so-good retrievers based on their scores on the simple arithmetic measure and a cluster-analysis procedure. In the second experiment, the undergraduate students were administered the same tests, but between 7 to 10 days elapsed between the two conditions (no choice) and (choice).

Results for each group were reported in terms of percent of times retrieval of facts was used as the strategy. Not-so-good retrievers had no statistically significant differences between the choice and no-choice conditions. Good retrievers increased their retrieval by 9% and 4% in Experiment 1 and 4% and 2% in Experiment 2, respectively. Perfectionists used a retrieval strategy more often when they were not given a choice than when they were given the opportunity to choose a strategy. Hecht (2006) used the results to suggest that good achievers rely on their working-memory skills to retrieve facts given choice or no-choice conditions. Perfectionists, however, chose to use other strategies when given the choice in addition to retrieval to be sure their answers are correct. Students in community-college mathematics classes are often unsure of the strategies that allow them to be successful in basic mathematics courses. From Hecht’s (2006) study, it was shown that the use of retrieval methods for basic facts is related to more than working memory skills. Anxiety about achieving the correct answer may prompt the use of other strategies that rely less on working memory. The central executive component of working memory should direct the use of strategies needed for computation and problem solving.

Summary

The studies in this section have highlighted a number of issues related to working memory and mathematics achievement. Verbal working memory was shown to be a statistically significant factor in studies of computation and problem solving for children (Fuchs et al, 2005; Keeler & Swanson, 2001; Swanson, 2006). Working memory was related to number sense and estimation skills, as well as strategies for improving mathematics skills (Hecht, 2006; Keeler & Swanson, 2001; Seethaler & Fuchs, 2006).

Working Memory and College Students

Studies conducted using college students as participants are often those that use samples recruited at college campuses from psychology classes. There are few studies that target samples of students in mathematics courses. In one example, McGlaughlin, Knoop, and Holliday (2005) conducted a study of postsecondary students with mathematics difficulties at a Midwestern university. There were 205 original participants, but only 76 participated in the study, 34 identified as having a mathematics disability (MD) and 42 as No Diagnosis (ND). The reason given by the researchers for the extreme drop in participants was that 129 were excluded because the researchers' criteria for inclusion did not fit into the students' diagnostic category after completing some of a psychoeducational battery. There were 53 females and 23 males; 68 European Americans, 7 African-Americans and one Hispanic American. The range of ages for MD students was 18 to 23, whereas the range for ND was 18 to 43.

The procedure involved the administration of a complete psychoeducational battery of assessments including a structured interview, the Wechsler Adult Intelligence Scale (WAIS-III), the Wechsler Memory Scale (WMS-III), Woodcock-Johnson III Tests of Achievement (WJ-III), and the Conners Adult ADHD Rating Scale (CAARS). Six individual variables were analyzed: WMS-III Working Memory Index score, WJ-III Math Fluency, WJ-III Passage Comprehension, WAIS-III Performance IQ, WAIS-III Verbal IQ, and CAARS DSM IV Total Symptoms Index score.

A MANOVA was used to test the statistical significance in the means of the two groups (MD and ND). After finding that statistically significant differences existed among the variables (an effect size of .43), the researchers computed separate ANOVAs to investigate which variables were statistically significantly different. Results showed statistically significant differences between the MD and ND groups on the following variables: Performance IQ an $F(1, 74)=6.96$, an effect size of .09, and power of .74; Working Memory an $F(1,74)=16.05$, an effect size of .18, and power of .98 ; Passage Comprehension an $F(1,74)=3.99$ an effect size of .05, and power of .51; and Mathematics Fluency an $F(1,74)=33.94$, an effect size of .31, and a power of 1.00. Two other variables, Verbal IQ and ADHD (attention) were not statistically significant.

The researchers concluded that working memory, mathematics fluency, reading comprehension, and nonverbal ability weaknesses (measured by the Performance IQ) were statistically significant contributors to and discriminators of mathematics disability. The effect sizes for only working memory and mathematics fluency were small, however, so this interpretation may be viewed with caution. The researchers used these findings to suggest that the underlying contributors of mathematics disability need to be addressed in any remedial program for mathematics difficulties. The researchers acknowledged that one of the limitations to this study was that all students involved had mathematics problems and so a random sample was not used. The finding that college students with mathematics disorder also had working memory deficits is an important area for further research for college students.

Does research in the area of working memory and college students also find that there are factors outside the cognitive domain that affect performance as well as working memory? Beilock and Carr (2005) examined pressure and effects on working memory in a study conducted with 93 undergraduate students at a major Midwestern university. No other information was given regarding the demographics of the students. The students were first given two working-memory tasks, Operation Span (OSPAN) and Reading Span (RSPAN), to construct a High Working Memory (HWM) and Low Working Memory (LWM) group. The two groups were administered a modular arithmetic task (MA) under high-pressure and low-pressure conditions, as well as high-demand and low-demand problems presented in random order. The data were analyzed using a 2 (LWM, HWM) x 2 (low demand, high demand) x 2 (low pressure, high pressure) ANOVA and were found to have a statistically significant three-way interaction, with a second ANOVA suggesting a Problem Demand by Pressure interaction for those students with High Working-Memory skills. The researchers used these results to suggest that students with High Working-Memory skills lose the advantage of those skills when high demand and high pressure combined on the mathematics items. Without further information regarding the students sampled, this study was limited to the conditions of the investigation and would be difficult to generalize to other postsecondary settings or as the researchers did to high-stakes tests like the SAT or Graduate Record Exam (GRE).

Ashcraft and Kirk (2001) designed a series of three experiments investigating the relationships between working memory, mathematics anxiety, and performance. In the first experiment, 66 participants (33 men, 33 women; 68% European American, 18% African American, 8% Hispanic American, and 5% Asian Pacific American) were selected from an undergraduate psychology course and were administered the short version of the Math Anxiety Rating Scale (sMARS) as well as a questionnaire to assess demographics. The questionnaire surveyed gender, age, ethnicity, class year, number of high-school and college courses, grades achieved in those courses, and self-reported mathematics enjoyment and anxiety level. To measure working memory capacity, two tasks, listening span (L-span) and computation span (C-span), were administered via a tape recording. The researchers first noted the correlations between the sMARS and number of high-school mathematics courses taken ($r=-.28$), high-school grades in mathematics ($r=-.29$), and self-rated mathematics anxiety ($r=.42$), along with both working-memory tasks, L-span ($r=-.36$) and C-span ($r=-.44$) were statistically significant. The results of the correlations of the remaining demographics with the sMARS were not reported.

From the sMARS data, the participants were put into three groups categorized as low, medium, and high mathematics anxiety with low at least one standard deviation below the mean and high at least one standard deviation above the mean ($M=36.3$, $SD=16.3$). There were 12 in the low group, 23 in the medium, and 15 in the high-mathematics-anxiety groups for a total of 50 participants' results that were analyzed using an ANOVA. There was no explanation given for excluding 16 of the participants from this analysis. A two-way ANOVA on the span scores (L-span and C-span) revealed a statistically significant main effect for anxiety group ($F(2, 47)= 11.22$), a small effect size of .32 and type of span task ($F(1, 47) = 10.17$), a small effect size of .30, but no statistically significant interaction ($F(2, 47) = 1.71$), an effect size of .06. The researchers interpreted these results as a finding that working memory was influenced negatively by mathematics anxiety.

In the second experiment, Ashcraft and Kirk (2001) used an addition task that ranged in problem difficulty from basic arithmetic facts to two digits with carrying. Participants in this experiment were 45 undergraduate students (15 male, 30 female; 67% European American, 22% African American, 7% Asian Pacific American, and 2% Hispanic American). Reaction time (RT) and errors were evaluated to establish when the working-memory system was overloaded, with a secondary task of holding either 2 or 6 letters in working memory for later recall. The task included three main events presented on a computer screen requiring a verbal response: first, letters were presented, next the addition problem to be solved, and third a prompt of the recall of letters. A mixed design ANOVA was used to analyze the data: mathematics anxiety as a between-subjects variable and the other factors as within-subjects variables. The within-subjects factors formed a 3 x 2 x 2 design (problem size: basic, medium, large; carry status: carry, no carry; and memory load: two letters, six letters). A statistically significant result was reported for the Anxiety by Carry interaction, $F(2, 42) = 6.04$, a small effect size of .22 (p. 230). There also was a statistically significant interaction among problem size, memory load, and carry, $F(2, 84) = 8.19$, a very small effect size of .16. Further results of Ashcraft and Kirk’s analysis interpreted the carrying factor to influence minimally in a control (read only, no letter recall) and light-memory condition (two-letter recall) but increased with a heavy-memory load (six-letter recall) across the anxiety groups. In their discussion, Ashcraft and Kirk (2001) argued that based on their findings carrying, along with high-working-memory task demands, produced a deterioration of performance for the high-anxiety group (p. 232). Ashcraft and Kirk had used control measures to compare results on simple naming tasks and those that required increasing memory loads to substantiate their findings. Limitations of this study, however, may be that the conditions of this experiment could not replicate anxiety experienced in a high-stakes setting such as a final examination or college entrance examination.

The third experiment conducted by Ashcraft and Kirk (2001) involved transformations of numbers and letters to simulate a task that was working-memory intensive but did not involve a learned-mathematics skill explicitly. Participants in this experiment were 45 undergraduate students (10 males, 35 females; 76% European American, 13% African American, 7% Asian Pacific American, and 2% Hispanic American) recruited from lower-level psychology classes. After they were given the sMARS and demographic questionnaire, the students were divided into the 3 mathematics-anxiety levels. The students were administered the L-span and C-span working memory tasks and then letter and number transformation trials using a computer. The dependent measures in this experiment were accuracy and latency collected for the transformation tasks. Results of the third experiment showed that latency (RT) increased and accuracy decreased for the high-mathematics-anxiety groups on both the letter and number-transformation tasks. Using an ANOVA, the results were that there was a statistically significant effect in both the two-item latency interaction between anxiety and position, $F(2,42) = 7.08$ a small effect size of .28, and the four-item effect of anxiety, $F(2,42) = 6.24$, a small effect size of .23. The researchers found that not only did the high-mathematics-anxiety participants spend more time on the transformation tasks but also were less accurate with mathematics anxiety that was the main effect $F(2,42) = 10.17$, a small effect size of .32.

Overall, this series of experiments provided some evidence of Ashcraft and Kirk's (2001) conclusion that, "math anxiety disrupts the on-going, task-relevant activities of working memory, slowing down performance and degrading its accuracy" (p. 236). A salient portion missing from their analysis was obtaining results from naturally occurring test situations within the mathematics classroom.

Summary

As students reach college level, there are additional factors related to mathematics achievement. Stress and anxiety and effect on working memory appear to have a negative effect on accuracy and processing of mathematics problem solving (Ashcraft & Kirk, 2001; Beilock & Carr 2005). In addition, working memory and mathematics fluency were shown to be predictors of mathematics disabilities (McGlaughlin, Knoop & Holliday, 2005).

Working Memory and Students with Learning Disabilities

Students with learning disabilities are present in the developmental levels of mathematics courses at the community-college level. This section focuses on characteristics of students with learning disabilities pertaining to mathematics and working memory.

In a meta-analysis of the literature from 1970 to 2003, Swanson and Jerman (2006) attempted to answer the following three questions related to students with mathematics disabilities (MD):

1. Are cognitive abilities in children with MD distinct from those in their average-achieving counterparts and in children with co-

morbid disorders such as Reading Disabilities (RD) (e.g., MD+RD)?
2. Are the cognitive deficits in children with MD a function of variations in age?
3. Do the cognitive deficits that emerge in children with MD vary as a function of definitional criteria? (p. 252)

Swanson and Jerman (2006) included 28 studies in their analysis that satisfied the following criteria: an MD group compared with an average-achieving group, no reported co-morbidity within the MD group, and the inclusion of a norm-referenced measure of intelligence and a norm-referenced measure of mathematics separated by comparison group. The researchers identified these studies through a search of the PsycINFO, MEDline, and ERIC databases using the following keywords and terms: “math disabled, math disabilities, dyscalculia, less skilled math, math disabled/reading disabled, arithmetic disabled, poor problem solvers, problem solving in math and problem solving and math” (p. 252). Additional articles were gathered from the names of primary researchers and a manual search of most-used journals.

The coding by the researchers included sample characteristics, classification measures, and cognitive measures. The subgroups for each study were classified as average achieving, mathematics disabled, reading disabled, and mathematics and reading disabled. Number of participants, mean age, gender, socioeconomic status, ethnicity, and primary language also were coded. All of the assessment measures were converted to raw scores and organized into 17 categories. These categories were aggregated into 10 variables to calculate effect sizes: literacy-reading, problem-solving verbal, naming speed, problem-solving-visual-spatial, long-term memory, short-term-memory-words, short-term-memory-numbers, working-memory verbal, working-memory visual-spatial, and attention. Effect size was computed using Cohen's d with the dependent measure defined as d/(1/v), where d is (mean of MD-mean of comparison group/average of standard deviation for both groups), and v is the inverse of the sampling variance, $v=(N_{md} + N_{ave})/(N_{md} \times N_{ave}) + d^2 [2(N_{md} + N_{ave})]$ (p. 257). Hierarchical linear modeling (HLM) was used to assess the extent to which individual studies and variation within studies influence outcomes, accommodate incomplete data, and solve for coefficients at two levels (p. 258). The two conditional models used in the HLM first tested if the effect-size difference was a function of IQ, mathematics, reading, or chronological age and random error. The second model tested whether specific cognitive domains moderated cognitive functioning (p. 259).

Generally, the results of the meta-analysis reported articles most frequently published in *Journal of Experimental Child Psychology, Journal of Clinical and Experimental Neuropsychology, Journal of Learning Disabilities*, and *Learning Disability Quarterly* from 1983 to 2002. Most (17) of the studies were conducted in the United States and contained information about socioeconomic status (SES) in only 8 studies and ethnic background in only 9 studies. Although 24 of the studies reported gender characteristics, none of the studies had separated mathematics performance as a function of gender, ethnicity, or SES, so these characteristics were not analyzed.

The relationship between MD and average achievers on 194 dependent measures in this analysis had a mean effect size of -.52, that is considered in the moderate range. The categories that yielded moderate effect sizes of .50 to .80 were in verbal and visual problem solving, speed, long-term memory (LTM), short-term memory (STM) for words, working-memory (WM) verbal, and WM visual-spatial. When comparing children with mathematics disabilities and reading disabilities, there were 58 effect sizes with a mean effect size of -.10. Although the researchers acknowledged that these effect sizes were in the low range, they still considered that the RD children had an advantage over MD children on measures of naming speed and visual-spatial WM. In the last analysis between groups, MD children were compared with those children who had both reading and mathematics difficulties (MD+RD). In this analysis, effect sizes in the moderate range were found on measures of literacy, visual-spatial problem solving, LTM and STM for words in which children with MD performed better than the other groups. MD+RD children did better on measures of attention and naming speed.

The analysis for cognitive variables associated with the classification measures revealed the effect size for IQ was correlated with verbal and visual-spatial STM, WM, and processing speed. Effect sizes for mathematics were correlated with verbal WM, STM for digits and LTM, and for reading were correlated with verbal WM. The age of the MD group also correlated with mathematics effect sizes, with older children (over age 8.5) having a more statistically significant deficit (r=-.57) than younger children (r=-.46).

In their discussion of the results of the meta-analysis, Swanson and Jerman (2006) stated that working memory was the variable that contributed most to cognitive functioning of children with mathematics disabilities. They further interpreted this result to suggest that semantic memory not memory for numbers was important. The researchers also found only weak support for differentiating MD from RD children suggesting that these disabilities have similar deficits.

Swanson and Jerman (2006) identified limitations to their analysis that have relevance for their findings. Some studies did not report intelligence levels for the participants so that this factor may have influenced some of the findings. Because criteria for inclusion as a student with mathematics disabilities may include students with reading disabilities, their findings of relationships between the two groups (MD and MD+RD) may be unfounded. Working memory shows statistically significant correlations with mathematics achievement for students with mathematics disability. This finding needs to be investigated further to include students at the community-college level, identified with and without mathematics disabilities.

Wilson and Swanson (2001) conducted a study of individuals across a broad range of ages (11 to 52 years) for the purpose of examining if age and domain (verbal vs. visual-spatial) involved in working memory was important for mathematics computation. There were 98 participants (43 male, 53 female, 2 unknown), who were separated into groups: those with mathematics disabilities (MD) and those without mathematics disabilities (non-MD) according to test scores on the Wide Range Achievement Test-Revised (WRAT-R). The cut-off for MD subjects was lower than the 25th percentile on the Mathematics subtest of the WRAT-R. The WRAT-R, however, assesses only computation not mathematics application skills, so this measure provides a very narrow identification of participants with mathematics disability. The sample contained 82.7% European American, 10.2 % Hispanic American, 3.1% African American, and 4.0% other. Researchers administered both the Arithmetic and Reading portions of the WRAT-R, as well as a series of working-memory tasks that included Visual Spatial (mapping and directions, visual matrix) and Verbal (story retelling, semantic association) measures. Results of a number of statistical analysis including correlations and multiple regression indicated that the statistically significant factors in mathematics achievement were first verbal working memory and next visual-spatial working memory. The multiple regression was accomplished through five models that entered variables in different order to view those that influenced the dependent measure. The only other factor that consistently contributed variance throughout the analysis was that of gender with women performing mathematics tasks better than men. The researchers suggested that the results of this study indicated that domain-specific and domain-general working memory systems are linked to mathematics performance.

Before reporting results, the researchers divided the participants into 3 age categories: age range 11 to 14, with a mean of 11.90 (n=27); age range 14 to 19, with a mean of 15.74 (n=37); and age range 21 to 52, with a mean of 30.93 (n=34). Wilson and Swanson (2001) used this quadratic age variable to interpret their findings in order to discover if there was a relationship between mathematics and working memory as a function of age. The researchers found no statistically significant correlations between the factors as a function of age or ability group. Results also suggested that there was no pattern of the magnitude of correlations between mathematics and working memory that are stronger in younger children than adults. The last result reported that raw scores for mathematics were statistically significant related to the verbal-working-memory composite score for the group of individuals in the mathematics disability group but not for the nonmathematics disability group. At the same time, reading scores were not related to the working-memory measure because ability groups had been matched on this variable. Because students in community colleges often have reading disabilities as well as mathematics disabilities, the findings in Wilson and Swanson’s study would suggest that this factor would not need to be part of the analysis. Mathematics disability, however, was found to contribute to the working-memory score and should be part of the analysis.

A regression analysis was used to investigate the unique variance in computational ability to address two questions. The first question investigated whether working memory moderated age-related changes in computation ability. The second question was used to examine whether verbal and visual-spatial working memory contributed independent variance to mathematics performance. The researchers looked at four issues to assess the amount of variance: Quadratic and linear age-related variables alone, age-related performance after working memory was partialed out, age-related contrast after working memory and reading were partialed out, and age-related contrast after working memory, reading, and gender were partialed out.

A general result of this analysis was that regardless of presentation order, verbal (22%) and visual-spatial working memory (14%) contributed statistically significant variance to mathematics when entered first into the multiple regression. Reading ability and gender only accounted for 4% and 3% of the variance when these variables were entered first into the analysis.

Wilson and Swanson (2001) concluded that the results of the regression analysis suggested an independent contribution of reading ability, age, gender, verbal working memory, and visual-spatial working memory to the individuals' mathematics computation skills. The researchers interpreted that conclusion to mean that verbal and visual-spatial working memory contributed statistically significant variance irrespective of their order of entry in the regression analysis.

The researchers cited two clear findings in their study and their interpretation of results. The first was that (contrary to their expectations) there is a relationship between working memory and mathematics ability but that it is no higher in children than adults. The second finding was that groups without mathematics disabilities scored higher than groups with mathematics disabilities on scores that included variance in domain-general, verbal working memory, and visual-spatial working memory. Wilson and Swanson (2001) also concluded that both verbal and visual-spatial working-memory scores predicted mathematics performance and that these results support other studies that suggested that the working-memory system plays a critical role in mathematics development.

Wilson and Swanson (2001) attempted to control extraneous variables by limiting the study to people who exclusively had mathematics disabilities as opposed to overall learning disabilities across different domains (reading and mathematics). Their conclusions, therefore, suggested that there are some critical components of poor mathematics achievement that may not affect reading achievement. They suggested, with little data support, that poor conceptual knowledge may not be related to reading.

Internal validity of this study may be examined in two ways. The first is that of measures used to establish a mathematics disability. Traditional assessment for a learning disability should include measures of ability and achievement. The individuals in this study were only tested using an achievement measure of mathematics computation that may not reflect a mathematics disability accurately. Other factors such as ability level and opportunity to learn were not factored into the analysis. Second, this study was a very simplistic accounting of working memory and mathematics disabilities because only computation was measured and not applied mathematics skills.

External validity could be questioned because the sample was not described adequately. There was no information on the number of people in each age category, so some of the age-related data might have been based on a small sample within a particular age range. A random sample was not suggested that would have included a comparison group of participants without mathematics disabilities.

Geary, Hamson, and Hoard (2000) investigated the differences between students with learning disabilities and children without disabilities on measures of number comprehension and production skills, counting knowledge, arithmetic skills, working memory, phonetic representations of words and numbers, and spatial abilities. There were 114 participants (50 boys, 64 girls; M=82 months) from 5 elementary schools in working class neighborhoods. Children whose IQ were lower than a standard score of 80 or higher than 120 were excluded from the study. The 84 remaining participants (29 African American, 48 European American, and 7 other) were classified into 3 learning disabilities groups-- mathematics disabilities (MD), reading disabilities (RD), and reading and math disabilities (MD/RD). There was also a variable group and a normal group. These groups were assigned based on scores from the subtest of mathematics reasoning from the Wechsler Individual Achievement Test (WIAT) and the word attack and letter-word identification subtests of the Woodcock-Johnson Psycho-Educational Battery-Revised (WJR). Children in the MD/RD category had mean achievement below the 20^{th} percentile in both first and second grade; children in the MD category had mean mathematics achievement below the 25^{th} percentile; and the RD category had mean word-attack skills below the 22nd percentile. The normally achieving group had mean achievement scores above the 66th percentile and the variable group had at least one achievement score below the 35th percentile in one grade and one that was above the 34th percentile in the other grade.

The students were assessed using the following measures: number production and comprehension that required students to write numbers using visual and auditory presentations, counting knowledge with left-to-right and right-to-left presentations by a puppet, addition strategy assessment presented on a computer screen with single digit problems, articulation speed of one-syllable words, and Digit span and Mazes subtests of the Wechsler Intelligence Scale for Children (WISC-III). The participants were assessed in the Fall and Spring of both first and second grades.

There were a number of results reported investigating each aspect of the assessment. Those most relevant to the current study included strategy choice and errors on addition tasks in which an analysis of covariance (ANCOVA) revealed a statistically significant group difference for counting fingers, $F(4, 26)=3.08$, a small effect size of .38, confirmed by a post hoc contrast that found children in the MD/RD and MD groups, $F(1, 32)=12.53$, a small effect size of .28, committed more errors than the three remaining groups. When children were required to use a retrieval strategy only, there was a statistically significant group difference in the percentage of retrieval errors, $F(4, 76)=10.71$, a small effect size of .36, with a post hoc contrast indicating that children in the MD/RD group had more retrieval errors than the four remaining groups, $F(1, 78)=33.78$, a small effect size of .30. The measure used for working memory, Digit Span was analyzed using a mixed ANCOVA with a 5 (group) by 2 (type) by 2 (grade) design.

The ANCOVA indicated that there were statistically significant group, $F(4, 77)=4.84$, a small effect size of .20 and type effects $F(1, 77)=11.30$, a very small effect size of .13, with the type differences favoring digits forward rather than digits backward, and group favoring children with normal achievement contrasted with the MD/RD group.

The results of the study by Geary et al. (2000) provided information related to working memory for students with learning disabilities, particularly those who were classified as MD. These students displayed statistically significant differences in working memory skills as well as the resulting increase in errors in simple arithmetic calculation. This study, however, did not investigate the relationship of working memory with problem-solving skills or include information regarding different age levels. The current study will use two measures of achievement, calculation and applied problems.

Summary

Researchers have found statistically significant results when investigating working memory and its relationship with mathematics achievement in students with learning disabilities when compared with students without learning disabilities (Geary et al.,2000; Swanson & Jerman, 2006). The verbal-working-memory relationship with mathematics achievement seems to be consistent across ages (Swanson & Jerman, 2006; Wilson & Swanson, 2001). Geary et al. (2000) also suggested that MD students use developmentally immature strategies and have more procedural errors than students who are academically average. They have suggested that children with MD have more difficulty retaining information in working memory when engaged in other processes such as counting.

Working Memory and Second Language Learning

For students who are English Language Learners (ELL) at the community-college level, there are a number of issues involved in successfully completing a mathematics course. Cummins (1980) made a distinction between two levels of language learning that are relevant to content-area instruction. The first level is Basic Interpersonal Communicative Skills (BICS) that takes approximately 2 years to develop and Cognitive Academic Language Proficiency (CALP) that in many cases, takes 5 to 7 years to develop. CALP is necessary for tasks in mathematics, particularly comprehending textbooks. Opportunity to learn is also a factor in whether students who are second-language learners are afforded the opportunity in middle and high school to select classes that will provide adequate instruction to compete with native English speakers on assessments of mathematics skills (Lindholm-Leary & Borsato, 2005; Wang & Goldschmidt, 1999). Brown (2005) identified 8 reasons for the difficulty ELL students face in mathematics' achievement:

1. Mathematics is a language all its own,
2. Mathematics learning must be accrued and the achievement gap widens as students get older,
3. Mathematics vocabulary is not commonly used, so students are not exposed to the vocabulary in everyday life,
4. Syntax used in mathematics is complex and may be confusing,
5. ELL students are slower readers which affects mathematics performance,
6. Different cultures solve mathematics problems differently,

7. The interpretation of mathematics questions may be related to socioeconomic status, and
8. The cultural context of a mathematics problem may not be recognizable to ELL students.

Brown (2005) examined results of literacy-based performance assessments (LBPAs) that asked students to explain in writing how they solve mathematics problems. The sample included third-grade students from 25 Maryland school districts with four subgroups identified: ELL students with free and reduced meals (FARMs), fully English proficient (FEP) students with FARMS, ELL students with non-FARMS, and FEP students with non-FARM. Of the total 65,536 students who were administered the Maryland School Performance Assessment Program (MSPAP) in the year 2000, 742 were ELL students. After excluding students who had received special-education services and those with incomplete test scores in mathematics, the sample of ELL students included 260 with FARMs status and 232 with non-FARMS status (2 Native Americans, 168 Asian Americans, 218 Hispanic Americans, and 56 European Americans). The FEP group consisted of 53,025 students who took the mathematics portion of the MSPAP who were selected randomly to include 260 FARMs students and 232 non-FARMs students.

Independent-samples t tests were used by Brown (2005) to answer two research questions that examined the performance differences between ELL student and FEP students in first the overall mathematics examination and second in the mathematics communication subskill. The results indicated that there was no statistically significant difference between the ELL students and FEP students with the same FARMS status on the mathematics portion of the MSPAP. There was a statistically significant and very large practically important difference between ELL students and FEP students with the same non-FARMS status in mathematics (t[462] = -13.70, d = 1.27). For the second research question, Brown (2005) found a similar result, with no statistically significant mean difference between the ELL students and FEP students with the same FARM status on the mathematics-communication subskill but a statistically significant difference and large practical difference for those students with non-FARM status (t [319] = -7.66, d=.85). A two-tailed t test was used to analyze the significance of predictor variables and revealed that reading, writing, language usage, and FARMS were statistically significant. Ethnicity and gender were not statistically significant for both the ELL and FEP groups. A multiple linear regression indicated that FARMs status was a statistically significant predictor of mathematics achievement for FEP and ELL students. For ELL students, reading was a stronger predictor, followed by usage.

Brown (2005) interpreted the results of the study to indicate that SES (as determined by FARMs) was the strongest predictor of mathematics achievement. In the high-SES groups, however, the ELL factor was important in determining success on mathematics measures. Brown further attributed the lack of success in mathematics achievement to lack of background knowledge and academic language. The reported educational implications called into question the validity of the literacy-based performance assessments (LBPAs) to determine mathematics achievement in ELL students.

Students with English as a Second Language are a segment of the population in developmental mathematics courses at community colleges. There have been studies conducted to investigate the effect learning a second language has on working memory in both the students' first and second language, as well as the effect working memory in the students' first and second language has on learning in content areas such as mathematics.

Gutierrez-Clellen, Caladeron, and Weismer (2004) conducted a study of bilingual children (Spanish and English) and those with limited second-language proficiency to investigate if verbal-working-memory tasks differed across and within these groups. There were 44 participants in the study (26 boys and 18 girls) ranging in age from 7.3 to 8.7 years of Mexican American descent, 22 considered bilingual, 11 Spanish proficient-limited English, and 11 English proficient-limited Spanish. The categories of proficiency were established by parent and teacher ratings of both language use and perceived proficiency in both languages.

The children were administered two tasks: Competing Language Processing (CLPT) and Dual Processing Comprehension (DPCT), Spanish versions adapted by Gutierrez-Clellen and English versions developed by other researchers. The tasks involved recalling the last word spoken in a sentence for the CLPT and manipulating tokens to demonstrate comprehension in the DPCT. An ANOVA revealed that there were no statistically significant differences for the three groups in comprehension on the CLPT. On the DPCT, there were no statistically significant group differences. Cross-language performance was evaluated using paired-samples t tests in the bilingual children and unpaired-samples t tests between the Spanish-proficient and English-proficient groups that showed no statistically significant difference. The researchers also used the Pearson product-moment correlation to investigate the relationship between the languages (Spanish and English) and found the languages were statistically significantly intercorrelated for the same task ($r=.44$) for the CLPT and ($r=.48$) for the DPCT. For the different tasks, the DPCT was strongly correlated with the CLPT for Spanish (r=-.70). The DPCT, however, was not correlated with the CLPT for English (r=.31). The researchers found no statistically significant differences between fluent bilingual children and those with proficiency in only one language. They also found that differences in language processing between the two languages had no effect on processing. The researchers used this finding to support the idea that verbal working memory is related to a general ability to process information rather than a language-specific ability.

Other studies investigated mathematics and its relationship with working memory. Ardila et al. (2004) were interested in the effect that second-language interference had on 3 linguistic elements: syntactic comprehension, verbal memory, and calculation abilities. They designed two studies: in the first they examined syntax and in the second working memory and calculation skills of adults with Spanish as their first language (L1) and English as their second language (L2). In the second study, 69 graduate students ranging in age from 18 to 49 years (M=30.28, SD=7.97) were participants. The sample included 34 men and 35 women who had Spanish as their L1 but had used English for more than 3 years. Of these participants, 27 stated they preferred using Spanish and 39 said they preferred to use English, but all claimed to be fluent in both languages, rating themselves as 3 or above on a self-evaluation scale ranging from 1 (virtually nothing) to 5 (excellent).

There were eight verbal memory tests administered in both Spanish and English that were from the Wechsler Memory Scale (WMS) English version and Spanish translations: Logical Memory, Digits Forward-Backward, and Associate Learning, as well the Serial Verbal Learning Test (SVL) developed by the researcher in both languages. There were 8 calculation tasks that required verbal answers in the language indicated: successive subtraction from 100 (Spanish), one two-digit multiplication (Spanish), one division (Spanish), successive subtractions from 100 (English), one two-digit multiplication (English), one division (English), one numerical word problem (Spanish), and one numerical word problem (English). After the calculation test was administered, delayed recall of the WMS and SVL assessments was given in both Spanish and English.

The data were analyzed using t tests that compared Spanish and English performance and that assessed age of acquisition and preferred language effect. Although the researchers suggested that performance in Spanish was greater on the verbal memory tests, there were no statistically significant effects that supported that assumption. In fact, there were three statistically significant effects that favored English verbal memory: Delayed Logical Memory, Digits Forward, and Total Digits. The analysis of the calculation tasks included only the amount of time used to respond to the item prompt. It took longer for responses in English, with 3 of the tasks statistically significant: multiplication, division, and the numerical problem. The language effect was statistically significant for four tasks: Delayed Logical Memory, Digits Forward, Total Digits, and Total Words. Age of acquisition of L2 favored Spanish in the WMS Delayed Logical Memory and Delayed Associative Learning and the SVL Delayed Recall and Total Words. Age effect was seen in only WMS Digits Forward, with a statistically significant interaction in WMS Logical Memory and SVL total words. The analysis of preferred language revealed a statistically significant language effect for six measures: Delayed Logical Memory, Delayed Associate Learning, Digits Forward, Total Digits, SVL Total

Words, and SVL Delayed Recall. The preference effect was evident in WMS Digits Forward, Backward and Total, whereas a statistically significant interaction effect was seen in WMS Logical Memory, Delayed Logical Memory, Total Digits, and SLV Total Words. On the calculation tasks, 3 measures (multiplication, division, and numerical problems) were statistically significant for language effect in Spanish, with an interaction effect for multiplication and division.

The researchers interpreted the results of the calculation tasks to support their theory that bilinguals will use their native language to count and perform other numerical operations because these skills are not influenced by social linguistic context. In particular, the researchers emphasized the numerical word problem as an example of a task that required calculation as well as understanding of the problem conditions. They extended the results of their study to include a caution on the method of testing Spanish-English bilinguals in either language because results of the verbal memory and calculation tasks were mixed. Their suggestion that bilinguals be tested using both or either languages on some subtests would be difficult to implement and may not be supported by participant self-reporting of preference. Although there are some limitations associated with this study including the inclusion of only graduate students, the results may help to point out the influence of competing languages in a content area course that requires high levels of working memory such as mathematics.

Summary

Students who are second-language learners in mathematics classes face challenges in interpretation as well as working memory. An increased demand for working memory capacity to include holding computation in memory while trying to translate information presented in English were found to influence accuracy in mathematics tasks (Gutierrez-Clellen et al. 2004).

Conclusion

The studies in this review have established that there is a relationship between working memory, particularly verbal working memory, and mathematics. It is apparent that variables such as age, ethnicity, opportunity to learn (previous mathematics courses), second-language proficiency, mathematics anxiety, and learning disability status may be factors, along with working memory, that influence mathematics achievement. For this reason, measured variables investigated in this study involving students in community colleges will include working memory, mathematics anxiety and mathematics achievement.. There have been no other studies investigating the relationship of working memory and mathematics achievement in community-college populations. Because students in this group form a unique population with diverse abilities, ethnicity, ages, and language proficiency, this study will provide information regarding these student factors as well as working memory and influence on mathematics achievement and successful course completion. Future research can then target strategies that can assist in increasing success of students in community-college mathematics courses by using the information gained regarding variables that most influence mathematics achievement.

CHAPTER III

METHODOLOGY

This study investigated the relationship between working memory and mathematics achievement in developmental mathematics courses at the community-college level. In addition, mathematics anxiety that may contribute to extraneous load and affect access to working memory and mathematics achievement was considered. This section contains a restatement of the research questions, a description of the study design, sampling and data-collection procedures, data analysis, and human-subjects considerations. Included are sections on the reliability, validity, scoring, and administration procedures for the instruments used in the study.

Research Questions

The major research questions that were investigated are as follows:

1. What are the characteristics of students enrolled in two levels of community-college developmental mathematics courses?
2. Is there a difference between working memory measures for students in two levels of community-college developmental mathematics courses?
3. Is there a difference between mathematics anxiety for students in two levels of community-college developmental mathematics courses?
4. What is the relationship between mathematics achievement and working memory in students in community-college developmental mathematics courses?
5. Is the relationship between working memory and Applied Problems stronger than the relationship between working memory and Calculation for students in community-college developmental mathematics courses?

6. What is the relationship between mathematics achievement and mathematics anxiety in students in community-college developmental mathematics courses?
7. What is the relationship between mathematics anxiety and working memory in students in community-college developmental mathematics courses?

Research Design

The study was designed as correlational and used an intact sample of students enrolled in the first two developmental mathematics courses at a community college: Preparing for Algebra and Elementary Algebra. Participants were assessed for working-memory skills using the Wechsler Adult Intelligence Scale (WAISIII) subtests needed to obtain the Working Memory Index: Arithmetic, Digit Span, and Letter-Number Sequencing. Mathematics achievement was assessed using the Woodcock-Johnson Revised Tests of Achievement: Calculation and Applied Problems. The independent variable for the first two questions was the level of developmental mathematics courses. The dependent variables were working memory, mathematics achievement, and mathematics anxiety. A questionnaire regarding demographic information was administered that included inquiries regarding ethnicity, age, gender, number of mathematics courses completed successfully in high school, learning disability status, disability services in college, and English-as-a-second language status. A measure of mathematics anxiety, the revised Math Anxiety Rating Scale (MARS) was given in a group setting.

Participants

Participants in this study consisted of students in two levels of developmental mathematics courses: Preparing for Algebra (Level 1) and Elementary Algebra (Level 2), at a community college in Northern California. Student enrollment at the college is approximately 20,000 students, with 75 to 80% of the students entering the college without college-level skills in mathematics or language arts (Annual Report to the Community, 2007). The 150 potential participants form a diverse group of students regarding age, ethnicity, socioeconomic status, and gender. It was estimated that a sample of 60 students (30 from each level) would participate in the study. Table 1 displays the characteristics of students who participated in the study.

Most students were placed in these courses based on a college administered (individualized) placement test. The first course (Preparing for Algebra) focuses on arithmetic skills including whole number operations, fractions, decimals, negative numbers and pre-algebraic concepts. The second course, Elementary Algebra, focuses on fundamental algebraic operations, real numbers, first degree equations and inequalities, operations on polynomials, and factoring. The two courses are basic skills mathematics courses because their content is considered below college level.

Protection of Human Subjects

Protection of human subjects complied with the standards set by the University of San Francisco’s Institutional Review Board and the standards set by the American Psychological Association (2002). Written permission from the vice-president of instruction at the community college was obtained, as well as from the five instructors in the courses. Students in the selected sections of the developmental mathematics course were informed by a cover letter that their participation was voluntary and results were confidential. Students were asked to sign a consent form indicating their voluntary participation

Table 1

Characteristics of Community College Students Who Volunteered for the Study Broken Down by Course Level

Factor	Characteristic	Level 1 (n=28)	Percent	Level 2 (n=35)	Percent
Gender	Female	16	57	18	51
	Male	11	39	16	46
	Unreported	1		1	
Age (years)	Mean	28.48		21.78	
	SD	11.71		3.99	
	Range	18-55		18-31	
Ethnicity	African American	3	11	0	0
	Asian American	3	11	5	14
	European America	13	46	13	37
	Hispanic Americar	8	29	10	29
	Native American	0	0	1	3
	Other	0	0	3	9
	Unreported	1	4	3	9
ESL Status	Yes	8	29	12	34
	No	20	71	23	66
LD Status	Yes	11	39	8	23
	No	17	61	26	74
	Unreported			1	
Previous Math Courses	High School				
	Mean	2.44		2.97	
	SD	1.09		1.12	
	Range	0-4		0-5	
	College				
	Mean	1.43		1.66	
	SD	.69		.73	
	Range	1-3		1-3	

ESL=English as a Second Language; LD= Learning Disabilities

(Appendix C). In order to maintain anonymity, an identification number was assigned to each student, and all information was kept in a secure location accessible only by the researcher. Students who decided not to participate in the study completed an alternative activity during group-administered assessments that were conducted during one hour of the class sessions.

Instrumentation

The instrumentation for this study included a questionnaire that self-reported ethnicity, learning disability status, English proficiency, and number of mathematics courses previously completed (Appendix B). Mathematics anxiety was measured by a 25-item Math Anxiety Rating Scale (Alexander & Martray, 1989); The three subtests of the Wechsler Adult Intelligence Scale III (WAISIII); (The Psychological Corporation, 1997). Arithmetic, Digit Span and Letter-Number Sequencing were administered individually to derive the Working Memory Index. The mathematics achievement was measured by the Woodcock-Johnson Tests of Achievement-Revised (Woodcock & Mather, 1989) subtests of Calculation and Applied Problems.

Mathematics Anxiety Rating Scale

Suin, Edie, Nicoletti, and Spinelli (1972) developed a 98-item Mathematics Anxiety Rating Scale (MARS) that used a 5-point self-rating scale. The scale's descriptors (not at all, a little, a fair amount, much or very much) were designed to measure a person's level of anxiety on mathematics-related items. The score for the scale is the total for all of the items. The Cronbach's coefficient alpha for the MARS, as reported by Suin et al. (1972) was .97.

Alexander and Martray (1989) revised the MARS to include only the 25 items that had the highest factor pattern coefficients on three factors: mathematics test anxiety, numerical task anxiety, and mathematics course anxiety. Cronbach's coefficient alpha examining internal consistency was .96 for the 15 items on the mathematics test anxiety factor, .86 for the five items on the numerical task anxiety factor, and .84 for the five items on the mathematics course anxiety factor. The correlation between this abbreviated MARS and a 69-item revision of the original MARS was .93, with a two-week, test-retest reliability of .86. According to Alexander and Martray, a validity study demonstrated relatively high-to-moderate correlations between MARS-Abbreviated and the Speilberger's Test Anxiety Inventory and Spielberger's State-Trait Anxiety Inventory, and the math anxiety subscale of the Fennema-Sherman Mathematics Attitude Scales. although no correlations were given.

Wechsler Adult Intelligence Scale III (WAISIII)

The Wechsler Adult Intelligence Scale was intended as a measure of intelligence to assess a broad range of cognitive abilities. Wechsler defined intelligence as the "capacity of the individual to act purposefully, to think rationally, and to deal effectively with his environment" (The Psychological Corporation, 1997, p. 1). Measuring working memory, the ability to process and retrieve information is an important purpose of the WAIS-III because it is seen as a "moderating variable of learning" (The Psychological Corporation, 1997, p. 7). The three tests that make up the Working Memory Index are Arithmetic, Digit Span, and Letter-Number Sequencing. Arithmetic requires a verbal response (no visual prompt), and the examiner may repeat an item once. Digit Span requires a verbal response after the presentation by the examiner, with no repetition allowed of digits forward and backward. Letter-Number Sequencing requires a verbal response that sequences numbers and letters after oral presentation by the examiner; numbers in order first and then letters in alphabetical order. No repetition is allowed. To keep standardization of the test at its optimal, a script for the examiner is provided to be read verbatim. For more difficult to understand subtests, practice of some items is given. Practice is allowed for Digit Span (backwards only) and Letter-Number Sequencing.

The three subtests in the Working Memory Index are designed to provide information regarding essential aspects of verbal working memory. The Arithmetic subtest measures auditory attention and numerical reasoning. The Digit Span subtest measures short term auditory memory. The Letter-Number Sequencing subtest measures mental manipulation and sequencing of auditory information. The three subtests are believed to be components of a single measure of verbal working memory, the Working Memory Index. A standard score (Mean=100, SD=15) is computed for the Working Memory Index when used for diagnostic purposes. The current study used raw scores for each of the subtests and a total raw score for the correlation analysis.

The test-retest stability coefficient was calculated based on a sample of 394 participants in four pooled age groups with the mean retest interval of 34.6 days. The average test-retest stability coefficient across all age groups for the working memory index was .89. The coefficient for the arithmetic subtest is .86, the digit span subtest is.83, and the letter-number sequencing subtest is. 75. The split-half reliability of the Working Memory Index is .94.

The intercorrelations of the Working Memory Index subtests are as follows: Arithmetic and Digit Span, .52; Arithmetic and Letter-Number Sequencing, .55; and Digit Span and Letter-Number Sequencing, .57. The publishers of the WAIS III concluded that this is evidence of convergent validity of both the IQ and Index scores (The Psychological Corporation, 1997).

Woodcock-Johnson Tests of Achievement-Revised

The calculation subtest of the Woodcock-Johnson Tests of Achievement-Revised (Woodcock & Mather, 1989) measures the students' skill in mathematical calculation. Included in the subtest are the operations of addition, subtraction, multiplication, and division as well as combinations of these operations involving decimals, fractions, and whole numbers. Some geometric, trigonometric, logarithmic, and calculus operations are also included (Woodcock & Mather, 1989, p. 13). The calculation subtest includes concepts that are taught in both levels of developmental mathematics courses: whole number operations and pre-algebraic as well as algebraic calculations. Test reliability calculated by the split-half procedure was .94 for the norming sample that included 18 year olds and .93 for 30 to 39 year olds.

The Applied Problems subtest "measures the subject's skill in analyzing and solving practical problems in mathematics" (Woodcock & Mather, 1989, p. 14). In the Applied Problems subtest, the person is required to decide on appropriate operations as well as determine which extraneous information should be excluded in the calculations. Items are read to the students as well as the prompt presented in text. For some items, diagrams or pictures are part of the visual prompt. The reliability based on 250 participants at age 18 is .93 and .91 for 351 participants age 30 to 39.

The Calculation and Applied Problems subtests combine to provide the Broad Mathematics cluster score that is considered a broad measure of mathematics achievement. This score is reported as a standard score (Mean=100, SD=15). In this study raw scores for each of the subtests and a total raw score for the two subtests were used in the correlation analysis. The WJ-R provides a quick reference to age and grade equivalents in the test protocols related to the raw scores. An expected raw score for college-age students would be 39 and above for Calculation and 46 and above for Applied Problems.

The median reliability of Broad Mathematics is .97 for adults. Concurrent validity for the age 17 group was evidenced using a sample of 51 participants with various tests including the Peabody Individual Achievement Test (PIAT-Math, .74) and Wide Range Achievement Test (WRAT-R Math, .72)

Procedures

The researcher made an announcement at the beginning of the developmental mathematics class asking for student participation in the study. After the announcement, students received a cover letter and an informed consent form. The investigator remained in the classroom to answer questions from the students and instructors. All students were asked to return the informed consent form in an envelope that was placed at the front of the classroom, regardless of their decision to participate or not participate in the study. Students who participated in the study were assigned a number beginning with 1 to 100 to insure confidentiality. Thereafter, only the number appeared on the questionnaire and assessments in place of a name. Participating students were given a copy of the consent form to keep and a copy of the Research Subject's Bill of Rights.

Students who agreed to participate were given the following assessments in one group session and one individual session. The group sessions consisted of the following:

1. A questionnaire that provided demographic information (Appendix B).
2. The Mathematics Anxiety Rating Scale-Abbreviated (MARS)
3. The Calculation subtest of the Woodcock-Johnson Tests of Achievement-

Revised as one measure of mathematics achievement. (WJ-R).

During the individual sessions, the following were administered:

1. The three subtests of the Wechsler Adult Intelligence Scale-III (WAIS III) (Arithmetic, Digit Span, Letter-Number Sequencing) that determine the Working Memory Index, were administered in a one-to-one session with the researcher.

2. The Applied Problems subtest of the Woodcock-Johnson Tests of Achievement administered in a one-to-one session with the researcher or research assistant.

Data collection occurred over a period of 3 weeks (Table 2). The five sections of the first level, Preparing for Algebra (Math 230) met at the same time 5 days per week. The two sections of Elementary Algebra (Math 101) met for two hours at different times, one in the morning, one in the afternoon. The first week consisted of recruitment of students, and completion of the student questionnaire and Math Anxiety Rating Scale-Abbreviated during a class session that lasted 15 minutes. During the second week, the group session comprised a total of 15 minutes during a class period for the WJ-R Calculation subtest. Also during the second week, the individual sessions began and consisted of approximately 30 minutes per student for both the WAIS-III and WJ-R. The researcher and research assistant met with each student in a separate location near the student's classroom to complete the administration of these two measures. Approximately eight students per day from the 2-hour classes participated from the Preparing for Algebra (level 1) classes. Because testing occurred over 4 days, approximately 32 students were assessed during that time. The two sections of the Elementary Algebra class met at different times so 16 students were assessed each week. During Week 3, any students who were not assessed scheduled a time to meet with the researcher and complete the assessment.

Qualification of Researchers

The primary researcher is qualified to administer the WAISIII through a required training session and 7 years of experience administering and interpreting the WAISIII to students in community college. She is a Learning Disability Specialist recognized by the State of California as a qualified administrator of assessments designed to determine eligibility for learning disability services at the community-college level. Additionally,

Table 2

Procedures for Administration of the Components of the Study

Classes	Week 1	Week 2	Week 3
Math 230 Preparing for Algebra (Level 1) (5 sections meet at the same time)	Recruitment Student Questionnaire Math Anxiety Rating Scale	A-Group: WJ-R Calculation Subtest R-Individualized WAIS III	A-Individualized WJ-R: Applied Problems. R-Remaining students who need to complete WAIS III.
Math 101 Elementary Algebra (Level 2) (1 morning section, 1 afternoon section)	Recruitment Student Questionnaire Math Anxiety Rating Scale	R-Group: WJ-R Calculation Subtest A-Individualized WJ-R Applied Problems	R-Individualized WAIS-III. A-Remaining students who need to complete WJ-R.

R=Researcher, A= Assistant

the researcher has administered the WJ-R for 20 years as a special education teacher in the K-12 system and Learning Disability Specialist at the community-college level. She has trained various assistants in the use and administration of the WJ-R. The researcher has a master's degree in Special Education and is a doctoral candidate in Learning and Instruction.

The research assistant has an associate's degree in special education and a bachelor's degree in educational technology. She has worked as an assistant on various projects at the community college with the researcher. The research assistant has been trained in the administration of the WJ-R by the researcher and assisted in the group administration of the calculation subtest and individual administration of the applied problems subtest.

Data Analysis

This study was designed to investigate the relationship between working memory and mathematics achievement. The first analysis conducted was an independent-samples t test to investigate if there was a statistically significant difference in the working memory total score between the two groups (first and second developmental mathematics courses). The t test provided an answer to the second research question investigating whether there was a statistically significant relationship between working memory and level in developmental mathematics courses.

The second analysis was an independent-samples t test to investigate if there was a statistically significant difference between the two groups on the Math Anxiety Rating Scale (MARS)-Abreviated. The t test provided an answer to the third research question.

The third analysis was a correlation using the following variables: raw scores from the working memory subtests of the WAISIII: arithmetic, digit span, letter-number sequencing; raw scores from the broad mathematics subtests of the WJ-R: calculation, and applied problems. A Pearson product-moment correlation coefficient was computed between each possible pair of variable measurement. The Pearson product-moment correlation coefficient was used to show the relationship between the variables. The fourth research question was answered by examining the relationship between working memory and mathematics achievement. For the fifth question, the correlations between subtests of working memory and calculation and between working memory and problem solving were compared using the test for dependent-sampled correlation coefficients.

The sixth question was answered by viewing the relationship between mathematics anxiety and mathematics achievement. The seventh question required the examination of the relationship between mathematics anxiety and the working memory measure.

Summary

This study investigated the relationship between working memory and mathematics achievement for students in developmental-level mathematics courses in community college. For this reason, an intact sample of students in two levels of developmental mathematics courses was used so that there should be variance in both working memory and mathematics achievement measurements. The students completed a demographic survey that provided information on the age, gender, ethnicity, previous mathematics courses completed, placement level, English language proficiency, and disability status. Portions of two standardized assessments that are valid for adult students, WAIS III and WJ-R, were administered in individual and group sessions. An abbreviated mathematics rating scale was completed by the students in a group session. The scores from these measures were correlated to view the relationship between working memory, mathematics anxiety, and mathematics achievement for this sample of community-college students. Further research may address this relationship to include other factors that influence working memory and the effect on mathematics achievement. In addition, it may be possible to design instructional and personal strategies that increase accessibility to working memory, decrease mathematics anxiety, and increase success in mathematics classes at the developmental level.

CHAPTER IV

RESULTS

The purpose of the study was to investigate the relationship between working memory and mathematics achievement for students in developmental mathematics classes in community college. This study examined differences between two levels of mathematics classes regarding working memory and mathematics anxiety and the relationship between working memory and mathematics achievement. The independent variable for the first two questions was the level of developmental mathematics courses. The dependent variables were working memory, mathematics achievement, and mathematics anxiety. The remaining four questions used a correlational analysis investigating relationships between working memory, mathematics achievement, and mathematics anxiety. The results of the data analysis are reported for each of the six research questions in the following section that contains the descriptive and inferential statistics related to six research questions.

Descriptive Statistics

The analysis in this section is organized according to the research questions with supporting data contained in the tables.

Question 1: What are the characteristics of students enrolled in two levels of community-college developmental mathematics courses?

A total of 63 students volunteered to participate in the study: there were more students in Level 2 (35 in Elementary Algebra) than in Level 1 (28 in Preparing for Algebra). Both courses are considered developmental level (precollege level) at a Northern California community college. The description of the students in the sample is displayed in Table 1. The sample was representative of the population of the community college with slightly more female (53 %) than male students (43%). The mean age of students in the Level 1 class was approximately 28 years, and the Level 2 class approximately 22 years. The effect size for this age difference is .77, which is a medium to large effect size. In the Level 1 class, 29% of the students reported that English was not their first language and 34% of the students in Level 2. Of the 28 students in Level 1, 39% reported having a learning disability and 23% of 35 students in Level 2 reported having a learning disability. These percentages are much higher than the estimated 7% of the population in college (Henderson, 2001). The mean number of mathematics classes taken in high school for the Level 1 students was 2.44 (SD=1.90) and 2.97 (SD=1.12) for the students in Level 2. The effect size for this difference is .47, which is a small to medium effect size.

The Woodcock-Johnson Tests of Achievement-Revised provides a scoring table that allows comparisons with student scores and age and grade equivalents. The age and grade equivalents for the student raw scores measured in the sample are displayed in Table 3. For students in the developmental mathematics courses, the mean raw score for Calculation was 36.00, which corresponds to an age equivalent of 17-0 (17 years, 0 months) and a grade equivalent of 10.7. The mean raw score for Applied Problems was 41.95, which approximately corresponds to an age equivalent of 15-3 (15 years, 3 months) and a grade equivalent of 10.1. Level 1 students’ mean raw score for Calculation was 33.70, which approximately corresponds to an age equivalent of 14-9 (14 years, 9 months) and a grade equivalent of 9.4; and Applied Problems mean raw score of 40.58, which approximately corresponds to an age equivalent of 14-6 and grade equivalent of

9.4. Level 2 students mean raw score for Calculation was 37.69 that corresponded approximately to age 21 and grade equivalent of 12.2. The mean raw score for Applied Problems for Level 2 students was 43.04 that corresponded to age 16-1 and a grade equivalent of 10.8. In all cases the mathematics achievement scores of the Level 2 students were higher than the achievement scores of the Level 1 students.

Table 3

Mean Raw Score Comparison of Sample of Students in Developmental Mathematics Courses with Suggested Age and Grade Levels for Mathematics Achievement from the Woodcock-Johnson Tests of Achievement by Course Level

	Calculation	Applied Problems
Level 1 (Mean)	33.70	40.58
Age Equivalent	14-9	14-6
Grade Equivalent	9.4	9.4
Level 2 (Mean)	37.69	43.04
Age Equivalent	21-0	16-1
Grade Equivalent	12.2	10.8
Total (Mean)	36.00	40.58
Age Equivalent	17-0	15-3
Grade Equivalent	10.7	10.1

Inferential Statistics

In this section, results of data analysis examining differences between Level 1 and Level 2 students regarding working memory, mathematics achievement and mathematics anxiety are presented.

Question 2: Is there a difference between working memory measures in students in two levels of community-college developmental mathematics courses?

The second research question examines if there is a statistically significant difference between the working memory measures of the sample in the two levels of developmental courses Level 1: Preparing for Algebra and Level 2: Elementary Algebra. Table 4 displays the means and standard deviations for the total and 3 subtests of the WAIS III. On average, the students in Level 2 have a higher mean on each of the subtests and the total working memory test. The range of scores for the total working memory measure for the Level 1 students was 24 to 49; the range of scores for Level 2 was 23 to 59. None of the differences for the measures of working memory (total or subtests) are statistically significant. For the rest of the analysis, therefore, the scores of the students for Level 1 and Level 2 were combined to examine the relationship between working memory and mathematics achievement as well as working memory, mathematics anxiety, and mathematics achievement.

Table 4

Means, Standard Deviations, and t-test Results for Total Working Memory Raw Scores and Subtest Raw Scores

WM Measure (WAIS III)	Statistic	Level 1 (n=19)	Level 2 (n=25)	t	d
Arithmetic	Mean	10.47	11.00	0.54	.17
	SD	3.32	3.11		
Digit Span	Mean	15.84	17.16	1.19	.36
	SD	2.87	4.11		
Letter/Number	Mean	9.84	10.32	0.60	.19
Sequencing	SD	2.39	2.75		
Total Working Memory	Mean	36.16	38.48	1.02	.31
	SD	6.64	8.07		

*Statistically significant at the .05 level when overall error rate was controlled

Assumptions

Because a t test was used to view the differences between the two groups regarding working memory, mathematics achievement, and mathematics anxiety, key assumptions were met. Independent samples were used, that is there were no participants that were in both levels of the mathematics courses. The histograms showed normal distributions with Level 1 students having a slight positive skew in mathematics achievement. Homogeneity of variance was checked to determine if the standard deviations were within a 2 to 1 ratio. The ratios for the standard deviations of the working memory measures were all approximately within a 2 to1 ratio, as were those in the Mathematics Anxiety Rating Scale-Abbreviated.

Question 3: Is there a difference between mathematics anxiety in students in two levels of community-college developmental mathematics courses?

Table 5 displays the statistics for the third research question that examines whether there is a statistically significant difference between the two developmental levels regarding mathematics anxiety. The mean raw scores and standard deviation for the Math Anxiety Rating Scale (MARS) are displayed for each group. The range of raw scores for Level 1 students was 31 to 120; the range for Level 2 students was 25 to 96.

Table 5

Means, Standard Deviations, and t-test Results for the Mathematics Anxiety Rating Scale-Abbreviated (MARS)

Scale (Raw Score)	Statistic	Level 1 (n=28)	Level 2 (n=35)	t	d
MARS	Mean	66.71	57.34	-1.63	-.40
	SD	24.98	20.75		

*Statistically significant at the .05 level when overall error rate was controlled

Students in the sample reported by Alexander and Martray (1989) had a mean total score of 55.34 which suggests that the sample of students in the current study reported higher mathematics anxiety ratings. There was a difference between the two levels of mathematics courses on the mathematics anxiety measures, but it was not statistically significant indicating a small effect size on this measure (d=.40). Cohen's d was used to interpret the effect size in which d=.20 is a small effect size, d=.50, a medium effect size and d=.80 is a large effect size. The level of anxiety in students in the second level is less than that of students in the first level.

Correlational Analysis

The research questions in this section were answered using a correlational analysis.

Assumptions

Assumptions for the correlation coefficients were tested and met. The scatterplots for each level and total samples were linear for the working memory and mathematics achievement at both levels of courses and total. Inspection of the scatterplots for working memory and mathematics achievement revealed no outliers.

Question 4: What is the relationship between mathematics achievement and working memory in students in community-college developmental mathematics courses?

Before examining the correlation between mathematics achievement and working memory, the descriptive statistics for mathematics achievement that were used for research question 4 are presented in this section. Table 6 contains the means, standard deviations, t-test results, and effect sizes for the mathematics achievement measures based on the administration of the Woodcock-Johnson Tests of Achievement-Revised. The Mathematics Achievement score is the total raw score of both the Calculation and Applied Problems subtests. For all measures, the mean for the Level 2 students is higher than that of the Level 1 students, which would be expected. The range of raw scores for Level 1 students was 17 to 40 for Calculation and 34 to 49 for Applied Problems. For

Level 2 students, the range of raw scores was 32 to 46 for Calculation and 34 to 52 for Applied Problems.

Table 6

Means and Standard Deviations for Total Math Achievement (MA), Calculation, and Applied Problems Broken Down by Course Level

Measure		Level 1 (n=19)	Level 2 (n=25)	Total	t	d
Mathematics	Mean	74.32	80.58	77.81	2.71*	.83
Achievement	SD	8.14	7.04	8.09		
Calculation	Mean	33.70	37.69	36.00	3.08*	.91
	SD	5.29	3.59	4.75		
Applied	Mean	40.58	43.04	41.95	1.60	.49
Problems	SD	4.34	5.50	5.11		

*Statistically significant at the .05 level when overall error rate was controlled.

The correlations for all students are presented in Table 7. From this analysis, there is a strong relationship between total working memory and Applied Problems according to Cohen's classification of strength of r values: r=.50 strong; r=.30 moderate, and r=.10 weak (Weinberg & Abromowitz, 2002). Digit Span shows a moderate relationship with

Table 7

Correlations for All Students Between Working Memory and Mathematics Achievement Measures

Measure	Total MA (n=44)	Calculation (n=44)	Applied Problems (n=44)
Working Memory	.46*	.09	.64*
Arithmetic	.47*	.05	.71*
Digit Span	.30	.09	.39*
Letter/Number	.33*	.07	.45*

*Statistically significant at .05 level when overall error rate is controlled

Applied Problems. A strong correlation was evident between the Arithmetic subtest and the Applied Problems subtest and a moderate correlation between the Letter-Number Sequencing subtest and Applied Problems.

Question 5: Is the relationship between working memory and Applied Problems stronger than the relationship between working memory and calculation for students in community-college developmental mathematics courses?

Table 7 displays the correlation between working memory and applied problems as well as working memory and calculation. From the analysis, it is shown that the relationship between working memory and applied problems is statistically significant for total working memory as well as the three subtests. The correlation between total working memory and applied problems is .64, a strong correlation compared with the correlation between total working memory and calculation skills (r=.09), a weak correlation. Each of the subtests showed a similar pattern, with moderate to strong correlations with applied problems, but very weak correlations with calculation skills. The difference in the relationship between working memory and applied problems and the relationship between working memory and calculation skills was tested using the test for dependent correlation coefficients (Glass & Stanley, 1970, pp. 313-314). The result is statistically significant (z=3.67), which indicates that there is a stronger relationship between working memory and Applied Problems than between working memory and calculation for students in community-college developmental mathematics courses.

Question 6: What is the relationship between mathematics achievement and mathematics anxiety in students in community-college developmental mathematics courses?

Correlations for the relationship between mathematics achievement and mathematics anxiety measured by the Math Anxiety Rating Scale-Abbreviated (MARS) are displayed in Table 8. From this analysis, there was a weak correlation between Mathematics Achievement and the Math Anxiety Rating Scale. This relationship is not statistically significant.

Question 7: What is the relationship between mathematics anxiety and working memory in students in community college developmental mathematics courses?

The correlation between mathematics anxiety and working memory is displayed in Table 8. This relationship is not statistically significant.

Table 8

Correlation between Mathematics Anxiety and Mathematics Achievement and Mathematics Anxiety and Working Memory

Measure	Total (MARS) (n=44)
Mathematics Achievement	-.26
Working Memory	-.17

*Statistically significant at .05 level when overall error rate is controlled

Additional Analysis

One analysis, not contained in the research questions, was the correlation between age and the three measures used in the study. Because there was a statistically significant difference between the two classes in terms of age, it was important to view the relationship between age and the three measures. Results of this analysis are displayed in Table 9. From the table, it is shown that there are very weak correlations between age and mathematics anxiety and age and mathematics achievement. None of the correlations are statistically significant.

Table 9

Pearson Product-Moment Correlation Coefficient Between Age and Mathematics Anxiety, Working Memory, and Mathematics Achievement

	Mathematics Anxiety (n=55)	Working Memory (n=44)	Mathematics Achievement (n=44)
Age	.09	.18	.03

*Statistically significant at .05 level when overall error rate is controlled

Summary of the Results

There was no statistically significant difference between the working memory measures of the two groups of students. There was not a statistically significant difference between the mathematics anxiety measures of the students in the two levels of developmental mathematics classes. In all cases across all working memory and mathematics achievement measures, students in the Level 2 classes had a higher mean than those in the Level 1 classes. The mean of the mathematics anxiety ratings were lower in the Level 2 students than the Level 1 students

Because the t test examining the difference between the two levels of students was not statistically significant, the groups were combined for the correlations. The correlation for total working memory measures and Applied Problems mathematics achievement was statistically significant for all students. There was a statistically significant relationship between the arithmetic subtest, as well as the letter-number sequence subtest of working memory and mathematics achievement Applied Problems. There was not a statistically significant relationship between total working memory or any working memory subtests and the calculation portion of mathematics achievement.

There was a weak relationship between mathematics anxiety and working memory, as well as a weak relationship between mathematics anxiety and mathematics achievement. This relationship was negative suggesting higher mathematics anxiety correlated with lower scores in working memory and mathematics achievement.

Because the difference in age between the two classes was statistically significant, an additional analysis was conducted not related to the research questions. From this analysis there was not a statistically significant correlation between age and working memory, between age and mathematics achievement, or between age and mathematics anxiety.

CHAPTER V

DISCUSSION OF RESULTS

Results of the research regarding the relationship of working memory and mathematics achievement for the current study are discussed in this chapter. The purpose of the study was to investigate the relationship between working memory and mathematics achievement for students enrolled in developmental mathematics courses in community college. First, a restatement of the problem and summary of the study are presented. Next, the limitations of the study are explained. Finally, a discussion of the findings and the implications for mathematics instruction in community colleges are given.

Summary of the Study

A sample of 63 students from two levels of developmental mathematics courses at a Northern California community college were assessed using a variety of measures. First, the students were given a questionnaire in order to describe characteristics of students enrolled in developmental mathematics courses. Next, the working memory component of the Wechsler Adult Intelligence Scale (WAISIII) was administered individually to each student in order to determine auditory working memory abilities. Data analysis using an independent-samples t test indicated that there was no statistically significant difference between the mean of the working memory of the two levels of developmental mathematics courses. The next measure, the Mathematics Anxiety Rating Scale-Abbreviated (MARS) was completed by the students in a group session. There was no statistically significant difference between the two levels regarding this self-reported rating. Because neither the working memory nor mathematics anxiety measures evidenced a statistically significant difference between the two developmental mathematics levels, the remaining correlational analysis was based on the combined two levels.

Students were assessed for mathematics achievement using the Woodcock-Johnson Tests of Achievement-Revised (WJ-R) in a group session for the Calculation subtest and an individual session for the Applied Problems subtest. The correlations for the relationship between auditory working memory and applied problems were statistically significant suggesting that lower working memory scores correlated with lower problem-solving skills. The relationship between working memory and calculation skills was not statistically significant. There was also not a statistically significant relationship between mathematics anxiety and mathematics achievement or mathematics anxiety and working memory.

Many students in community colleges are unable to complete the required mathematics sequence in order to achieve an associate of arts degree or transfer to a 4-year college (Research and Planning Group, 2005). A number of factors may combine to contribute to this problem including learning disabilities (Swanson & Jerman, 2006), second language (Gutierrez-Clellen et al., 2004), number sense (Gersten & Chard, 2001), opportunity to learn (Bettinger & Long, 2006), and mathematics anxiety (Ashcraft & Kirk, 2001). The current study, however, concentrated on the relationship of working memory to mathematics achievement in the community college population placed in developmental (remedial) courses. A relationship was found between working memory and mathematics achievement in previous studies in elementary school populations as well as limited studies with college-age students (McGlaughlin, Knoop, & Holliday., 2005; Swanson, 2006). The current study examined auditory working memory of students placed in remedial mathematics courses. Most students in the classes completed a college-administered placement test that indicated that developmental courses needed to be completed before students could advance to required college-level mathematics courses. The first course, Preparing for Algebra (Level 1) begins at a basic, elementary-school level, with addition, subtraction, multiplication, and division of whole numbers. The course progresses through a series of modules that include fractions, decimals, positive and negative numbers, and beginning algebraic concepts. Elementary Algebra (Level 2) introduces fundamental algebraic operations, real numbers, first-degree equations and inequalities, operations on polynomials, and factoring. Student success rates in these courses (grade C or better) in the past have been approximately 50% (Research and Planning Group, 2005). This study included the students in these courses in order to view the relationship between working memory and mathematics achievement in the developmental courses in order to identify possible factors that influence success. Previous studies did not include students within developmental mathematics courses in postsecondary settings (Ashcraft, 2001; McGlaughlin et al., 2005), but rather volunteers from the college population.

There was a statistically significant difference in the mean age of the students in the two courses. Although the mean age of students in Level 1 was greater than that of the Level 2 students, the students over 25 years of age in Level 1 constituted 43% of the sample, similar to that of the statistics presented by the U.S. Bureau of Census (USBC) for 2002 for community colleges. Only 22% of the students in Level 2 were above age 25. This difference in age may have been due to the willingness of the older students in the Level 1 classes to volunteer for this study, as well as smaller numbers of older students in Level 2 classes. Wilson and Swanson (2001) suggested that there was no statistically significant correlation between working memory and mathematics as a function of age or ability group. Cherry, Elliott, and Reese (2007) contended that older adults show a greater decline in performance on visuospatial tasks than they do on verbal tasks. Because the tasks in the current study are verbal tasks and most of the students are under age 30, the difference in ages should not affect the results of the study. Figure 2 shows the percentage of students enrolled in community colleges by age.

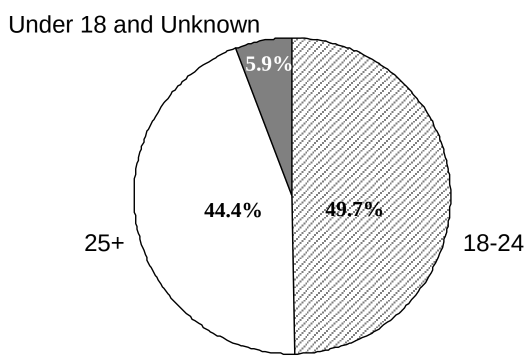

Figure 2. Enrollment in two-year colleges by age group (USBC, 2002).

A second statistically significant difference was the number of mathematics classes taken by the students in high school. Students in Level 1 enrolled in an average of 2.44 classes in high school, whereas the students in Level 2 enrolled in an average of 2.97 classes in high school. The difference in number of mathematics courses completed is an important factor in placement tests and eventual placement in basic skills courses. The following is a review by the Research Planning Group (2005) of the importance of consistency in mathematics requirements in high school:

> At the national level, 44% of those who started in community colleges in the 1990s did not reach Algebra 2 in high school, compared to 11% of

> those who entered four-year colleges. In addition, 55% of these students must take two or more remedial courses and 72% of those who take two or more remedial courses earn no credential whatsoever. The extent to which community colleges, working with high schools, can move more secondary school students to the level of Algebra 2 and beyond will signal a major change in academic momentum, and substantially reduce remediation at the postsecondary level. (p.5)

Placement is an issue that needs to be explored further as both a reason for success and a hindrance to success in remedial education. A study reported by Long (2005) showed that the likelihood of completing a degree increased by nine percent for students who took remediation classes when recommended. The hindrance was viewed in the approximate success rate of 10% for students beginning at the developmental level of mathematics courses (Research and Planning Group, 2005). An approach that Long (2005) recommended was early placement testing with results shared with students so that they can learn competencies before entering college to avoid remedial courses. The placement examination taken when entering college, in many cases, is the gatekeeper to success for students who are not prepared to enter college-level classes.

Limitations

The limitations of this study are discussed regarding the sample and methodology. These limitations may have affected the results of the study and should be considered in future research.

Sample

The first limitation of this study concerns the particular sample used. Students volunteered to participate in the study that limited the numbers as well as random sampling. Results, therefore, could be questioned as representative of the developmental mathematics population. For example, the large percentage of students who self-reported learning disabilities is far more than studies investigating prevalence of learning disabilities in postsecondary institutions (Henderson, 2001). There were 39% of the students in Level 1 who reported being diagnosed with learning disabilities. The highest estimate of students with learning disabilities in postsecondary institutions is 11% (Boswell & Wilson, 2004). There are, however, serious questions about the accurate numbers of students with disabilities in postsecondary settings because students choose not to take advantage of disability services in college. The questionnaire used in the study, because it was confidential, may have encouraged students to report previous identification of a learning disability. The alternative view is that students who previously had been identified as having a learning disability were more comfortable volunteering for a study in which individualized testing was used. Some students involved in the study expressed frustration with their lack of success in the developmental courses. These students may have volunteered for the study in an attempt to discover a reason for their lack of success, even though it was stressed that the study itself would not provide individual answers or results.

Students who self-reported that English was their second language made up 34% of the sample in Level 2 and 29% of the students in Level 1. This language difference could have an effect on the results of both the working memory and mathematics achievement measures because both assessments were normed on native English speakers and, therefore, could indicate test bias. Ardila et al. (2004) stressed the importance of testing in the student’s native language whenever possible to minimize language effects.

Another limitation is the sample size. Once students signed up for the study, many (15 students) did not complete all measures because the individualized portions required appointments of 30 minutes or more. This limitation did not allow the researcher to use a correlational analysis for individual levels (Level 1 and Level 2) because a sample size of 30 or more is needed for valid and stable correlations. This limitation was due in part to the intense level of assessment used in the study. The primary purpose of the study was to investigate the relationship between working memory and mathematics achievement. It was important, therefore, to obtain a complete measure of mathematics achievement that included individualized administration of a problem-solving measure, the Applied Problems subtest of the Woodcock-Johnson Tests of Achievement (WJ-R). The individualized administration minimized reading differences and allowed students to use as much time as needed to complete the measure.

The motivation for participating in this study and effort for the working memory and mathematics achievement measures may not be optimal because the reward for participation was not apparent to the student. In the future, consistently offering extra credit or individualized meetings with students regarding the results of the assessments, may increase motivation for participation.

Methodology

Auditory subtests were used to measure working memory and, therefore, did not provide information regarding the visuospatial component of working memory. This limitation was imposed in the study to view the effects specifically of auditory working memory on mathematics achievement. Research in the area of working memory has shown the effects of visual working memory as well in mathematics (Swanson & Jerman, 2006). This limitation should not have any adverse effects if the working memory measures in the study are reported consistently as auditory working memory or verbal working memory.

Because the measures in this study are norm-referenced standardized assessments, and not curriculum-based, all skills presented in the developmental mathematics courses may not be represented. It is probable that higher-level mathematics skills were present in the assessments than are expected competencies in the developmental mathematics courses.

Discussion of Research Questions

The first research question was answered with a description of students in two levels of developmental mathematics courses at a community college in California. The student sample had a high proportion of students with self-reported learning disabilities (30 %) and English as a Second Language status (32 %). The students in both levels of developmental mathematics courses had mathematics achievement scores that were lower than would be expected for students regarding age and grade level according to the Woodcock-Johnson Tests of Achievement-Revised (Woodcock & Mather, 1989). This age equivalent is a “level of development” score that “reflects the subject’s performance in terms of the age level in the norming sample at which the average score is the same as the subject’s score” (p. 60). For students in the Level 1 class, the mean raw score for Applied Problems corresponded to an age equivalent of 14-6. For Level 2 students the mean raw score for Applied Problems corresponded to an age equivalent of 16-1.

The second research question regarding differences in working memory between the students in both courses did not show a statistically significant difference. Students in the first two developmental courses did differ in their working memory abilities, but this difference did not reach statistical significance. The effect size for this analysis, .31, for the total score did show there was some weak practical significance. It had been expected that working memory might be a factor in predicting placement in levels of courses in the community college. Because this difference did not reach statistical significance, the two levels of courses were combined for the correlational analysis.

The third research question examined the difference between the two groups regarding mathematics anxiety. It was hypothesized that the groups would differ in how the students rated themselves concerning the anxiety felt in mathematics-related situations. Students in both groups rated themselves higher in mathematics anxiety than the students in the Alexander and Martray (1989) study that constructed the Mathematics Anxiety Rating Scale-Abbreviated. The 517 participants in that study were enrolled in psychology courses at the university level and had a mean raw score of 55.34. For the students in the developmental mathematics classes, there was some difference between the two groups, with Level 2 students rating themselves lower in overall mathematics anxiety, but the difference did not reach statistical significance. The ratings of the students in the two groups, Level 1 and Level 2, were combined for the correlational analysis with working memory and mathematics achievement for the remaining research questions

The fourth and fifth research questions involved the relationship between working memory and mathematics achievement. It is apparent that there is a difference, not only between student groups, but also within the measures used. Cohen’s classification of Pearson product-moment correlation values was used to interpret the correlations (Weinberg & Abromowitz, 2002), in which r=.50 and above is strong, r=.30 and above is moderate, and r=.10 and above is weak. For example, the correlation between working memory and total mathematics achievement is .46, a moderate correlation. The correlation between working memory and applied problems for all students, however, is .64, a strong correlation.

Both the working memory and mathematics achievement assessments require that a person manipulate information. In the case of pure recall, as in the digit span subtest of working memory, there is a weak correlation between that subtest and calculation and applied problems. Digit span can be considered a short-term memory task because there is no manipulation of information except in the digits backward portion of the subtest. All of the correlations with calculation skills are weak across the total sample. The weak relationship could be interpreted as calculation having less in common with the working memory tasks in that much of the memory used for the calculation tasks is stored in long-term memory. Some of the calculations are simple addition-and-subtraction tasks that are often automatic tasks. Tasks in the applied problems subtest require the student to concentrate on information to solve the problems and disregard irrelevant information. The central executive (Baddely, 2000) controls the storage and manipulation aspects of working memory needed for solving these problems. There are no time limits and students are presented the prompts in written as well as oral formats to reduce effects of reading ability. As in research conducted by Pawley et al. (2005), the tasks in the Applied Problems subtest have high element interactivity that is associated with working memory overload. For students in the developmental courses, all problem-solving tasks would be considered high in element interactivity. In cognitive load theory, high element interactivity affects the intrinsic load in that one element relies on the other elements in order to solve the problem. Any extraneous load (due to poor instructional design for example) placed on the student, therefore, would have a detrimental effect on learning (May, 2005).

The last two research questions involved mathematics anxiety and the relationship between self-reported anxiety and working memory and anxiety and mathematics achievement. There was a negative relationship (r=-.26) between mathematics anxiety and mathematics achievement: higher mathematics anxiety was correlated to lower mathematics achievement. This finding supports those from Ashcraft and Krause (2007) studies that have shown this relationship. The correlation between mathematics anxiety and working memory (r=-.17) was not as strong as the correlation (r=-.40) found by Ashcraft and Krause (2007). This difference supports a study conducted by Beilock and Carr (2005) that suggested that students only at the higher levels of working memory and under pressure to perform in mathematics are influenced by mathematics anxiety. The researchers' study of 93 college undergraduates showed that only students who were high in working memory capacity experienced decrements in problem solving and also working memory under pressure situations.

Conclusions

One of the key findings in this study is that working memory does have a relationship to mathematics achievement in community-college developmental mathematics courses. The correlation is strongest regarding applied-problem skills that require manipulation of information in order to solve problems. Although there was not a statistically significant difference between levels of mathematics courses regarding working memory, disregarding the individual courses, the relationship of working memory and Applied Problems was statistically significant.

There was a statistically significant difference between the two levels of mathematics courses regarding mathematics achievement. Although this difference was assumed based on placement tests previously administered to the two groups qualifying the students for the two levels, it was confirmed with the current assessment, the Woodcock-Johnson-Revised Tests of Achievement (WJ-R). Within the groups, however, it appeared the relationship between working memory and mathematics achievement was strong, supporting the idea that working memory has a direct relationship with mathematics achievement. The impact of this finding may be that even in mathematics classes that are higher in complexity, students with lower working memory skills will have difficulty with tasks that impose increased cognitive load. Future studies, therefore should include intermediate algebra as well as “college-level” courses to view this relationship and its impact on mathematics instruction.

There was a relationship between mathematics anxiety and working memory, as well as between mathematics anxiety and mathematics achievement, but the relationship was not statistically significant. This relationship, measured by a self-reporting measure the Math Anxiety Rating Scale-Abbreviated, was weak and indirect indicating higher mathematics anxiety was related to lower working memory ($r=-.17$) and lower mathematics achievement ($r=-.26$). Mathematics anxiety did not contribute significantly to either working memory or mathematics achievement for this sample of students.

Implications

The implications of the current study are discussed in the following sections regarding future research and education.

Research Implications

The current study suggests that there is a relationship between working memory and mathematics achievement. By using students currently enrolled in developmental mathematics courses as the participants in the study, the results of the study could be applied to increasing success for students at the community-college level. This research could suggest strategies that promoted increased mathematics skills by reducing demands on working memory such as those that scaffold learning and employ efficient instructional design.

Further research in the area of working memory and mathematics achievement should include the factors that influence working memory: learning disabilities, (Swanson & Jerman, 2006), second language (Gutierrez-Clellen et al., 2004), number sense (Gersten & Chard, 2001), and opportunity to learn (Bettinger & Long, 2006). This research should be conducted in the community-college setting because the population of students differs from traditional college students regarding those factors (Boswell & Wilson, 2004). It is important to find solutions to increase student success rates in mathematics courses at the community college that address individual student needs, especially related to low-working-memory skills.

The assessment of calculation skills only does not provide an effective methodology for investigating relationships between working memory and mathematics achievement. The correlation between working memory and calculation was very weak (r=.09), suggesting that using a mathematics achievement assessment focusing on calculation only would not be an appropriate measure for investigating the relationship between working memory and mathematics achievement.

A further implication is that concentrating on calculation in basic skills mathematics courses does not address adequately the needs of the students who have working memory deficits. When students progress through more difficult courses that require problem solving, the load placed on working memory may affect success. Finding methods that reduce cognitive load in problem solving would be an effective intervention strategy in developmental mathematics courses in community colleges. In cognitive load theory, intervention strategies would require reduction of cognitive load, both extraneous and intrinsic, as well as increasing germane load. Extraneous load can be reduced through the control of factors that compete with the presentation of materials. Intrinsic load may be reduced through scaffolding skills needed to solve problems using instructional strategies. For students at the lower levels of mathematics in community college, research by Ayres (2006) suggests that tasks that are high in element interaction be taught using an isolated elements strategy. Using a variety of examples and practice exercises in different contexts may help to increase germane cognitive load necessary to build mental models (Clark et al., 2006).

Educational Implications

Instructional methods to reduce cognitive load for students with low-working-memory abilities are suggested as a result of this study. Students in developmental courses in community college, in particular students in this study have demonstrated low-working-memory abilities that also suggest low mathematics achievement. Because all students in community college are required to complete courses in intermediate algebra in order to attain a degree or transfer to a 4-year college, it is necessary to address the success of students who have low-working-memory ability. The students in the beginning developmental levels will be required to complete three to four additional mathematics courses in order to attain their degree. Mastering concepts and strategies that can be applied throughout these courses is necessary so that sequential learning can take place. Three strategies that show promise for more efficient use of working memory for community college students are presented in this section.

The first strategy is one developed for elementary students, but given the level of mathematics presented in the first developmental course, may address the beginning level of problem solving needed. This strategy, Schema-Based Instruction (SBI), has been shown to increase elementary students' proficiency with problem solving using specific schemas developed for different types of word problems (Jitendra, 2007). The essential elements of this strategy involve: teacher-led instruction followed by paired learning and independent learning activities and tasks that begin with story situations followed by word problems with unknown information. Throughout the program, visual diagrams and checklists are used. Gradually the diagrams are faded when students learn to apply the strategies learned for different problem types independently. SBI incorporates adequate practice during the acquisition and fluency stages as well as mixed reviews of problem types, such as change, comparison, and group problems (Jitendra, 2007). As in research conducted by Cognitive Load theorists, Jitendra used worked examples to develop the schema needed for problem solving. According to Clark, Nguyen, and Sweller (2006), worked examples are "step-by-step demonstrations used to illustrate how to complete a task" (p. 352). Worked examples are thought to reduce cognitive load in that students study problems that will not impose a cognitive load "associated with problem solving search" (Pawley, Ayres, Cooper, & Sweller, 2005 p. 77). Instead, students consider relevant problem states that can be integrated into a schema stored in long-term memory. By using worked example problem pairs, this technique "decreases extraneous cognitive load by replacing some practice exercises with a series of worked examples, each followed by a similar practice exercise" (Clark et al., 2006, p. 352).

The second strategy, investigated by Pawley et al. (2005), uses explicit instruction combined with worked examples in translating sentences into equations. This method, similar to Jitendra's (2007), is more effective with students who have a low level of knowledge in mathematics. Also, as in Jitendra's strategies, the important idea is schema construction that allows working memory to retrieve the schema as a single element from long-term memory in order to solve a problem. Cognitive load is reduced "because interacting elements of information that previously imposed a heavy working memory load are treated as a single element that can easily be handled by working memory" (Pawley et al., 2005, p.76). The focus of their study, translating words into equations is high in element interactivity and, therefore, puts a high demand on working memory. Learners have to identify relevant information, match specific words to symbols, and construct a relationship between the variables. The researchers in this study introduced a schema for checking work after solving the problem. They found that nonexpert learners benefited most from this strategy after being explicitly instructed in its use. Because the students in the current study would be considered nonexpert learners, this strategy may aid students in increasing the accuracy of their responses.

The third approach, Enhanced Anchored Instruction (EAI) uses problems presented in a multimedia format to help minimize low reading and mathematics skills (Bottge, Rueda, Serlin, Hung, & Kwan, 2007). This strategy was investigated for three different levels of middle-school students (n=128) including students with learning disabilities. Video-based anchors were used to present a problem to the students that encouraged participation in activities the student designs to solve the problem. Concepts connected to the problem area also are taught in a unit developed by the mathematics teachers. The researchers used multiple measures in repeated waves to assess the effects of this intervention and to examine the effects of this strategy on achievement of students by ability level and disability status. The results of their study showed that students with learning disabilities in inclusive settings (included in a classroom with students without disabilities) scored slightly below the students without disabilities, but improvements made from pretest to maintenance were larger. Because students with learning disabilities are included in mathematics classes at the community-college level, this finding may help in designing learning situations that promote active learning of mathematics skills for all students. Bottge et al. (2007) contended that using story contexts, visual representations, and multimedia applications will reduce cognitive load by allowing students to process information high in element interactivity simultaneously.

Summary

The purpose of this research was to identify individual student factors that may contribute to success in developmental mathematics courses in community college. In particular, working memory and mathematics anxiety were examined to view their relationship to mathematics achievement.

The current study shows that there was a relationship between auditory working memory and mathematics achievement. The most statistically significant relationship was between working memory and applied problem skills. It appears that the skills needed to solve problems require high-working-memory abilities. These skills increase cognitive load (intrinsic and extraneous) that negatively impact a person's ability to access working memory.

The current study confirmed previous studies conducted with elementary-school-age students, as well as college-age students (McGlaughlin, Knoop, & Holliday, 2005; Swanson & Jerman, 2006). There is a statistically significant relationship between working memory and mathematics achievement.

The results of the current study contradict studies that show a statistically significant relationship between mathematics anxiety and working memory. The correlation for all students between working memory and mathematics anxiety was -.17, a weak correlation. Research by Ashcraft and Krause (2007) had shown correlations between mathematics anxiety and working memory of r=.40.

This study extends previous research to include students currently enrolled in developmental mathematics courses at the community-college level. Students in these classes usually are classified as nontraditional students, but increasingly more students are choosing community colleges for the first 2 years of their college experience. "According to the National Center for Education Statistics, the definition of a nontraditional student is one who is financially independent, attends part time, works full time, delays enrollment after high school, has dependents, is a single parent, or does not have a high school diploma" (Boswell & Wilson, 2004, p. 9). It is important that research is extended to include students in developmental courses for two reasons. First, community colleges enroll almost half of all undergraduate students attending college. Beyond demographic information, there are few studies conducted in community college to increase success rates for nontraditional students. Second, the demand for skilled workers who can use mathematics effectively in their jobs has increased competency requirements in community college (Golfin et al., 2005). These strict competency requirements may contribute to the lack of degree attainment for a number of students in community college (Brown & Niemi, 2007; Golfin et al., 2005; Research and Planning Group, 2005).

The implications of this research may be viewed best in terms of instructional strategies and instructional design. Strategies that reduce extraneous cognitive load should be examined in order to increase access to working memory abilities needed for problem solving. Intrinsic cognitive load, influenced by element interaction of the content, may be “artificially reduced by instructional designers through chunking and sequencing of content” (Clark et al., 2006, p. 347). Three strategies for accomplishing this reduction (Bottge et al., 2007; Jitendra, 2007; Pawley et al., 2005) should be investigated with students at the community-college level in developmental mathematics courses. Future studies should be conducted within the developmental mathematics classes at community colleges to verify results from this study as well as expand the research to include other individual factors that influence success in basic skills mathematics courses.

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Appendix A

State Exit Exams

STATE HIGH SCHOOL EXIT EXAMS: States Continue Trend Toward Higher-Level Exit Exams, More Subjects Tested

State high school exit exams are state-mandated tests that high school students must pass to receive a high school diploma. In 2006, 22 states required students to pass an exam to receive a high school diploma. Three additional states are phasing in exit exams, so that by 2012, 25 states expect to have these exams in place. (These totals do not include states in which local school districts decide for themselves whether to make the exams a condition of graduation.)

Generally, states have moved toward more rigorous exams by including higher-level content in their tests and testing additional subjects. Between 2002 and 2006, the number of states administering minimum competency exams (MCE), which generally focus on basic skills below the high school level, fell from 10 to 3. During the same period, the number of states administering standards-based exams (SBE), which are generally aligned with high-school-level standards, rose from 7 to 15, while the number administering end-of-course exams (EOC), which assess the content of specific high school courses, increased from 2 to 4. By 2012, only 2 states plan to continue administering minimum competency exams. All states with mandatory exit exams assess students in English language arts and mathematics, but more states are adding tests in science and social studies. Between 2006 and 2012, the number of states assessing science is expected to rise from 11 to 19 and the number assessing social studies from 9 to 13.

Main Features of State Exit Exams

State	Current Exam	Withholding Diplomas Began/ Will Begin	Subjects Tested	Type of Test	Grade Level of Alignment
Alabama	Alabama High School Graduation Exam 3^{rd} Edition	2001*	Reading, language, math, science, social studies	SBE	11^{th}
Alaska	Alaska High School Graduation Qualifying Exam	2004	Reading, writing, math	MCE	8^{th}-10^{th}
Arizona	Arizona's Instrument to Measure Standards	2006	Reading, writing, math	SBE	10^{th}
California	California High School Exit Examination	2006	ELA, math	SBE	ELA (through 10^{th}), math (6^{th}– 7^{th} and Algebra I)
Florida	Florida Comprehensive Assessment Test	2003*	Reading, math	SBE	10^{th}
Georgia	Georgia High School Graduation Tests	1994*	ELA, writing, math, science, social studies	SBE	11^{th}
Idaho	Idaho Standards Achievement Test	2006	Reading, language usage, math, science	SBE	10^{th}
Indiana	Graduation Qualifying Exam	2000	ELA (through 9^{th}), math (through pre-algebra and Algebra I)	SBE	9^{th}
Louisiana	Graduation Exit Examination	2003*	ELA, math, science, social studies	SBE	9^{th}-12^{th}
Maryland	Maryland High School Assessment	2009**	English II, algebra/data analysis, biology, government	EOC	10^{th}
Massachusetts	Massachusetts Comprehensive Assessment System	2003	ELA, math, science (2010)	SBE	10^{th}
Minnesota	Minnesota Comprehensive Assessments Series/ Graduation Required Assessments for Diploma	2010**	Reading, writing, math	MCE	Writing (9^{th}), reading (10^{th}), math (11^{th})
Mississippi	Mississippi Subject Area Testing Program	2006***	English II (with writing component), Algebra I, Biology I, U.S. History from 1877	EOC	Aligned to course content
Nevada	High School Proficiency Examination	2003*	Reading, writing, math, science (2008)	SBE	8^{th}-12^{th}
New Jersey	High School Proficiency Assessment	2003*	Language arts literacy, math, science (2007)	SBE	11^{th}
New Mexico	New Mexico High School Competency Examination	1990	Reading, language arts, composition, science, social studies	MCE	8^{th}
New York	Regents Examinations	2000*	ELA, math, science, social studies, language other than English	EOC	9^{th}-12^{th}
North Carolina	North Carolina Competency Tests and Tests of Computer Skills	1982 (math/reading); 2001 (computer skills)	Reading comprehension, math, computer skills; and starting 2010, end-of-course exams in English I, Algebra I, biology, U.S. history, civics and economics	SBE; EOC (2006-07)	8^{th}
Ohio	Ohio Graduation Tests	2007**	Reading, writing, math, science, social studies	SBE	10^{th}
Oklahoma	Oklahoma End-of-Instruction Exams	2012	English II, English III, Algebra I, Algebra II, geometry, Biology I, U.S. history	EOC	High school standards
South Carolina	High School Assessment Program	2006***	ELA, math, science (2010), U.S. history (2010)	SBE	Through 10^{th}
Tennessee	Gateway Examinations	2005*	English II, Algebra I, Biology I	EOC	10^{th}
Texas	Texas Assessment of Knowledge and Skills	2005*	ELA (reading/ writing), math, science, social studies	SBE	Aligned to course content
Virginia	Standards of Learning End-of-Course Exams	2004*	English (writing/ reading), Algebra I, Algebra II, geometry, biology, earth science, chemistry, world history to 1500, world history from 1500, Virginia and U.S. history, world geography	EOC	Aligned to course content
Washington	Washington Assessment of Student Learning	2008	Reading, writing, math, science (2010)	SBE	10^{th}

Table reads: Alabama currently administers the Alabama High School Graduation Exam, 3^{rd} Edition for which consequences began for the class of 2001. The exam assesses reading, language, math, science, and social studies, and is considered by the state to be a standards-based exam aligned to 11^{th} grade standards.

* These states have had earlier versions of their exams. The year listed reflects the first year diplomas were withheld based on the test currently administered by each state.

** Maryland, Minnesota, and Ohio are in transition to more rigorous exams. Maryland is not currently withholding diplomas, but the class of 2009 will be the first required to pass the HSA. Minnesota currently withholds diplomas based on the Basic Skills Test, and the class of 2010 will be the first required to pass the MCA-II/GRAD. Ohio currently withholds diplomas based on the 9^{th} Grade Proficiency Test, and the class of 2007 will be the first required to pass the OGT.

*** For most graduating seniors in South Carolina and Mississippi, 2006 was the first year that diplomas were withheld based on the current exam; prior to 2006, students had to pass the earlier versions of the exams.

Note: ELA = English language arts

Source: Center on Education Policy, exit exam survey of state departments of education, June 2006.

Appendix B

Student Demographic Questionnaire

Student Questionnaire

Please answer the following questions regarding your background. All answers will be confidential.

1. Number assigned: ______________________ Male ___ Female ___ Age ____

2. Ethnic Background : please choose one answer that best describes your ethnicity

African-American ___
Asian-American ___
Hispanic-American ___
European-American ___
Native-American ___
Other __________________________________

3. Number of Math courses taken in high school:

1 ___	4 ___	Other (number) ___
2 ___	5 ___	Were these classes
3 ___	6 ___	Semester? __Trimester? __ Yearlong? __

4. Number of Math courses taken in college: _____ Quarter? ____ Semester? ____

5. Did you take the College Placement test for mathematics? Yes ___ No ___

If yes, placement recommended: Math 230 ___ Math 101 ____ Math 105 ____

6. Is English your first language? Yes ___ No ___

If English is **not** your first language, how many years have you been speaking English __

7. Have you taken ESL (English as a Second Language) classes at this or any other college?
Yes ___ No ___ How many classes have you taken? ___ Semester? ____ Quarter? ____

[OVER]

8. Have you ever been diagnosed with a learning disability? Yes ___ No ___

If you were diagnosed with a learning disability was it in:

Elementary school ___
High school ___
College ___

9. Have you ever been diagnosed with a disability other than a learning disability?
Yes ___ No ___ What disability? ____

10. Do you receive services (accommodations) from the Disability Resource Center (DRC)?
Yes ____ No ____

If yes, what accommodations have you used?

Extra Time on Tests ____

Testing at the DRC ____

Specialized tutoring ___

Other ____

Thank you for completing this questionnaire!

Appendix C

Student Cover Letter and Consent Form

My name is Janet Spybrook and I am a doctoral student in the College of Education at the University of San Francisco. I am conducting a study on student factors that influence mathematics achievement and success in developmental math courses. I am especially interested in how working memory affects your ability to solve problems in math.

You are being asked to participate in this research study because you are a student in Math 230, or 101 at College. If you agree to be in this study, you will complete a short questionnaire giving basic information about yourself, including age, gender, ethnicity, socioeconomic status, and number of previous math courses taken in high school and/or college. You will also complete a survey about math anxiety. In addition, the researcher will administer a short test designed to assess your working memory skills, as well as a math calculations and applied problems test.

It is possible that some of the questions on the survey may make you feel uncomfortable, but you are free to decline to answer any questions you do not wish to answer, or to stop participation at any time. Although you will not be asked to put your name on the survey, a number that identifies you will be included so that all parts of this study can be examined. Participation in research may mean a loss of confidentiality. Study records will be kept as confidential as is possible. No individual identities will be used in any reports or publications resulting from the study. Study information will be coded and kept in locked files at all times. Only Janet Spybrook will have access to the files. Individual results will not be shared with your instructors.

While there will be no direct benefit to you from participating in this study, the anticipated benefit of this study is a better understanding of the effect of working memory and student demographics on mathematics achievement at the community college level. In turn this study may help to increase success rates in community college mathematics classes. There will be no costs to you as a result of taking part in this study, nor will you be reimbursed for your participation in this study.

If you have questions about the research, you may contact me at (650) 949-7618. If you have further questions about the study, you may contact the IRBPHS at the University of San Francisco, which is concerned with protection of volunteers in research projects. You may reach the IRBPHS office by calling (415) 422-6091 and leaving a voicemail message, by e-mailing IRBPHS@usfca.edu, or by writing to the IRBPHS, Department of Psychology, University of San Francisco, 2130 Fulton Street, San Francisco, CA 94117-1080.

PARTICIPATION IN RESEARCH IS VOLUNTARY. You are free to decline to be in this study, or to withdraw from it at any point. College is aware of this study but does not require that you participate in this research and your decision as to whether or not to participate will have no influence on your present or future status as a student at College.

Thank you for your attention. If you agree to participate, please complete the attached consent form and return it to me by placing it in the envelope at the front of the room.

Sincerely,

Janet Spybrook
Doctoral Student
University of San Francisco

INFORMED CONSENT FORM
UNIVERSITY OF SAN FRANCISCO

CONSENT TO BE A RESEARCH SUBJECT

Purpose and Background

Janet Spybrook, a doctoral student in the School of Education at the University of San Francisco, is conducting a study of student factors that affect mathematics achievement in developmental mathematics courses in community colleges. One of the factors is a student's working memory: that is a person's ability to store and retrieve information needed to solve mathematics problems.

I am being asked to participate in this study because I am a student in Math 230, or 101 , the first developmental math courses at College.

Procedures

If I agree to be a participant in this study, the following will happen:

1. I will complete a short questionnaire giving basic information about me, including age, gender, ethnicity, socioeconomic status, and number of previous mathematics courses taken in high school and/or college.

2. I will complete a survey about mathematics anxiety.

3. I will participate in a test of working memory skills and mathematics achievement.

Risks and/or Discomforts

1. It is possible that some of the questions on the working memory test or math achievement test may make me feel uncomfortable, but I am free to decline to answer any questions I do not wish to answer or to stop participation at any time.

2. Participation in research may mean a loss of confidentiality. Study records will be kept as confidential as is possible. No individual identities will be used in any reports or publications resulting from the study. Study information will be coded and kept in locked files at all times. Only the researcher, Janet Spybrook will have access to the files.

3. Because the time required for my participation may be up to one hour, I may become uncomfortable or bored.

Benefits

There will be no direct benefit to me from participating in this study. The anticipated benefit of this study is a better understanding of the effect of student factors that influence mathematics achievement. This may help to design strategies for success in mathematics courses.

Costs/Financial Considerations

There will be no financial costs to me as a result of taking part in this study.

Payment/Reimbursement

There is no payment for my participation.

Questions

I have talked to Ms. Spybrook about this study and have had my questions answered. If I have further questions about the study, I may call her at

If I have any questions or comments about participation in this study, I should first talk with the researcher. If for some reason I do not wish to do this, I may contact the IRBPHS, which is concerned with protection of volunteers in research projects. I may reach the IRBPHS office by calling and leaving a voicemail message, by e-mailing IRBPHS@usfca.edu, or by writing to the IRBPHS, Department of Psychology, University of San Francisco, 2130 Fulton Street, San Francisco, CA 94117-1080.

Consent

I have been given a copy of the "Research Subject's Bill of Rights" and I have been given a copy of this consent form to keep.

PARTICIPATION IN RESEARCH IS VOLUNTARY. I am free to decline to be in this study, or to withdraw from it at any point. My decision as to whether or not to participate in this study will have no influence on my present or future status as a student or employee at College.

My signature below indicates that I agree to participate in this study.

Subject's Signature Date of Signature